- 1. A shop advertises everything is 'half price in today's sale.' In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price?
- 2. The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?
- 3. How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?
- 4. The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, and x are all equal. What is the value of x?
- 5. What is the units digit ( ones place digit ) of  $13^{2012}$ ?
- 6. Jamar bought some pencils costing more than a penny each at the school bookstore and paid \$1.43. Sharona bought some of the same pencils and paid \$1.87. How many more pencils did Sharona buy than Jamar?
- 7. In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?
- 8. Find the smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?
- 9. Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?

- 10. A square with integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?
- 11. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?
- 12. In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?
- 13. What is the correct ordering of the three numbers  $\frac{5}{19}$ ,  $\frac{7}{21}$ , and  $\frac{9}{23}$ , in increasing order?
- 14. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?
- 15. Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4,6, 9, and 14. What is the number of possible values of the median of R ?
- 16. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?
- 17. A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



18. A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length *a*, and the other of length *b*. What is the value of *ab*?



- 19. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?
- 20. A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles?
- 21. A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?
- 22. Let  $\angle ABC = 24^{\circ}$  and  $\angle ABD = 20^{\circ}$ . What is the smallest possible degree measure for angle CBD?

- 23. Last year 100 adult cats, half of whom were female, were brought into the Smallville Animal Shelter. Half of the adult female cats were accompanied by a litter of kittens. The average number of kittens per litter was 4. What was the total number of cats and kittens received by the shelter last year?
- 24. The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?
- 25. In a bag of marbles,  $\frac{3}{5}$  of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?
- 26. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?
- 27. A pair of six-sided dice are labeled so that one die has only even numbers (two each of 2, 4, and 6), and the other die has only odd numbers (two each of 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?
- 28. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?
- 29. Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?
- 30. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

- 31. An iterative average of the numbers 1, 2, 3, 4, and 5 is computed the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?
- 32. If a,b, and c are digits for which

then a + b + c =

- 33. In the year 2001, the United States will host the International Mathematical Olympiad. Let I, M, and O be distinct positive integers such that the product  $I \cdot M \cdot O = 2001$ . What is the largest possible value of the sum I + M + O?
- 34. A positive integer is called *referential* if each digit occurs the same number of times as the value of the digit. For example, 1, 22, and 313223 are referential. How many integers between 0 and 100000 are referential?
- 35. Referring to the last question, how many six digit numbers are referential?
- 36. Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?
- 37. The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, ... starts with two 1s, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

- 38. How many positive integers b have the property that  $\log_b 729$  is a positive integer?
- 39. Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last score Mrs. Walter entered?
- 40. The point P = (1, 2, 3) is reflected in the *xy*-plane, then its image Q is rotated by 180° about the *x*-axis to produce R, and finally, R is translated by 5 units in the positive-*y* direction to produce *S*. What are the coordinates of *S*?
- 41. Let A, M, and C be nonnegative integers such that A + M + C = 12. What is the maximum value of  $A \cdot M \cdot C + A \cdot M + M \cdot C + A \cdot C$ ?
- 42. One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?
- 43. When the mean, median, and mode of the list

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

44. Let f be a function for which  $f(x/3) = x^2 + x + 1$ . Find the sum of all values of z for which f(3z) = 7.

- 45. A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered  $1, 2, \ldots, 17$ , the second row  $18, 19, \ldots, 34$ , and so on down the board. If the board is renumbered so that the left column, top to bottom, is  $1, 2, \ldots, 13$ , the second column  $14, 15, \ldots, 26$  and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).
- 46. In year N, the 300<sup>th</sup> day of the year is a Tuesday. In year N + 1, the 200<sup>th</sup> day is also a Tuesday. On what day of the week did the 100th day of year N 1 occur?
- 47. Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property— the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winning ticket?
- 48. A block of cheese in the shape of a rectangular solid measures 10 cm by 13 cm by 14 cm. Ten slices are cut from the cheese. Each slice has a width of 1 cm and is cut parallel to one face of the cheese. The individual slices are not necessarily parallel to each other. What is the maximum possible volume in cubic cm of the remaining block of cheese after ten slices have been cut off?