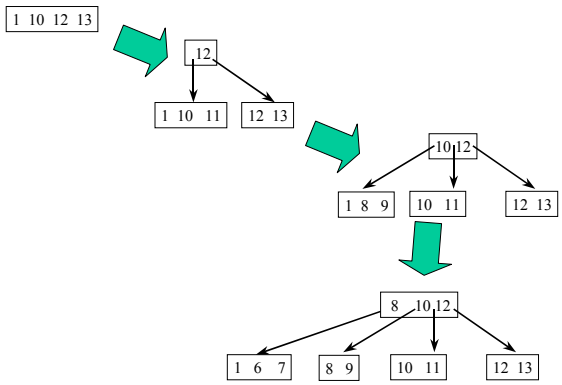
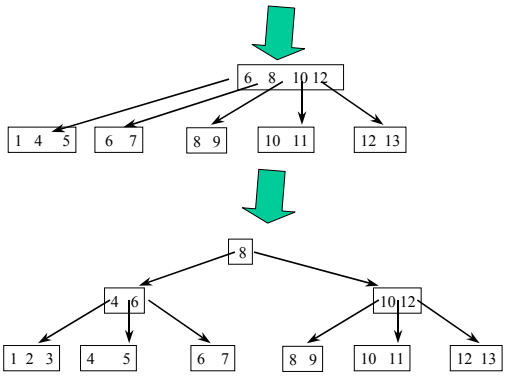
3. **8 points**[1] Consider the set of keys $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$. Write down a sequence of inserting the keys of S such that at the end the resulting B+ tree has 3 levels and is as empty as possible, i.e., as many nodes as possible have the minimum number of nodes. Provide the B+ tree snapshots that correspond to the points right after node splits. (Use the white space at the end of the page and the back side of this page.)

Solution: The sequence $(1), 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2$ produces the sequence of splits and the resulting B+ tree shown below, assuming the splits are "3 nodes to the left, 2 nodes to the right".





This is just one of the possible solutions. However, the resulting tree is the only one that is possible if the splits are "3 nodes to the left, 2 nodes to the right". There is eactly one possible resulting tree in the case of "2 nodes to the left, 3 nodes to the right".

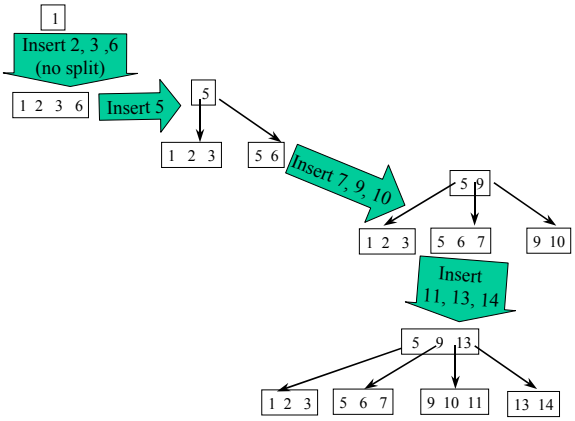
[1] May be time consuming.

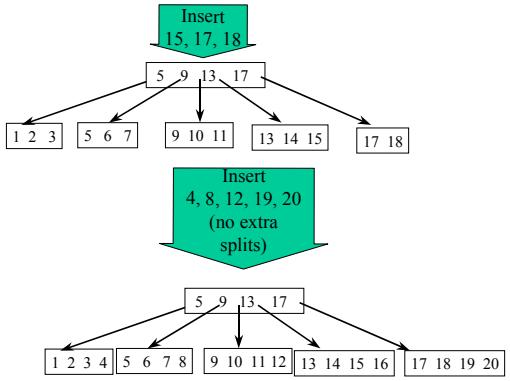
4. **8 points**[2] Consider the set of keys $S' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$. Write down a sequence of inserting the keys of S' such that at the end the resulting B+ tree is as full as possible, i.e., as many nodes as possible have the maximum number of nodes. Provide the B+ tree snapshots that correspond to the points right after a node split. (Use the next blank page.)

Remarks

- Be consistent in the way you split nodes.
- Assume that there are no duplicate nodes in the internal leaves.

Solution: There is exactly one resulting tree that has just two levels and, hence, is as full as possible. It is produced as follows





[2]May be time consuming