1 Alphametics.

Thanks to Sian Zelbo and Tony Gardiner for several of these cute problems.[1]

1. Find digits `A` and `B` such that `A + A + A = B A`.
2. Find digits `A, B` and `C` such that `A + B B = A C C`.
3. Find digits A, B, C and D such that
$$AB + A = CDC.$$
4. Find digits A, B and C such that
$$AB + BC = BCB.$$
5. Find digits A, B and C such that
$$AB + CB = BBA.$$
6. Find digits A, B and C such that
$$4 \cdot AB = CA.$$
7. Find digits A, B and C such that
$$AA + BB = CBC.$$

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8. Find digits A, B and C such that

$$AB + AB = CBB.$$

9. Find digits A, B and C such that

$$AAA + BB + A = CAB.$$

10. Find digits A, B and C such that

$$ABC + ACB = CBA.$$

11. Find digits A, B, C and D such that

$$ABC + ABC = CDDB.$$

12. Find digits A, B, C, D and E such that

$$ABC + CBC = CDEB.$$

13. Use eight different digits to build four two-digit number AB, CD, EF, GH whose sum is a three-digit number JJJ. What is J and how many solutions are there? Suppose that in addition, J is not in the set $\{A, B, \ldots, H\}$. Is J uniquely determined?

14. Use all ten digits to build five two-digit numbers, AB, CD, EF, GH and IJ. How many sums $AB + CD + EF + GH + IJ$ are possible?

15. Find values for each of the digits `A, B` and `C`

```
  B A
  A B
+ A B
-----
C A A
```

16. Find values for each of the digits `A, B` and `C`

```
  A B C
  A B C
+ A B C
-------
  C C C
```

17. Find values for each of the digits `A, B, C` and `D`

```
  A B C
  A B C
  A B C
+ A B C
-------
  C D A
```

18. Find values for each of the digits `F, I, V, E, O, U, R` and `N`

```
  F O U R
+ F I V E
---------
  N I N E
```

19. Find values for each of the digits `A, B` and `C`

```
  A B C
  A B C
+ A B C
-------
  B B B
```

20. Find values for each of the digits.

```
  S E N D
+ M O R E
---------
M O N E Y
```

21. Find values for each of the digits `a,b,c` and `d`. Have you seen this before?

```
  a b c d
  a b c d
  a b c d
+ a b c d
  d c b a
```

22. Find values for each of the digits `A, B` and `C`

```
  A B
  B C
+ C A
A B C
```

23. Find values for each of the digits `A, B` and `C`

```
    A
  B B
+ C C C
  B A B
```

24. Find values for each of the digits `A, B` and `C`

```
    A
  B B
+ C C C
  B C B
```

25. Find values for each of the digits `A, B, C` and `D`

```
  B A A
  B A A
  B A A
+ B A A
C A A D
```

26. Find values for each of the digits `A, B, C` and `D`

```
  A B A
```

```
  A B A
  A B A
  A B A
+ A B A
-------
C D B A
```

27. There are lots of solutions to this one. Can you find them all?

```
  F O U R
+   O N E
---------
  F I V E
```

28. The addition below is incorrect. The display can be made correct by changing one digit d, wherever it occurs, to another digit e. Find the sum of d and e.

```
  7 4 2 5 8 6
+ 8 2 9 4 3 0
-------------
1 2 1 2 0 1 6
```

29. The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

```
  6 4 1
  8 5 2
+ 9 7 3
-------
2 4 5 6
```

30. Letters A, B, C, and D represent four different digits selected from $0, 1, 2, \ldots, 9$. If $(A + B)/(C + D)$ is an integer that is as large as possible, what is the value of $A + B$?

31. Find a five-digit number $\underline{abcde}$ such that $4 \cdot \underline{abcde} = \underline{edcba}$.

32. In equation $AB + CD = EFG$, each letter represents a different digit. What is the greatest possible value of the three-digit number EFG?

33. Find values for A and B so that $AB + BA + B = AAB$.

34. If a,b, and c are digits for which

$$\begin{array}{r} 7\ a\ 2 \\ -\ 4\ 8\ b \\ \hline c\ 7\ 3 \end{array}$$

what is a + b + c?

2 More Challenging Problems

35. The next two problems are due to my dear friend George Berzsenyi in 1993. Find different values for all the letters so that

$$\begin{array}{r} H\ A\ R\ R\ I\ E\ T \\ M\ A\ R\ R\ I\ E\ D \\ +\qquad H\ E\ R \\ \hline D\ E\ N\ T\ I\ S\ T \end{array}$$

36. Find different values for all the letters so that

$$\begin{array}{r} D\ I\ A\ N\ A \\ \quad A\ N\ D \\ S\ A\ R\ A\ H \\ +\quad A\ R\ E \\ \hline R\ E\ B\ E\ L\ S \end{array}$$

37. Find different values for all the letters so that

$$\begin{array}{r} \quad H\ A\ L\ F \\ F\ I\ F\ T\ H \\ T\ E\ N\ T\ H \\ T\ E\ N\ T\ H \\ +\ T\ E\ N\ T\ H \\ \hline W\ H\ O\ L\ E \end{array}$$

38. Find digit values for the letters T, W, O, E, I, G, and H so that 2 (T W O)^2 = E I G H T.

39. Try SLOW + SLOW + OLD = OWLS.

40. Find four digits `a,b,c,d` which satisfy 3 $\underline{\texttt{abcd}}=\underline{\texttt{cbad}}$.

41. Try `BLASE + LBSA = BASES`.

42. Try `LOVES + LIVE = THERE`. What is the largest possible value of `THERE`?

43. Find the letter E so that $\underline{YE} \cdot \underline{ME} = \underline{TTT}$

44. Here's a slightly different type

$$\texttt{E V E} \div \texttt{D I D} = .\overline{\texttt{T A L K}}$$

45. Find values for each of the digits `F, T, H, R, E, O, U, R, V, S` and `N`. The arithmetic is in base 11.

$$\begin{array}{r} \texttt{T H R E E} \\ + \quad \texttt{F O U R} \\ \hline \texttt{S E V E N} \end{array}$$

46. If `O, N, E, T,` and `W` are digits for which

$$\begin{array}{r} \texttt{O N E} \\ + \texttt{O N E} \\ \hline \texttt{T W O} \end{array}$$

what are the possible values of `T`? How many solutions are there?

The following six problems are taken from Tony Gardiner's Mathematical Puzzling.

47. Find all solutions to $TWO + TWO = FOUR$.

48. Explain why $FIVE + TWO = SEVEN$ has no solutions.

49. Try $THIS + IS = HARD$.

50. $FOUR + FIVE = NINE$.

51. How about $\frac{HAPPY}{XMAS} = A$.

52. Try $\frac{MERRY}{XMAS} = A$.

53. Can you solve $SEVEN + THREE + TWO = TWELVE$?

54. Can you solve $TEN + TEN + FORTY = SIXTY$?

55. Here's a cute one: $\sqrt{PASSION} = KISS$

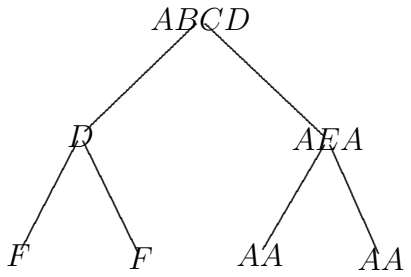
56. Find numbers A and B such that $6AB = BA$ in base 1000.

57. Find two-digit numbers a, b, c such that $4 \cdot abc = cab$ in base 100.

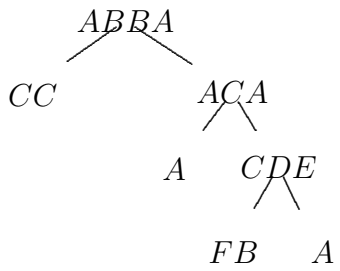
3 Factor Trees.

Thanks to many students for these problems. I'm looking now for base 6 factor trees, the kind they might like on Mars.

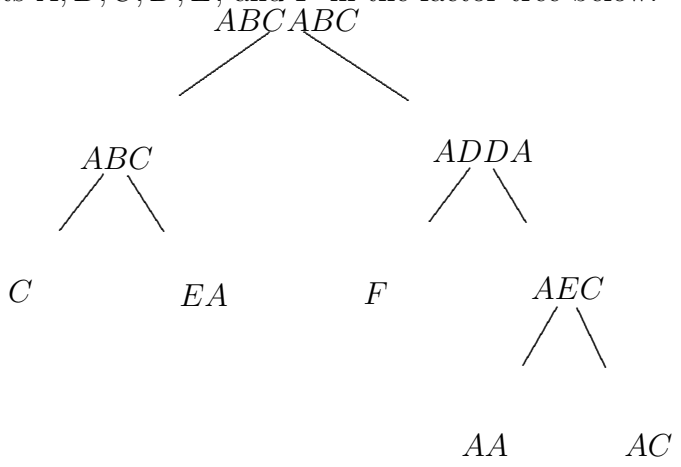
1. Each letter stands for a different digit. The leaves are prime numbers.



2. Find digit values for all the letter in the factor tree. As before, all the leaf nodes are prime numbers.


ABBA
CC
ACA
A
CDE
FB
A

3. Find each of digits A, B, C, D, E, and F in the factor tree below.



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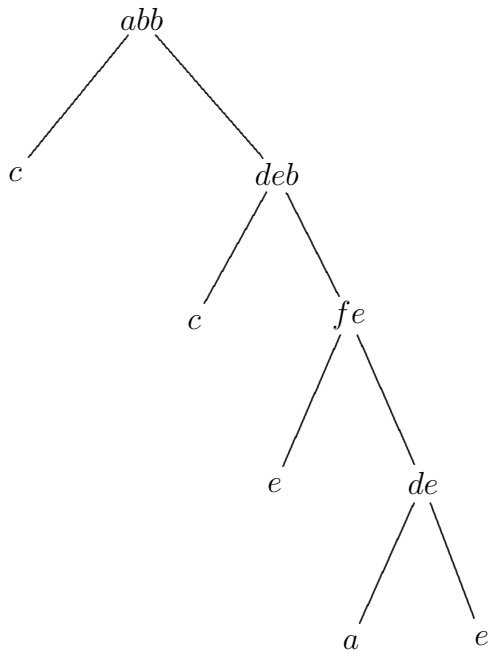
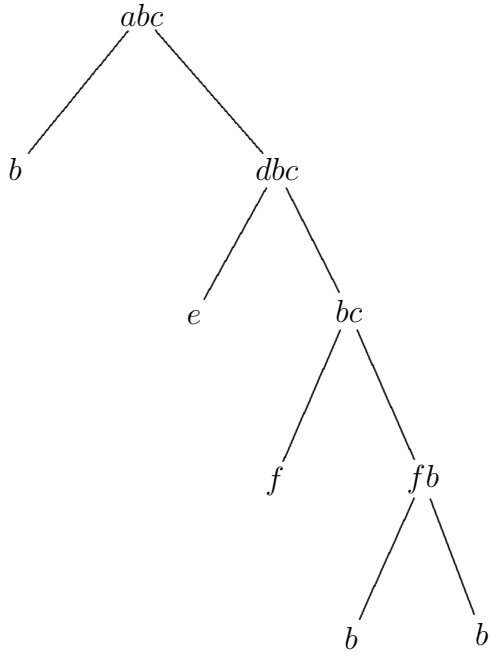
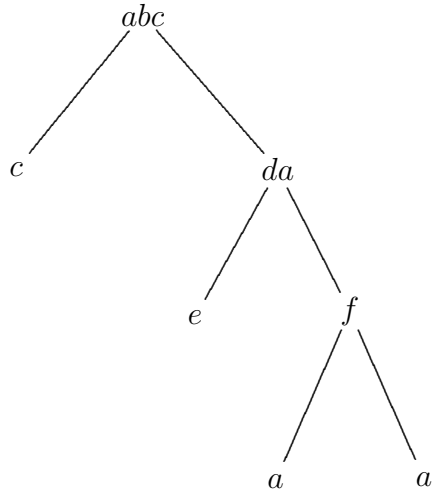
4. This item is due to Ryan Chiu.



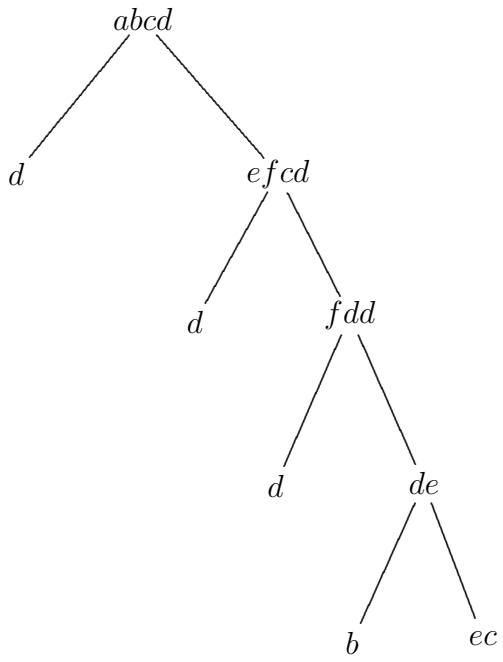
5. This item is due to Christina Zhao, a sixth grader at JM Robinson Middle.



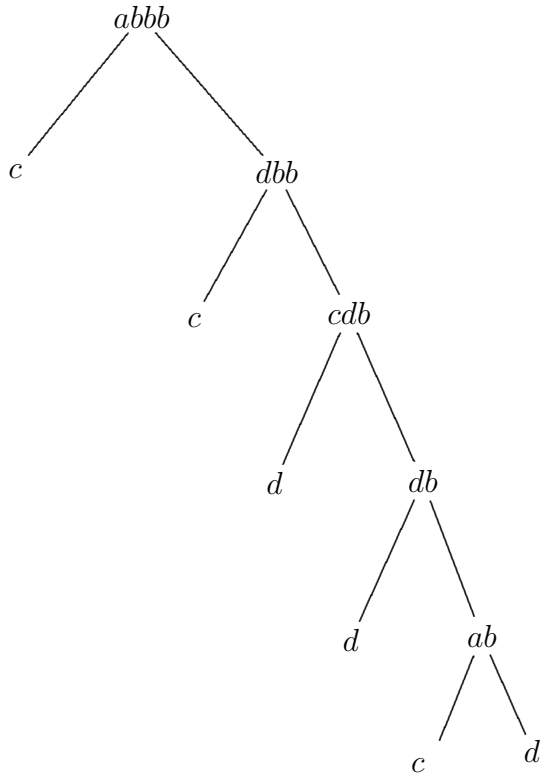
6. This item is due to Anthony Yang, a fifth grader at Barringer Academic Center.



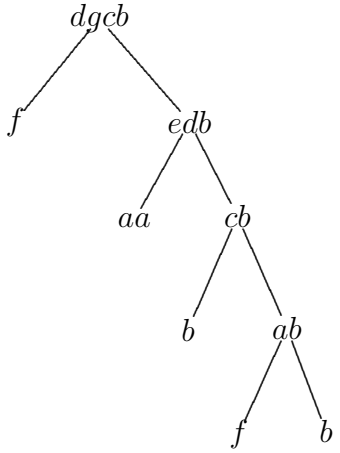
7. This item is due to David Tian, a fifth grader at .



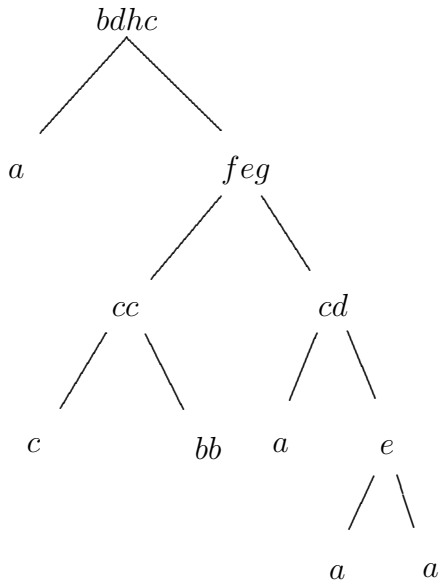
8. This item is due to Soumyadeep Bhattacharjee, a homeschooled fourth grader in Charlotte, NC.



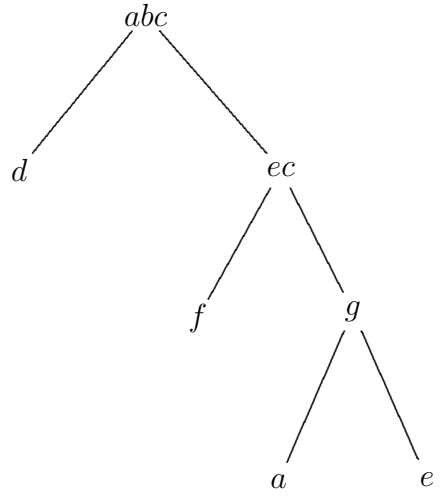
9. This item and the next one are due to 8 year old Nelson Huang of Charlotte, NC.



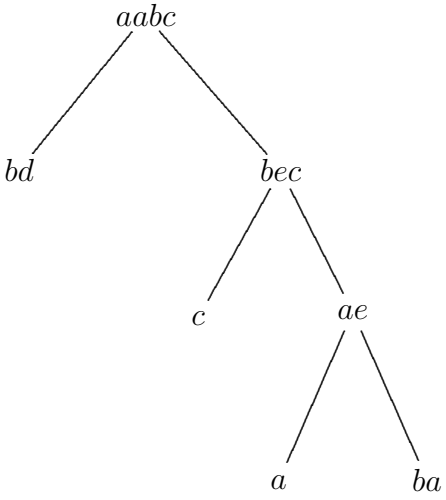
10. Find the number $bdhc$. As usual two different letters represent two different digits.



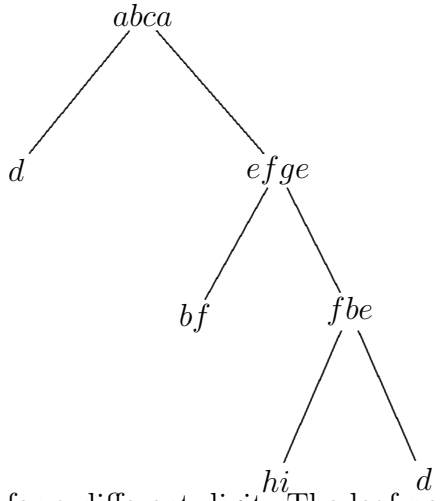
11. This item and the next three are due to Sarah Carter `@mathequalslove`, a middle school teacher in Tulsa, OK.



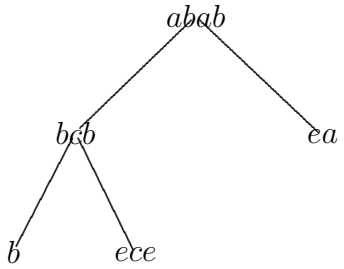
12.



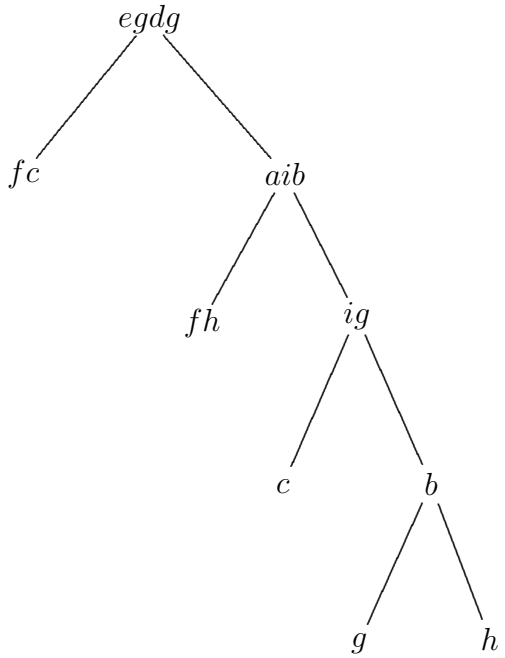
13.



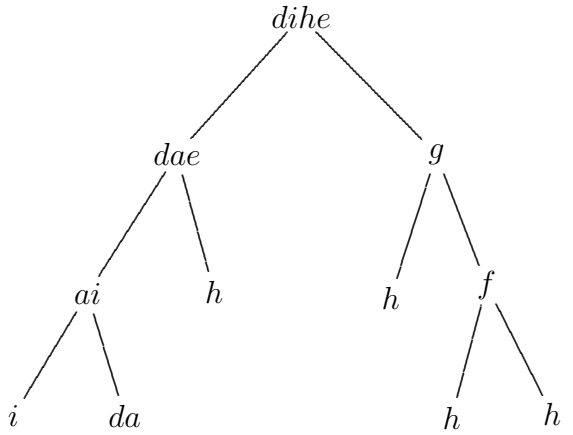
14. As usual, each letter stands for a different digit. The leaf nodes are all prime numbers.



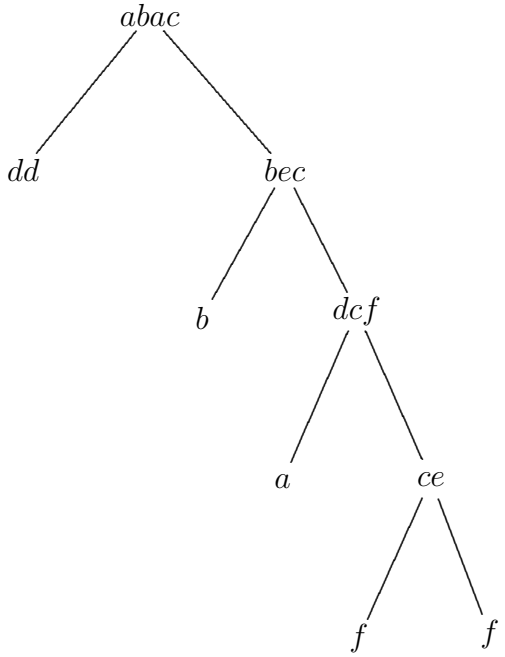
15. This item is due to Kayla Dean, a 6/7 teacher at Union Center, Tulsa, OK.



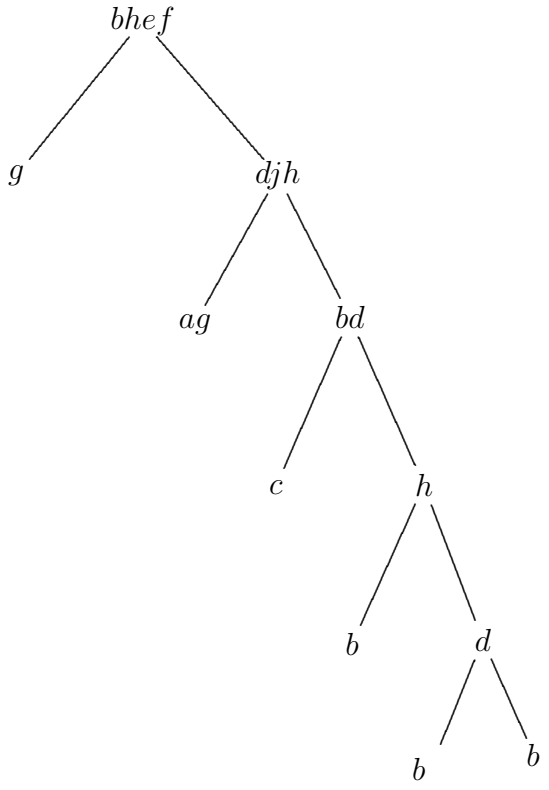
16. This item is due to 6th grader Audrey Loomis of Hope Middle of East Lansing, MI.



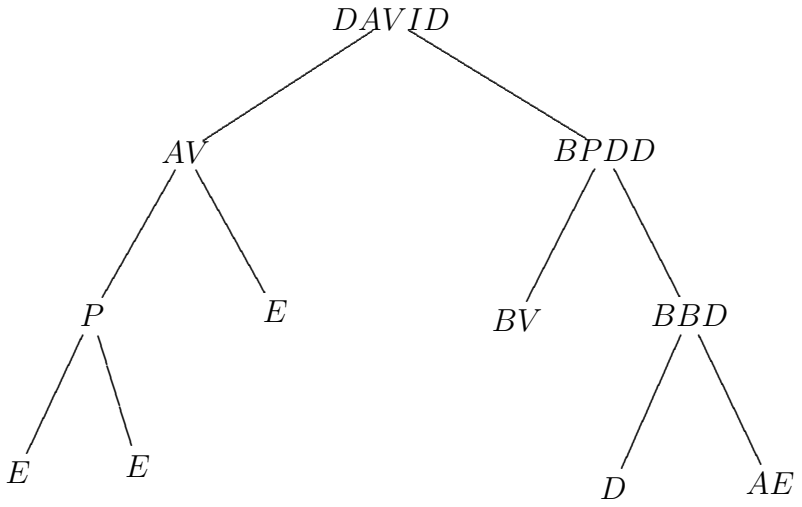
17. This item is due to Aayush Bharti, a seventh grader at Randolph IB Middle, Charlotte, NC.



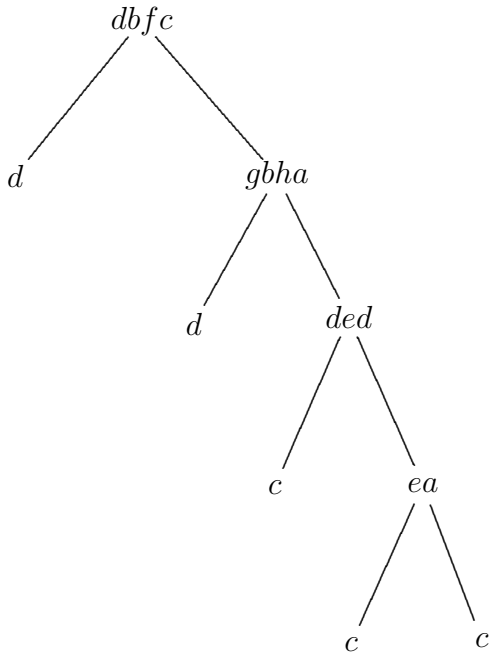
18. This item is due to Jiatong 'Crystal' Zhai, McDonald Middle School in East Lansing, MI.



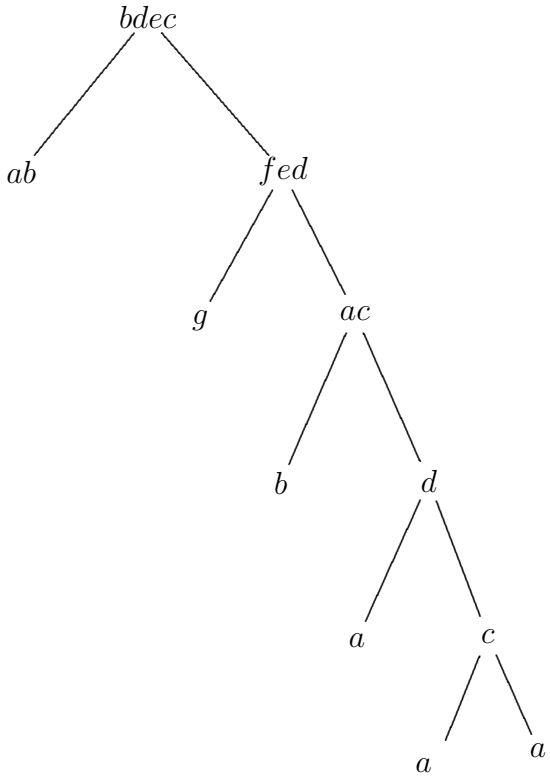
19. This item is due to David Ried, a fourth grader at Okemos Public Montessori at Central in Okemos, MI.



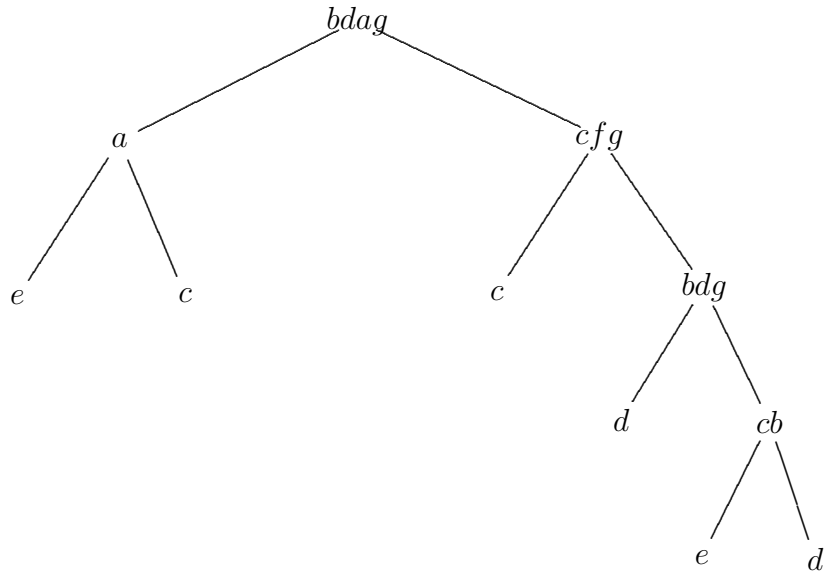
20. This item is due to Alex Paddock, a sixth grader at McDonald Middle in East Lansing, MI.



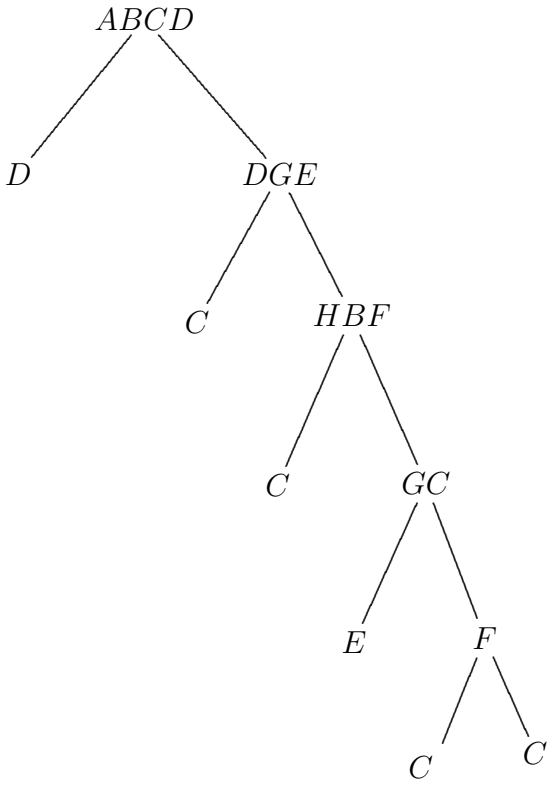
21. This item and the next one are due to Aaron Longuski, an eighth grader at Chippewa Middle in East Lansing, MI.



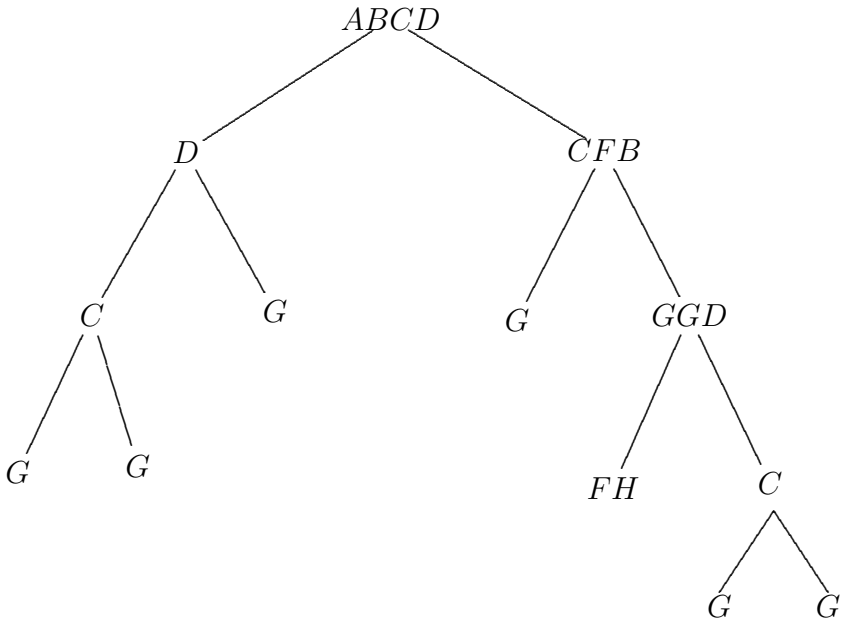
22. As before, each leaf node is prime.



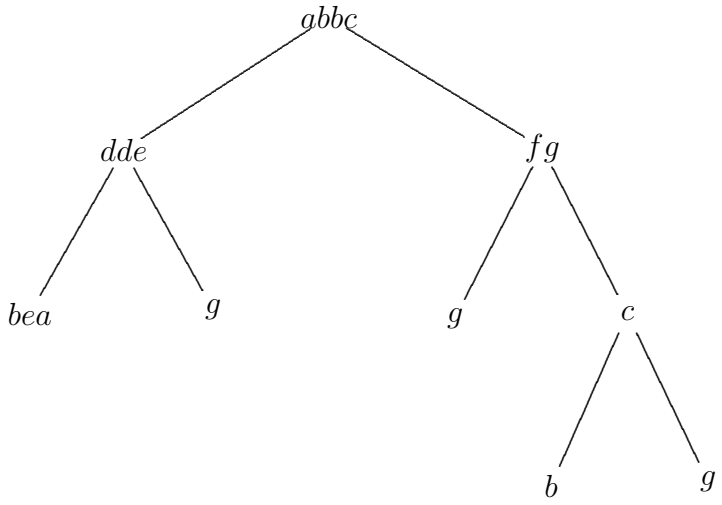
23. This item is due to Jun Ahn, McDonald Middle School seventh grader.



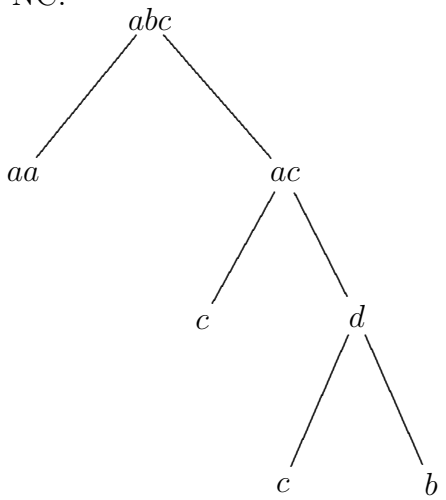
24. This item is due to Anjali Nelatoor, an eighth grader at Community House Middle in Charlotte, NC.



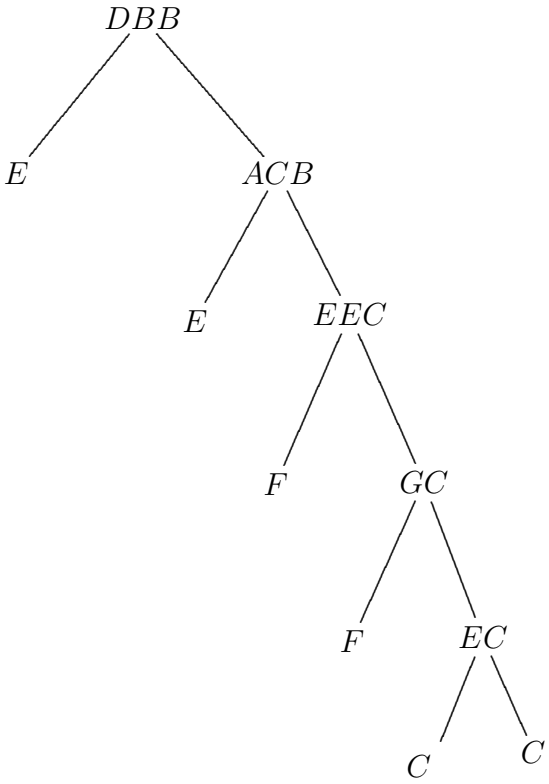
25. This lovely puzzle is due to Dylan Hernquist, an eighth grader at Community House Middle in Charlotte, NC.



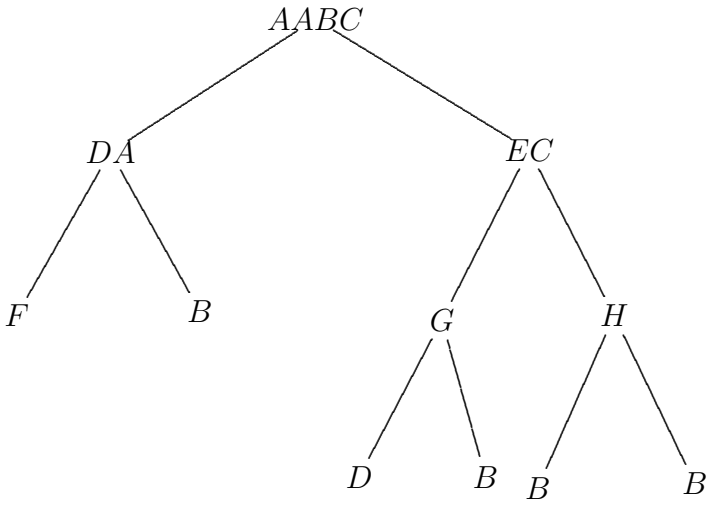
26. This item is due to Aditya Vadakattu a seventh grader at Community House Middle in Charlotte, NC.



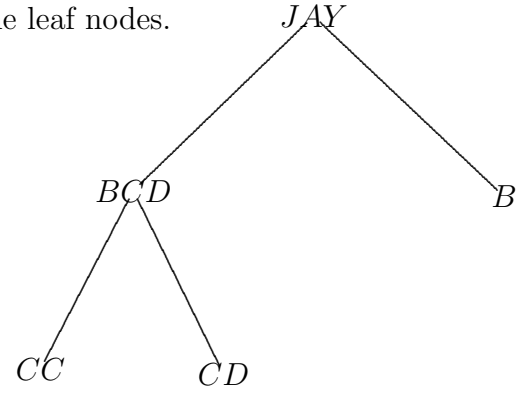
27. This item is due to Parvathy Unnikrishnan, a fifth grader at Metrolina Regional Scholars.



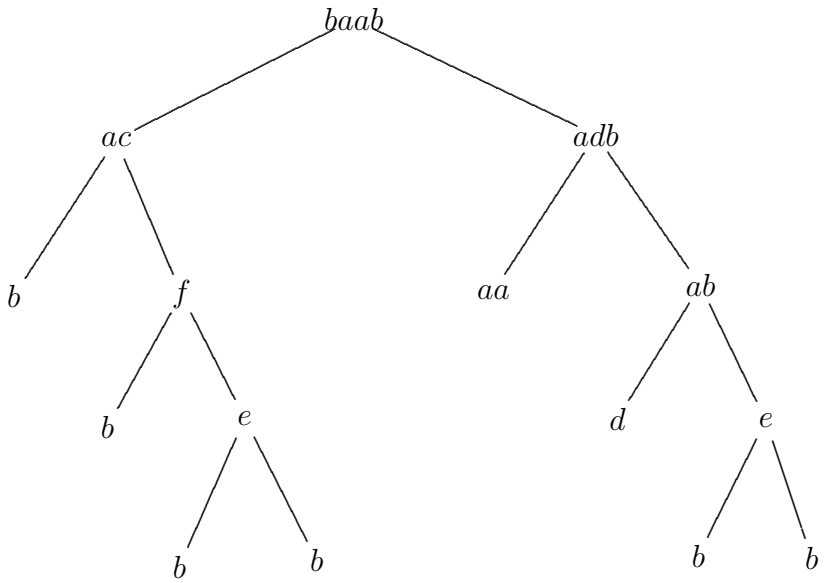
28. This item is due to Christina Zhao, now (summer 2018) a seventh grader at JM Robinson Middle.



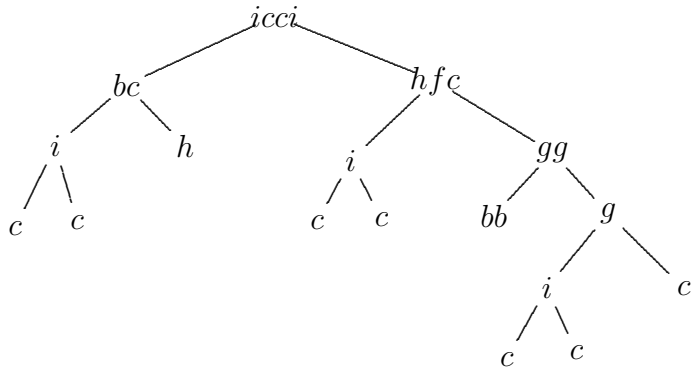
29. This item is due to Jay Sundar, a seventh grader at Metrolina Scholars in Charlotte NC. Each letter stands for a different digit. The prime numbers are the leaf nodes.



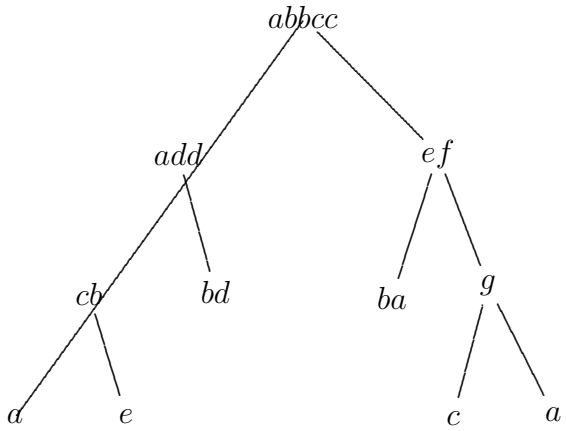
30. This item is due to Jonathan Zeng, now (summer 2018) a seventh grader at JM Robinson Middle.



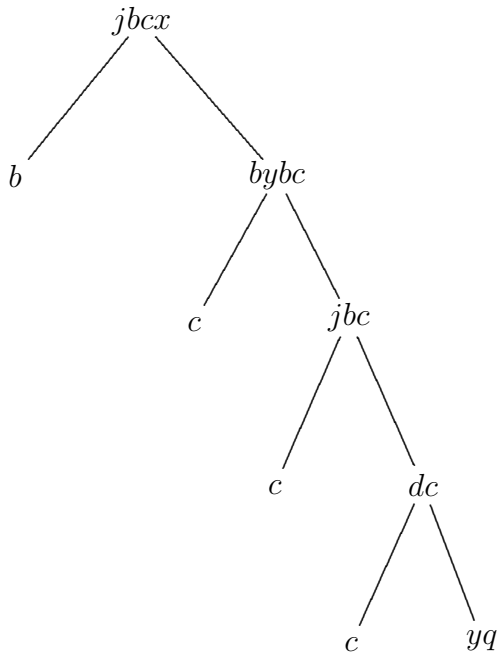
31. This item was contributed by Radhika Unnikrishnan, a student at Metrolina Scholars in Charlotte, NC.



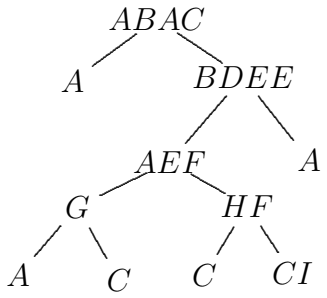
32. This item was contributed by Swara Repala, a student at J M Robinson Middle in Charlotte, NC.



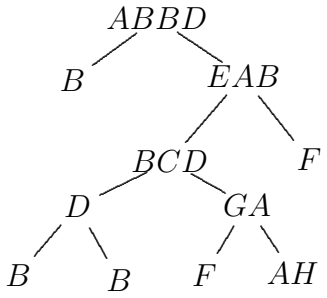
33. This item is due to Regina Penny, grade 7, St Peter & Paul School, Tulsa, OK.



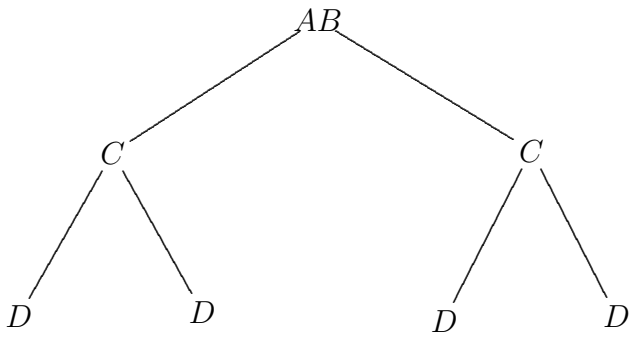
34. This item is due to Jadyn Goodson, grade 6; Bristow, OK Middle School.



35. This item is due to Carley Manus, grade 7; Bristow, OK Middle School.



36. This item is due to Nandana Marpadaga, Charlotte, NC. This is an answer to my question, 'can you find an ambiguous factor tree problem?' Can you find multiple solutions to this problem?



4 Factor Trees in Reverse Polish Notation.

Here's a new type problem that is accessible after you've learned Reverse Polish Notation (RPN). The idea is to build the factor tree puzzle using the right side of the equality. But you need to see the left side to know which items are at the same level. There is no guarantee that the RPN can be unabiguously translated into the multiplication tree, but by playing with it, you can often make this happen. Its not a hard exercise to build a string of words that translates into two different trees. of course this is quite unlike the case for arithmetic expressions where the POINT is that the RPN translates uniquely into the tree.

1. $eg\ ge\ afg\ eh\ he\ edfi\ abcdef$
2. This lovely puzzle is due to Leo Hong, grade 3, Elizabeth Lane, Charlotte, NC.
$$fd\ hi\ gb\ dgbe\ abcde$$
3. This puzzle is due to Harava Rahardjo
$$A\quad A\quad C\quad D\quad AC\quad B\quad B\quad F\quad AGA$$
4. This puzzle is due to Aaron Wahid
$$A\quad B\quad D\quad A\quad A\quad G\quad A\quad F\quad A\quad A\quad G\quad B\quad A\quad CEA$$
5. This puzzle is due to Ryan Wangidjaja
$$A\quad A\quad D\quad B\quad A\quad F\quad B\quad A\quad F\quad AE\quad B\quad ABC$$
6. This puzzle is due to Aniruddh Dayananda, then a third grader at Barringer Academic, Charlotte, NC.
$$d\quad i\quad aa\quad ii\quad fhd\quad b\quad fecd\quad f\quad ada\quad edf\quad adcbced$$

7. This beauty is due to Shreyas Srinivasan, a seventh grader at Community House Middle in Charlotte.

$$c \quad e \quad ae \quad fbc \quad abcde$$

5 Factor Trees in base b, $b \neq 10$.

8. In base 6,

$$dd \quad dc \quad ed \quad bcc \quad abac$$

9. Base 8. This one is from Watson Houck at Barringer Academic.

$$f \quad a \quad bg \quad ga \quad bdef \quad eg \quad abcda$$

10. This lovely problem is due to Akshaj Arora. Its a factor tree in base 6.

$$f \quad b \quad ad \quad e \quad f \quad f \quad b \quad f \quad ad \quad ed \quad ed \quad c \quad bfdd$$

6 Factor Trees over subsets of the positive integers.

What makes factor tree problems work? Or you could ask, what make factoring interesting? It's the fact that each integer has a *unique* factorization into primes. Are there other sets of integers for which factor tree problems might be interesting? First we need a set of numbers **closed** under multiplication. A set of positive integers S is closed under multiplication if for each pair $x, y \in S$, $xy \in S$. Now see if you can

think of some subsets of the set Z^+ of positive integers which are closed under multiplication. Then build a factor tree problem over that set. Here's the kind of thing what I'm thinking about. The set E of even numbers is closed under multiplication. Why?

11. Each letter stands for a different digit. The 'prime' numbers are the leaf nodes.