Problems on Games

1. For each of the following games, label each position with its value O for oasis and P for poison.

   (a) Consider the game G_1 which starts with one pile of 20 counters. The rules allow a player to take $1, 3$, or 5 counters on each turn. The player who makes the last move wins. Denote this game by $N(20; 1, 3, 5)$.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

   (b) Consider the game G_2 which starts with one pile of 20 counters. The rules allow a player to take $1, 2$, or 5 counters on each turn. Denote this game by $N(20; 1, 2, 5)$.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

   (c) Consider the game G_3 which starts with one pile of 20 counters. The rules allow a player to take $1, 2$, or 6 counters on each turn. Denote this game by $N(20; 1, 2, 6)$. As usual, the player who makes the last move wins.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

   (d) Consider the game G_4 which starts with one pile of 20 counters. The rules allow a player to take a prime number of counters on each turn. Denote this game by $N(20; 2, 3, 5, 7, 11, 13, 17)$. The move 19 is purposely left out. As usual, the player who makes the last move wins.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

   (e) Consider the game G_5 which starts with one pile of 20 counters. The rules allow a player to take an integer power of 2 counters on each turn. Denote this game by $N(20; 1, 2, 4, 8, 16)$. As usual, the player who makes the last move wins.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

   (f) Consider the game G_6 which starts with one pile of 20 counters. The rules allow a player to take an integer power of three counters on each turn. Denote this game by $N(20; 3^0, 3^1, 3^2)$. As usual, the player who makes the last move wins.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

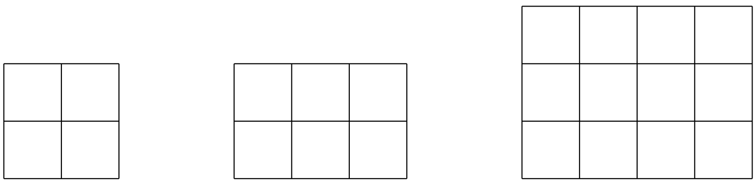
2. **The Crippled Rook Game.** This game, also known as two-pile nim, is at the foundation of combinatorial games. The rook in question can move either up or to the left. Label each position with its O, P value.

$\oplus$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0															
1															
2															
3															
4															
5															
6															
7															
8															
9															
10															

3. **Puppies and Kittens.** An animal shelter specializes in puppies and kitten. You and your opponent can alternately take any number of kittens, any number of puppies, or the same number of kittens and puppies. The objective is once again to make the last legal move. Play the game $(11, 14)$, where the first number is the number of kittens and the second is the number of puppies. The grid below will be helpful in playing this game. Then find the O, P value of each position.

$\oplus$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0															
1															
2															
3															
4															
5															
6															
7															
8															
9															
10															
11															

4. **Cram.** In this game, we start with a grid, usually rectangular. The players alternately place either a square □ or a domino □□ on the grid but not overlapping any of the previously played squares. The first player who is unable to move is the loser. Try this game with your partner for each of the grids below.



5. **Divisor Game** We're playing the misere version of the game, so the last player loses. Start with a pile on N counters. Players are allowed to take a divisor number of counters. Lets play this game with $N = 100$ counters. Who wins with optimal play?

6. Numbers that are "unsafe" when playing the game Dollar Nim, which is a Nim game where users can remove 1, 5, 10, or 25 cents from an initial pile of money. The most common version of the game is played with an initial amount of \$1, hence the name. 1