

# On Reaching Consensus by a Group of Collaborating Agents

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**Abstract.** In this paper, an agent is defined as a triple  $(S, R_S, L_S)$ , where  $S$  is a multi-hierarchical decision system,  $R_S$  is a set of rules extracted from  $S$  defining values of its decision attributes, and  $L_S$  is a language which the agent can use to communicate with other agents.  $L_S$  is built from values of decision attributes in  $S$  which are treated as agent's external attributes. Classification attributes in  $S$  are treated as agent's internal attributes which for all agents are the same. If objects stored in two decision systems representing different agents are the same, then their descriptions in terms of internal attributes are the same as well. Agents can learn from each other definitions of their external attributes. If these definitions differ or are contradictory then agents may try to propose a new definition which is more acceptable to both of them. Standard semantics and agent centered semantics are introduced and used to describe the strategy of reaching consensus among agents. Expressions in the language  $L_S$  are called analytical questions. Music information retrieval is taken as an application domain.

## 1 Introduction

One of the main goals in data mining area is to describe knowledge hidden in data sets by means of classifiers. In the world of agents, which construction is grounded on multi-hierarchical decision systems, classifiers are used to describe semantics of decision attributes, called external attributes in this paper. Language of an agent is defined as a set of terms built from values of its external attributes and logical connectives  $\ast, +, \sim$  which informally are interpreted as *and*, *or*, *not*, respectively.

As an example, we take agents for automatic indexing of music by instruments and emotions (called music-indexing agents). In [19], a multi-hierarchical decision system  $S$  with a large number of attributes built for describing music sound objects was presented. The decision attributes in  $S$  are hierarchical and they include Hornbostel-Sachs classification and classification of instruments with respect to a playing method. Also, there are well known models describing emotions which can be invoked in people when they are listening to music. For instance, Thayer model [16] is a two-dimensional model in which the main elements are stress and energy laid out on 2 perpendicular axes. Stress can change from happy to anxious, and energy varies from calm to energetic. This way, 4 main categories are formed: exuberance, anxious, depression and contentment. Instruments and emotions are defined as hierarchical decision (or external) attributes. All

values of these two decision attributes (for instance aerophone, aero\_double-reed, an instrument, energetic-negative, angry) are taken as atomic expressions of the language used by a music-indexing agent. Example of two analytical questions which can be asked by an agent is given below:

“identify a few examples of music pieces which in your opinion are in happy mood and they are played by guitar and piano” or “what moods invokes a given piece of music and what instruments are playing”.

These two analytical questions can be presented either to a single agent or to a group of agents. Because answers in this case are subjective then agents can be asked to reach a consensus before the answer to any of these two analytical questions is given by them.

There is a number of papers published in the area of Multi-Agent Consensus [3], [5], [1] but in most of them authors do not consider the issue of different semantics of languages used by these agents. In this paper, we present a model of an agent, language used for communication between agents and its user-specific semantics, analytical questions representing expressions in that language, and the process of reaching consensus by a group of agents.

## 2 Agents and Their Communication Language

In this section we introduce the notion of an agent which is defined as a triple  $(S, R_S, L_S)$ , where  $S$  is a multi-hierarchical decision system [14],  $R_S$  is a set of rules extracted from  $S$  defining values of its decision attributes, and  $L_S$  is a language which the agent can use to communicate with other agents.  $L_S$  is built from values of classification and decision attributes in  $S$ . Since decision attributes (external) and their interpretations may differ between agents, then only classification attributes (internal) in  $S$  which semantically have the same meaning for all agents can be used as a safe communication bridge between them. Rules describing values of decision attributes in  $S$  in terms of its internal attributes are used to define the agent centered semantics of the language  $L_S$ . Agent  $S$ -centered semantics assigns a weighted set of objects to a query  $t$  of  $L_S$  which match  $t$  according to knowledge  $R_S$  of agent  $(S, R_S, L_S)$ . For each query  $t$ , agent  $S$ -standard semantics identifies all objects in  $S$  which precisely match  $t$ .  $S$ -standard semantics can be used to check the precision and recall of agent  $S$ -centered semantics.

If a multi-hierarchical decision system contains only one decision attribute, then it is called a hierarchical decision system.

By a decision system we mean a triple  $S = (U, A \cup \{g\}, V)$ , where:

- $U$  is a nonempty, finite set of objects,
- $A \cup \{g\}$  is a nonempty, finite set of attributes i.e.  $a : U \rightarrow V_a$  is a function for any  $a \in A \cup \{g\}$ , where  $V_a$  is the domain of  $a$ ,
- elements in  $A$  are called classification attributes and  $g$  is a distinguished attribute called the decision,
- $V = \bigcup\{V_a : a \in A \cup \{g\}\}$ .

As an example of a decision table we take  $S = (\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{a, b, c\} \cup \{g\}, V)$  represented by Table 1. Decision attribute  $g$  is hierarchical with

**Table 1.** Decision System  $S$ 

	$a$	$b$	$c$	$g$
$x_1$	3	1	2	$g[1, 1]$
$x_2$	3	1	3	$g[1, 1]$
$x_3$	1	1	0	$g[1, 2]$
$x_4$	1	1	3	$g[1, 2]$
$x_5$	2	2	2	$g[2, 1]$
$x_6$	2	2	1	$g[2, 1]$
$x_7$	1	2	3	$g[1, 1]$

$V_g = \{g[1], g[2], g[1, 1], g[1, 2], g[2, 1], g[2, 2]\}$ . Value  $g[i, j]$  should be seen as a child of value  $g[i]$ , for  $i, j \in \{1, 2\}$ .

By a multi-hierarchical decision system we mean a triple

$S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ , where  $X$  is a nonempty, finite set of objects,  $A$  is a nonempty finite set of classification attributes,  $\{d[1], d[2], \dots, d[k]\}$  is a set of hierarchical decision attributes and  $V = \bigcup\{V_a : a \in A \cup \{d[1], d[2], \dots, d[k]\}\}$  is a set of their values. We assume that:

- $V_a, V_b$  are disjoint for any  $a, b \in A \cup \{d[1], d[2], \dots, d[k]\}$ , such that  $a \neq b$ ,
- $a : X \rightarrow V_a$  is a partial function for every  $a \in A$  and total function for every  $a \in \{d[1], d[2], \dots, d[k]\}$

By the set of  $S$ -centered analytical questions of type 1 we mean a least set  $L1_S$  such that:

- $0, 1 \in L1_S$ ,
- if  $w \in \bigcup\{V_a : a \in \{d[1], d[2], \dots, d[k]\}\}$ , then  $w, \sim w \in L1_S$ ,
- if  $t_1, t_2 \in L1_S$ , then  $(t_1 + t_2), (t_1 * t_2) \in L1_S$ .

If decision system is represented by Table 1, then  $g[1]* \sim g[1, 2]$  is an example of  $S$ -centered analytical question of type 1. The answer to  $S$ -centered analytical question  $q$  of type 1 is the set of weighted objects in  $S$  satisfying  $q$ .

By the set of  $S$ -centered analytical questions of type 2 we mean a least set  $L2_S$  such that:

- $B(x) \in L2_S$ , if  $x$  is an object (not necessarily in  $X$ ) and  $B \subseteq \{d[1], d[2], \dots, d[k]\}$
- $B(x) \in L2_S$ , if  $x \notin X$  and  $B \subseteq A$

Informally, we are looking for values of attributes in  $B$  supporting object  $x$ . The result is a set of weighted attribute values.

We assume that  $L_S = L1_S \cup L2_S$ .

Analytical question  $t \in L1_S$  is called simple if  $t = t_1 * t_2 * \dots * t_n$  and  $(\forall j \in \{1, 2, \dots, n\})[(t_j \in \bigcup\{V_a : a \in \{d[1], d[2], \dots, d[k]\}\}) \vee (t_j = \sim w \wedge w \in \bigcup\{V_a : a \in \{d[1], d[2], \dots, d[k]\}\})]$ .

By the set of  $S$ -standard analytical questions of type 1, where  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ , we mean a least set  $T1_S$  such that:

- $0, 1 \in T1_S$ ,
- if  $w \in \bigcup\{V_a : a \in A\}$ , then  $w, \sim w \in T1_S$ ,
- if  $t_1, t_2 \in T1_S$ , then  $(t_1 + t_2), (t_1 * t_2) \in T1_S$ .

By the set of  $S$ -standard analytical questions of type 2, where  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ , we mean a least set  $T2_S$  such that:

- $B(x) \in T2_S$ , if  $x \notin X$  and  $B \subseteq A$

We assume that  $T_S = T1_S \cup T2_S$ .

Question  $t \in T1_S$  is called simple if  $t = t_1 * t_2 * \dots * t_n$  and  $(\forall j \in \{1, 2, \dots, n\})[(t_j \in \bigcup\{V_a : a \in A\}) \vee (t_j = \sim w \wedge w \in \bigcup\{V_a : a \in A\})]$ .

By a classification rule in  $S$  we mean any expression of the form  $[t_1 \longrightarrow t_2]$ , where  $t_1 \in T1_S, t_2 \in L1_S$ , and both  $t_1, t_2$  are simple.

Agent  $S$ -centered semantics  $M_S$  of analytical queries in  $T_S$ , where  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ , is defined as follows:

- $M_S(0) = 0, M_S(1) = X$ ,
- $M_S(w) = \{x \in X : w = a(x)\}$  for any  $w \in V_a, a \in A$ ,
- $M_S(\sim w) = \{x \in X : (\exists v \in V_a)[(v = a(x)) \wedge (v \neq w)]\}$  for any  $w \in V_a, a \in A$ ,
- if  $t_1, t_2 \in T1_S$ , then  $M_S(t_1 + t_2) = M_S(t_1) \cup M_S(t_2), M_S(t_1 * t_2) = M_S(t_1) \cap M_S(t_2)$ ,
- if  $B \subseteq A, x \in X$ , then  $M_S(B(x)) = \{[a, a(x)] : a \in B\}$

Now, we introduce the notation for values of decision attributes at different granularity levels. Assume that  $d[i]$  is a hierarchical decision attribute which is also interpreted as its first or highest granularity level.

The set  $\{d[i, 1], d[i, 2], d[i, 3], \dots\}$  represents the values of attribute  $d[i]$  at its second granularity level. The set  $\{d[i, 1, 1], d[i, 1, 2], \dots, d[i, 1, n_i]\}$  represents the values of attribute  $d$  at its third granularity level, right below the node  $d[i, 1]$ . We assume here that the value  $d[i, 1]$  can be refined to any value from  $\{d[i, 1, 1], d[i, 1, 2], \dots, d[i, 1, n_i]\}$ , if necessary. Similarly, the set  $\{d[i, 3, 1, 3, 1], d[i, 3, 1, 3, 2], d[i, 3, 1, 3, 3], d[i, 3, 1, 3, 4]\}$  represents the values of attribute  $d$  at its fourth granularity level which are finer than the value  $d[i, 3, 1, 3]$ .

Let us assume that by  $R_{[S, v]}^x$  we denote the set of all rules  $[t \longrightarrow v]$  in  $R_S$  supporting object  $x$ . Agent  $S$ -centered semantics  $M_S$  of analytical queries in  $L_S$  is defined as follows:

- if  $t$  is a simple expression in  $L1_S$  and  $\{r_j = [t_j \longrightarrow t] : j \in J_t\}$  is the set of all rules defining  $t$  which for instance are extracted from  $S$  by RSES [15] or WEKA [17], then  $M_S(t) = \{(x, p_x) : (\exists j \in J_t)(x \in M_S(t_j)[p_x = \sum\{conf(j) \cdot sup(j) : x \in M_S(t_j) \wedge j \in J_t\} / \sum\{sup(j) : x \in M_S(t_j) \wedge j \in J_t\}]\}$ , where  $conf(j), sup(j)$  denote the confidence and the support of  $[t_j \longrightarrow t]$ , correspondingly.
- $M_S(B(x)) = \bigcup\{\{(v, p_v) : [v \in V_b] \wedge p_v = \frac{\sum\{conf(j) \cdot sup(j) : [t_j \longrightarrow v] \in R_{[S, v]}^x\}}{\sum\{conf(j) \cdot sup(j) : [t_j \longrightarrow v] \in \bigcup\{R_{[S, v]}^x : v \in V_b\}\}}\} : b \in B\}$ , where  $conf(j), sup(j)$  denote the confidence and the support of  $[t_j \longrightarrow v]$ , correspondingly.

Attribute value  $d[j_1, j_2, \dots, j_n]$  in  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$  is dependent on  $d[i_1, i_2, \dots, i_k]$  in  $S$ , if one of the following conditions hold: (1)  $n \leq k \wedge (\forall m \leq n)[i_m = j_m]$ , (2)  $n > k \wedge (\forall m \leq k)[i_m = j_m]$ . Otherwise,  $d[j_1, j_2, \dots, j_n]$  is called independent from  $d[i_1, i_2, \dots, i_k]$  in  $S$ .

For example, let us notice that the attribute value  $d[2, 3, 1, 2]$  is dependent on the attribute value  $d[2, 3, 1, 2, 5, 3]$ . Also,  $d[2, 3, 1, 2, 5, 3]$  is dependent on  $d[2, 3, 1, 2]$ .

Let  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ ,  $w \in V_{d[i]}$ , and  $IV_{d[i]}$  be the set of all attribute values in  $V_{d[i]}$  which are independent from  $w$ . Standard semantics  $N_S$  of analytical queries in  $L1_S$  is defined as follows:

- $N_S(0) = 0, N_S(1) = X,$
- if  $w \in V_{d[i]}$ , then  $N_S(w) = \{x \in X : d[i](x) = w\}$ , for any  $1 \leq i \leq k$
- if  $w \in V_{d[i]}$ , then  $N_S(\sim w) = \{x \in X : (\exists v \in IV_{d[i]})[d[i](x) = v]\}$ , for any  $1 \leq i \leq k$
- if  $t_1, t_2$  are terms, then  $N_S(t_1 + t_2) = N_S(t_1) \cup N_S(t_2), N_S(t_1 * t_2) = N_S(t_1) \cap N_S(t_2)$ .

Standard semantics  $N_S$  of analytical queries in  $T1_S$  is defined in a similar way.

Let  $S = (X, A \cup \{d[1], d[2], \dots, d[k]\}, V)$ ,  $t$  is an analytical query in  $L1_S$ ,  $N_S(t)$  is its meaning under standard semantics, and  $M_S(t)$  is its meaning under agent  $S$ -centered semantics. Assume that  $N_S(t) = X_1 \cup Y_1$ , where  $X_1 = \{x_i : i \in I_1\}$ ,  $Y_1 = \{y_i : i \in I_2\}$ . Assume also that  $M_S(t) = \{(x_i, p_i) : i \in I_1\} \cup \{(z_i, q_i) : i \in I_3\}$  and  $\{y_i : i \in I_2\} \cap \{z_i : i \in I_3\} = \emptyset$ .

By precision of an agent  $S$ -centered semantics  $M_S$  on analytical query  $t \in L1_S$ , we mean  $Prec(M_S, t) = [\sum \{p_i : i \in I_1\} + \sum \{(1 - q_i) : i \in I_3\}] / [card(I_1) + card(I_3)]$ .

By recall of an agent  $S$ -centered semantics  $M_S$  on analytical query  $t \in L_S$ , we mean  $Rec(M_S, t) = [\sum \{p_i : i \in I_1\}] / [card(I_1) + card(I_2)]$ .

Clearly, the precision and recall of an agent  $S$ -centered semantics can be improved by using classifiers of higher confidence.

To give example of an agent, assume that  $S = (\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}, \{a, b, c\} \cup \{g\}, V)$  is represented by Table 1 and  $R_S$  is a set of rules extracted from  $S$ .

Also, let us assume that  $g[1, 1]$  is analytical question. It can be easily checked that:

$$M_S(g[1, 1]) = \{(x_1, \frac{2 \cdot 1 + 2 \cdot (1/2) + 1 \cdot (1/2)}{2+2+1}), (x_2, \frac{2 \cdot 1 + 2 \cdot (1/2) + 2 \cdot (2/3) + 1 \cdot (1/2) + 1 \cdot 1}{2+2+2+1+1}), (x_7, \frac{1 \cdot (1/3) + 1 \cdot (1/3) + 2 \cdot (2/3) + 1 \cdot 1 + 1 \cdot (1/2) + 1 \cdot 1}{1+1+2+1+1+1})\} = \{(x_1, \frac{7}{10}), (x_2, \frac{35}{48}), (x_7, \frac{27}{42})\}$$

since

- $[a3 \rightarrow g[1, 1]]$  with sup=2 and conf=1,  $[b1 \rightarrow g[1, 1]]$  with sup=2 and conf=1/2,  $[c2 \rightarrow g[1, 1]]$  with sup=1 and conf=1/2 are rules in  $R_S$  supporting  $x_1$
- $[a3 \rightarrow g[1, 1]]$  with sup=2 and conf=1,  $[b1 \rightarrow g[1, 1]]$  with sup=2 and conf=1/2,  $[c3 \rightarrow g[1, 1]]$  with sup=2 and conf=2/3,  $[b1 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1/2,  $[a3 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1 are rules in  $R_S$  supporting  $x_2$
- $[a1 \rightarrow g[1, 1]]$  with sup=1 and conf=1/3,  $[b2 \rightarrow g[1, 1]]$  with sup=1 and conf=1/3,  $[c3 \rightarrow g[1, 1]]$  with sup=2 and conf=2/3,  $[a1 * b2 \rightarrow g[1, 1]]$  with sup=1 and conf=1,

$[a1 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1/2,  $[b2 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1 are rules in  $R_S$  supporting  $x_7$ .

To give another example of analytical question, assume that  $x$  is a new object satisfying the properties  $a(x) = 3, b(x) = 2, c(x) = 3$ . Now, it can be easily checked that the answer to analytical question  $g(x)$  will be:

$$M_S(g(x)) = \{[g(1, 1), \frac{2 \cdot 1 + 1 \cdot (1/3) + 2 \cdot (2/3) + 1 \cdot 1 + 1 \cdot 1}{22/3}], [g(2, 1), \frac{2 \cdot (2/3)}{22/3}], [g(1, 2), \frac{1 \cdot (1/3)}{22/3}]\} = \{[g(1, 1), \frac{17}{22}], [g(2, 1), \frac{4}{22}], [g(1, 2), \frac{1}{22}]\}$$

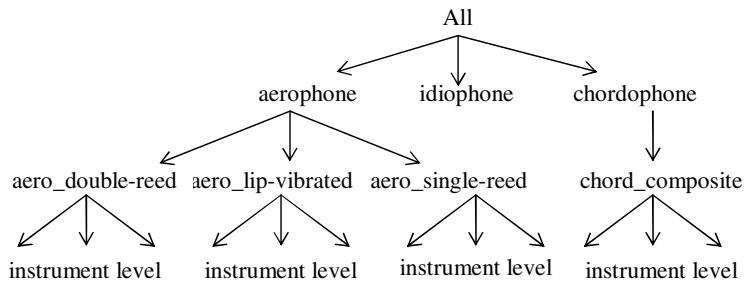
since  $[a3 \rightarrow g[1, 1]]$  with sup=2 and conf=1,  $[b2 \rightarrow g[1, 1]]$  with sup=1 and conf=1/3,  $[b2 \rightarrow g[2, 1]]$  with sup=2 and conf=2/3,  $[c3 \rightarrow g[1, 1]]$  with sup=2 and conf=2/3,  $[c3 \rightarrow g[1, 2]]$  with sup=1 and conf=1/3,  $[a3 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1,  $[b2 * c3 \rightarrow g[1, 1]]$  with sup=1 and conf=1, are the rules in  $R_S$  supporting  $x$ .

Assume now that another agent whose knowledge is based on a decision system  $S_1$  gave the answer  $\{[g(1, 1), \frac{12}{22}], [g(2, 1), \frac{6}{22}], [g(2, 2), \frac{4}{22}]\}$  to the same analytical question  $g(x)$ . The answer representing the consensus of this two agents on question  $g(x)$  will be  $\{[g(1, 1), \frac{17+12}{44}], [g(2, 1), \frac{4+6}{44}], [g(1, 2), \frac{1}{44}], [g(2, 2), \frac{4}{44}]\} = \{[g(1, 1), \frac{29}{44}], [g(2, 1), \frac{10}{44}], [g(1, 2), \frac{1}{44}], [g(2, 2), \frac{4}{44}]\}$  or  $\{[g(1), \frac{30}{44}], [g(2), \frac{14}{44}]\}$ .

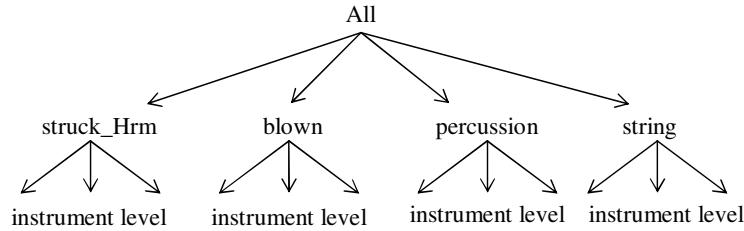
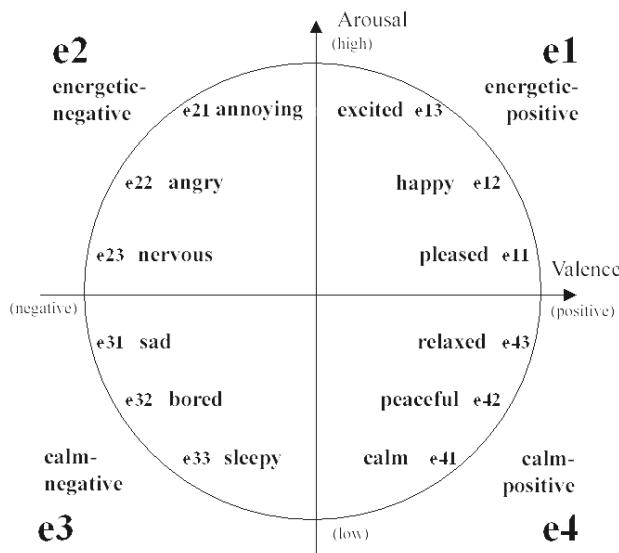
Assume also that his answer to the second analytical question  $g[1, 1]$  is  $\{(x_1, \frac{1}{2}), (x_7, \frac{7}{21}), (x_{10}, \frac{2}{5})\}$ . The answer representing the consensus of this two agents on objects  $x_1, x_2, x_7, x_{10}$  will be  $\{(x_1, \frac{12}{20}), (x_2, \frac{35}{96}), (x_7, \frac{41}{84}), (x_{10}, \frac{2}{10})\}$ .

### 3 Application Domain

Music instrument identification [8],[10],[18] is one of the important subtasks of a content-based automatic indexing, for which authors built a multi-hierarchical decision system  $S$  with the low-level MPEG7 descriptors as well as other popular descriptors for describing music sound objects. The decision attributes in  $S$  are hierarchical and they



**Fig. 1.** Hombostel-Sachs classification of instruments

**Fig. 2.** Classification of instruments with respect to playing method**Fig. 3.** Thayer's arousal-valence emotion plane

include Hornbostel-Sachs classification (see Figure 1) and classification of instruments with respect to playing method (see Figure 2).

Automatic indexing of music by emotions is even more difficult task because of its subjectivity. There is a number of models describing emotions contained in music. One of them is the model proposed by Hevner [6] which is made up of a list of adjectives grouped in 8 main categories. After modification it was used by Li et al. [9].

Another model is the two-dimensional Thayer model [16] in which the main elements are Stress and Energy laid out on 2 perpendicular axes. Stress can change from happy to anxious, and Energy varies from calm to energetic. This way, 4 main categories representing emotions can be used: Exuberance, Anxious, Depression and Contentment.

The model we have chosen for our music automatic indexing system is based on Thayer's model (Figure 3) and it was investigated in [4]. This hierarchical model of emotions consists of two levels, L1 and L2.

**Table 2.** Description of mood groups in L1, the first level

Abbreviation	Description
e1	energetic-positive
e2	energetic-negative
e3	calm-negative
e4	calm-positive

**Table 3.** Description of mood groups in L2, the second level

Abbreviation	Description
e11	pleased
e12	happy
e13	excited
e21	annoying
e22	angry
e23	nervous
e31	sad
e32	bored
e33	sleepy
e41	calm
e42	peaceful
e43	relaxed

The first level L1 contains 4 emotions. To ease the indexing of files, group names were replaced with compound adjectives referencing Arousal and Valence. Our mood model as well as the model used in [4] contains the following groups (Table 2).

In the first group (e1), pieces of music can be found which convey positive emotions and have a quite rapid tempo, are happy and arousing (Pleased, Happy, Excited). In the second group (e2), the tempo of the pieces is fast, but the emotions are more negative, expressing Annoying, Angry, Nervous. In the third group (e3), are pieces that have a negative energy and are slow, expressing Sad, Bored, Sleepy. In the last group (e4), are pieces that are calm and positive and express Calm, Peaceful, Relaxed.

The second level is related to the first, and is made up of 12 sub-emotions, 3 emotions for each emotion contained in the first level (Table 3).

Our multi-hierarchical decision system  $S$  has about 4,000 singular monophonic sound objects described by more than 1,000 features. Each sound object is represented as a temporal sequence of approximately 150-300 tuples which gives us a temporal database of about 1,000,000 tuples. This database is used to construct classifiers for music instruments recognition and for music automatic indexing [12]. A separate music database was built to construct classifiers for identification of emotions invoked by a music piece. If the resulting classifiers are of a rule-based type, then the system is searching for rules supporting each music segment. Supporting rules help to identify most dominating instruments and emotions to be assigned to each segment. If a music segment is denoted by  $x$ , then music indexing agents

will try to reach consensus before they answer the analytical question  $g(x)$ , where “ $g$ ” means “music instrument”. So, if  $(S, R1_S, L_S)$ ,  $(S, R2_S, L_S)$  are the collaborating music agents (their semantics of analytical queries may differ) and for instance  $\{[violin, 75\%], [guitar, 60\%], [happy, 70\%]\}$  is the answer to  $g(x)$  given by the first agent and  $\{[violin, 65\%], [piano, 50\%], [pleased, 60\%]\}$  is the answer given by second agent, then  $\{[violin, 70\%], [guitar, 30\%], [piano, 25\%], [energetic-positive, 60\%]\}$  represents these two agent’s consensus. Please observe that we have taken “energetic-positive” as the smallest generalization of “pleased” and “happy”. These two values are subjective so clearly we can not force any of these agents to change their emotions concerning a musical piece.

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