ABSTRACT

Title of dissertation: ENERGY HARVESTING COMMUNICATION NETWORKS WITH SYSTEM COSTS

Ahmed Arafa, Doctor of Philosophy, 2017

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This dissertation focuses on characterizing optimal energy management policies for energy harvesting communication networks with system costs. The system costs that we consider are the cost of circuitry to be on (processing cost) at the transmitters, cost of decoding at the receivers, cost of moving to harvest more energy in mobile energy harvesting nodes, and the cost of collecting measurements (sampling cost) from physical phenomena.

We first consider receiver decoding costs in networks where receivers, in addition to transmitters, rely on energy harvested from nature to communicate. Energy harvested at the receivers is used to decode their intended messages, and is modeled as a convex increasing function of the incoming rate. With the goal of maximizing throughput by a given deadline, we study single-user and multi-user settings, and show that decoding costs at the receivers can be represented as generalized data arrivals at the transmitters. This introduces a further coupling between the transmitters and receivers of the network and allows us to characterize optimal policies by moving all constraints to the transmitter side.

Next, we study the decoding cost effect on energy harvesting cooperative multiple access channels, where users employ data cooperation to increase their achievable rates. Data cooperation requires each user to decode the other user's data before forwarding it to the destination, which uses up some of the harvested energy. With the presence of decoding costs, we show that data cooperation may not be always helpful; if the decoding costs are relatively high, then sending directly to the receiver without data cooperation between the users achieves higher throughput. When cooperation is helpful, we determine the optimum allocation of available energy between decoding cooperative partner's data and forwarding it to the destination.

We then study the impact of adding processing costs, on top of decoding costs, in energy harvesting two-way channels. Processing costs are the amounts of energy spent for circuitry operation, and are incurred whenever a user is communicating. We show that due to processing costs, transmission may become bursty, where users communicate through only a portion of the time. We develop an optimal scheme that maximizes the sum throughput by a given deadline under both decoding and processing costs.

Next, we focus on online policies. We consider a single-user energy harvesting channel where the transmitter is equipped with a finite-sized battery, and the goal is to maximize the long term average utility, for general concave increasing utility functions. We show that fixed fraction policies are near optimal; they achieve a long term average utility that lies within constant multiplicative and additive gaps from the optimal solution for all battery sizes and all independent and identically distributed energy arrival patterns. We then consider a specific scenario of a utility function that measures the distortion of Gaussian samples communicated over a Gaussian channel. We formulate two problems: one with, and the other without sampling costs, and design near optimal fixed fraction policies for the two problems.

Then, we consider another aspect of costs in energy harvesting single-user channels, that is, the energy spent in physical movement in search of better energy harvesting locations. Since movement has a cost, there exists a tradeoff between staying at the same location and moving to a new one. Staying at the same location allows the transmitter to use all its available energy in transmission, while moving to a new one may let the transmitter harvest higher amounts of energy and achieve higher rates at the expense of a cost incurred through the relocation process. We characterize this tradeoff optimally under both offline and online settings.

Next, we consider different performance metrics, other than throughput, in energy harvesting communication networks. First, we study the issue of delay in single-user and broadcast energy harvesting channels. We define the delay per data unit as the time elapsed from the unit's arrival at the transmitter to its departure. With a pre-specified amount of data to be delivered, we characterize delay minimal energy management policies. We show that the structure of the optimal policy is different from throughput-optimal policies; to minimize the average delay, earlier arriving data units are transmitted using higher powers than later arriving ones, and the transmit power may reach zero, leading to communication gaps, in between energy or data arrival instances.

Finally, we conclude this dissertation by considering the metric of the age of information in energy harvesting two-hop networks, where a transmitter is communicating with a receiver through a relay. Different from delay, the age of information is defined as the time elapsed since the latest data unit has reached the destination. We show that age minimal policies are such that the transmitter sends message updates to the relay just in time as the relay is ready to forward them to the receiver.

Energy Harvesting Communication Networks with System Costs

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2017

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Dedication

To my family.

Acknowledgments

First and foremost, I express my utmost gratitude to Allah, The All-Knowing and The Most Generous, for being able to finish this dissertation, and for reaching this current stage of my career.

I owe everything in this dissertation to my adviser, Professor Sennur Ulukus. It has been through her help and guidance over the past five years that I was able to produce the works included in this dissertation. She has been a true role model when it comes to discipline and hard work; she pushed my mind to its limits, enhanced my writing and presentation skills, and I learned a lot from her intuitive approach to modeling and solving problems. I thank her for giving me the opportunity to work with her.

I would like to thank Professors Richard La, Nuno Martins, Prakash Narayan, and Amr Baz, for being in my dissertation committee and offering their valuable feedback. I enjoyed the technical discussions I had with Professor Narayan, whether they were while being a student in his classes, or while serving as his teaching assistant. I am thankful to all the professors I have engaged with over the past years at UMD. In particular, to Professor Andre Tits, whose classes taught me how to enhance my proof techniques and mathematical rigor; and for being his co-instructor in one of his classes, which gave me an invaluable teaching experience. I am also thankful to Professor Onur Kaya, of Isik University, for our collaboration in one of the works in this dissertation.

I am thankful to all my colleagues at UMD and to all my office mates at the

Communications and Signal Processing Lab with whom I shared the same working space and engaged in both technical and personal discussions throughout my time at UMD – I thank Professor Himanshu Tyagi, Dr. Jianwie Xie, Dr. Omur Ozel, Dr. Berk Gurakan, Dr. Pritam Mukherjee, Praneeth Boda, Abdulrahman Baknina, Karim Banawan, Yi-Peng Wei, Ajaykrishnan Nageswaran, Melih Bastopcu, and Baturalp Buyukates. I am especially thankful to Abdulrahman and Karim for bringing up the Egyptian spirit to our lab; I will truly miss the daily technical discussions and collaborations with Abdulrahman, that resulted in multiple works in this dissertation.

I am deeply indebted to my family, who have been continuously supporting me throughout this journey. I am grateful to my parents, who have always pushed me forward and encouraged me to continue my graduate studies. I have truly sensed the effects of their prayers upon my life so far, and I hope that seeing me in this stage would make them feel pleased. I thank my brother Yehia for his love and support. I am appreciative of the times when my parents came to visit along with my brother; these times have truly made me feel like I am back home.

I am grateful to my wife May, who has been with me from the first day, and chose to stay beside me over anything else, out of her faithfulness and love. We faced all the hard times and passed all the difficulties together. I will always say that this dissertation is a joint work with my wife, for I cannot imagine myself achieving this without her presence. I am thankful to my kids, Hamza and Maryam, who have brought joy into my life ever since they were born, and whose shiny smiling faces had always given me a fresh restart after tiring long days at work.

Finally, I would like to mention the great community that I have been a proud member of since I came to Maryland; the Tauba community. Our Friday nights gatherings and discussions have been the main source of nourishment to my sole, and have been where my heart could find the peace it needs to keep me going. Everyone I have met in this community have affected me, and my family, in a way that made this journey easier and more enjoyable.

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CHAPTER 1

Introduction

Energy harvesting communications offer the promise of energy self-sufficient, energy self-sustaining operation for wireless networks with significantly prolonged lifetimes. In this dissertation, we characterize optimal energy management policies in energy harvesting communication networks taking into consideration different aspects of system costs. Namely, we study the effects of the costs of circuitry to be on (processing costs) at the transmitters, the costs of decoding at the receivers, the costs of moving to harvest more energy in mobile energy harvesting nodes, and the costs of collecting measurements (sampling costs) from physical phenomena, on energy management policies that optimize certain utilities. Considering such system costs introduces new structures to optimal energy management policies, and in general takes the analysis of energy harvesting communication networks one step further into practicality.

In Chapter 2, we focus on receiver-side energy harvesting. Energy harvesting communications have been considered mostly for energy harvesting *transmitters*, e.g., [1–36], with fewer works on energy harvesting *receivers*, e.g., [37–41]. In Chapter 2, we consider energy harvesting communications with both energy harvesting transmitters and receivers. The energy harvested at the transmitters is used for data transmission according to a rate-power relationship, which is concave, monotone increasing in powers. The energy harvested at the receivers is used for decoding costs, which we assume to be convex, monotone increasing in the incoming rate [37,38,42–45]. The transmission energy costs and receiver decoding costs could be comparable, especially in short-distance communications, where high rates can be achieved with relatively low powers, and the decoding power could be dominant; see [42] and the references therein.

We model the energy needed for decoding at the receivers via *decoding causality* constraints: the energy spent at the receiver for decoding cannot exceed the receiver's harvested energy. We already have the *energy causality* constraints at the transmitter: the energy spent at the transmitter for transmitting data cannot exceed the transmitter's harvested energy. Therefore, for a given transmitter-receiver pair, transmitter powers need now to adapt to both energy harvested at the transmitter and at the receiver; the transmitter must only use powers, and therefore rates, that can be handled/decoded by the receiver. The most closely related work to the work in Chptaer 2 is [37], where the authors consider a general network with energy harvesting transmitters and receivers, and maximize a general utility function, subject to energy harvesting constraints at all terminals. Reference [37] carries the effects of decoding costs to the objective function. If the objective function is no longer concave after this operation, it uses time-sharing to concavify it, leading to a convex optimization problem, which it then solves by using a generalized water-filling algorithm. We consider a similar problem with a specific utility function which is throughput, for specific network structures, with different decoding costs informed by network information theory. First, we consider the single-user channel, and observe that the decoding costs at the receiver can be interpreted as a *gate keeper* at the front-end of the receiver that lets packets pass only if it has sufficient energy to decode. We show that we can carry this *gate* effect to the transmitter as a *generalized data arrival constraint*. Therefore, the setting with decoding costs at the receiver is equivalent to a setting with no decoding costs at the receiver, but with a (generalized) data arrival constraint at the transmitter [1]. We also note that the energy harvesting component of the receiver can be separated as a *virtual relay* between the transmitter and the receiver; and again, the problem can be viewed as a setting with no decoding costs at the receiver but with a *virtual relay* with a (generalized) energy arrival constraint [12–17].

We then consider several multi-user settings. We begin with a decode-and-forward two-hop network, where the relay and the receiver both have decoding costs. This gives rise to *decode-and-forward causality* constraints at the relay in addition to decoding causality constraints at the receiver and energy causality constraints at the transmitter. We decompose the problem into inner and outer problems. In the inner problem, we fix the relay's decoding power strategy, and show that *separable* policies are optimal [12, 13]. These are policies that maximize the throughput of the transmitter-relay link independent of maximizing the throughput of the relay-destination link. Thereby, we solve the inner problem as two single-user problems with decoding costs. In the outer problem, we find the best relay decoding strategy by a water-filling algorithm. Next, we consider a two-user multiple access channel (MAC) with energy harvesting transmitters and receiver, and maximize the departure region. We consider two different decoding schemes: simultaneous decoding, and successive cancellation decoding [46]. Each scheme has a different decoding power consumption. For the simultaneous decoding scheme, we show that the boundary of the maximum departure region is achieved by solving a weighted sum rate maximization problem that can be decomposed into an inner and an outer problem. We solve the inner problem using the results of single-user fading problem [3]. The outer problem is then solved using a water-filling algorithm. In the successive cancellation decoding scheme, our problem formulation is non-convex. We then use a successive convex approximation technique that converges to a local optimal solution [47, 48]. The maximum departure region with successive cancellation decoding is larger than that with simultaneous decoding. We conclude Chapter 2 by characterizing the maximum departure region of a two-user degraded broadcast channel (BC) with energy harvesting transmitter and receivers. With the transmitter employing superposition coding [49], a corresponding decoding power consumption at the receivers is assumed. We again decompose the weighted sum rate maximization problem into an inner and outer problem. We show that the inner problem is equivalent to a classical single-user energy harvesting problem with a time-varying *minimum power constraint*, for which we present an algorithm. We solve the outer problem using a water-filling algorithm similar to the outer problems of the two-hop network and the MAC with simultaneous decoding.

In Chapter 3, we study the decoding costs effects on an energy harvesting cooperative MAC. In a cooperative MAC, users decode the signals transmitted by the other user to form common information, and cooperatively send the previously established common information to the receiver to achieve beamforming gains [50]. This model has the unique property that the transmitters act as receivers as well, where transmission power and decoding costs simultaneously reflect on the total energy budget of each node. The energy harvesting cooperative MAC is considered in [51] for data cooperation only, and extended in [52] to the case of joint data and energy cooperation, without taking into account the decoding costs incurred at the nodes, and significant gains in departure regions are demonstrated. The goal of Chapter 3 is to incorporate the decoding cost of cooperation into the cooperative MAC model, and investigate the gains from cooperation in a more realistic setup. To this end, we model the decoding power as an increasing convex function in the incoming rate [37, 38], and in particular, we focus on exponential decoding functions [43, 44]. We characterize the optimal offline power scheduling policies that maximize the departure region by a given deadline subject to energy causality constraints and decoding costs.

In Chapter 4, we explore another aspect of system costs: the costs for circuitry operations, or processing costs. We study the effects of processing and decoding costs combined in an energy harvesting two-way channel. We design optimal offline power scheduling policies that maximize the sum throughput by a given deadline, subject to energy and decoding causality constraints at both users, with processing costs. In the two-way energy harvesting channel, each node transmits data to the other user, and receives data from the other user in a full duplex manner. Therefore, each node is simultaneously an energy harvesting transmitter and an energy harvesting receiver, and needs to optimize its power schedule over time slots by optimally dividing its energy for transmission and decoding. The power used for transmission is modeled through a concave rate-power relationship as in the Shannon formula; and the power used for decoding is modeled as a convex increasing function of the incoming rate. In particular, throughout Chapter 4, we focus on decoding costs that are exponential in the incoming rate [43, 44].

Even in the case of energy harvesting transmitters only and energy harvesting receivers only, the energy availability of one side limits the transmission and reception abilities of the other side; energy harvesting introduces coupling between transmitters and receivers. In the energy harvesting two-way channel, this coupling is even stronger. In addition, we assume that power consumption at a user includes power spent for processing as well, i.e., power spent for the circuitry. This is the power spent for the user to be *on* and communicating. Depending on the energy availability and the communication distance, processing costs at the transmitter could be a significant system factor. References [24–29] study the impact of processing costs on energy harvesting communications. As discussed in Chapter 2, decoding power at the receiver could be a significant system factor as well [37, 38, 41–43]. The differentiating aspect regarding processing costs and decoding costs is as follows: the processing cost is modeled as a constant power spent per unit time whenever the transmitter is on [53], whereas the decoding cost at a receiver is modeled as an increasing convex function of the incoming rate to be decoded [37, 38]. In Chapter 4, we consider *both* decoding and processing costs in a single setting.

In the first part of Chapter 4, we focus on the case with only decoding costs. We first consider the case with a single energy arrival at each user. We show that the transmission is limited by the user with smaller energy; the user with larger energy may not consume all of its energy. We next consider the case with multiple energy arrivals at both users. We show that the optimal power allocations are non-decreasing over time, and they increase synchronously at both users. We develop an iterative algorithm based on two-slot updates to obtain the optimal power allocations for both users that converges to the optimal solution. Next, we focus on the case with only processing costs. We assume that both users incur processing costs per unit time as long as they are communicating. We first consider the formulation for a single energy arrival. In this case, we show that transmission can be bursty [53]; users may opt to communicate for only a portion of the time. We also show that it is optimal for the two users to be fully synchronized; the two users should be switched on for the same portion of the time during which they both exchange data, and then they switch off together. Then, we generalize this to the case of multiple energy arrivals, and show that any throughput optimal policy can be transformed into a *deferred* policy, in which users postpone their energy consumption to fill out later slots first. We find the optimal deferred policy by iteratively applying a modified version of the single energy arrival result in a backward manner. We conclude Chapter 4 by studying the general case with *both* decoding and processing costs in a single setting. We formulate a sum throughput optimization problem that is a generalization of the setting with only decoding costs or only processing costs. We solve this general problem in the single energy arrival scenario, and then present an iterative algorithm to solve the multiple energy arrival case that is a combination of the algorithms used to solve the cases with only decoding and only processing costs.

In Chapter 5, we focus on *online* settings, where the amounts of energy harvested are revealed causally over time. We consider a single-user communication channel, where the transmitter has a battery of finite size to save its incoming energy, and achieves a reward for every transmitted message that is in the form of some general concave increasing utility function of the transmission power. The goal is to characterize *online* power control policies that maximize the long term average utility subject to energy causality constraints. One motivation for this setting is energy harvesting receivers studied in Chapter 2. Since power consumed in decoding is modelled as a convex increasing function of the incoming rate [37, 38], the rate achieved at the receiver is then a concave increasing function of the decoding power. Recently, [54] has introduced an online power control policy for a single-user energy harvesting channel that maximizes the long term average throughput under the AWGN capacity utility function $\frac{1}{2}\log(1+x)$. The proposed policy is near optimal in the sense that it performs within constant multiplicative and additive gaps from the optimal solution that is independent of energy arrivals and battery sizes. This is extended to broadcast channels in [55], multiple access channels in [56, 57], and systems with processing costs in [58–60] (for examples of earlier online approaches see, e.g., [61–63]). In Chapter 5, we generalize the approaches in [54] to work for general concave monotonically increasing utility functions for single-user channels. That is, we consider the design of online power control policies that maximize the long term average general utilities.

We first study the special case of Bernoulli energy arrivals that fully recharge the battery when harvested, and characterize the optimal online solution. Then, for the general independent and identically distributed (i.i.d.) arrivals, we show that the policy introduced in [54] performs within a constant multiplicative gap from the optimal solution for any general concave increasing utility function, for all energy arrivals and battery sizes. We then provide sufficient conditions on the utility function to guarantee that such policy is within a constant additive gap from the optimal solution. We then consider a specific scenario where a sensor node collects samples from an i.i.d. Gaussian source and sends them to a destination over a Gaussian channel, and the goal is to characterize online power control policies that minimize the long term average distortion of the received samples at the destination. We note that an offline version of this problem has been considered in [35]. We follow the approaches in [54–58] to extend the offline results in [35] to online settings. We formulate two problems: one with and the other without sampling energy consumption costs. In both problems, we show that fixed fraction policy achieves a long term average distortion that lies within a constant additive gap from the optimal achieved distortion for all energy arrivals and battery sizes.

In Chapter 6, we consider another aspect of power consumption in energy harvesting sensor nodes, namely, the power consumed in the process of harvesting energy. That is, there is a cost to taking actions to harvest energy. In Chapter 6, we model this cost via the energy consumed in physical movement. We consider an energy harvesting transmitter with the ability to move along a straight line. Two energy sources are located at the opposite ends of the line, and the amount of energy harvested at the transmitter from each source depends on its distance from the two sources. Movement is thus motivated by finding better energy harvesting locations. However, the transmitter incurs a moving cost per unit distance travelled. Therefore, a tradeoff arises between staying in the same position and using all available energy in transmission, and spending some of the available energy to move to another location where it harvests higher energy. In this work, we characterize this tradeoff optimally, by designing throughput optimal power and movement policies. We note that related system models are considered in [64,65] where some devices (energy-rich sources) move through a sensor network and refill the batteries of the sensors with RF radiation.

We study both offline and online settings in Chapter 6. In the offline setting, our goal is to maximize the throughput by a given deadline. We first study the case where each energy source has a single energy arrival, and then generalize it to the case of multiple energy arrivals. Although our problem formulation is non-convex, we are able to solve it optimally for the single energy arrival scenario. For the multiple energy arrivals scenario, we design an iterative algorithm with guaranteed convergence to a local optimal solution of our optimization problem. For each iteration, we first show that given the optimal movement energy expenditure in a given time slot, the movement policy is *greedy;* the transmitter moves to the better location (energy-wise) in that time slot only without considering future time slots. We then find optimal movement energy consumption using a water-filling algorithm.

In the online setting, we model the energy arrival processes at the two sources as two independent and i.i.d. processes. Only the means of the two processes are known before communication. Our goal is to maximize the long term average throughput. To that end, we propose an optimal *move-then-transmit* scheme where the transmitter first uses all its harvested energy to move towards the source with higher energy harvesting mean. After that, it stays at that source's position and starts communicating with the receiver. We show that the energy used in movement does not affect the throughput in the long term average sense. If the transmitter has an infinite battery, we use the best effort transmission strategy to optimally manage the harvested energy in transmission [66]. In this policy, the transmitter sends with the average harvesting rate whenever feasible and stays silent otherwise. On the other hand, if the transmitter has a finite battery, we use the fixed fraction policy [54], where the transmitter uses a fixed fraction of the amount of energy available in its battery for transmission in every time slot, to achieve a long term average rate that lies within constant additive and multiplicative gaps from the optimal solution for all energy arrival patterns and battery sizes.

In Chapters 7 and 8, we consider different performance metrics, other than the throughput metric considered mainly in previous chapters. First, in Chapter 7, we study the issue of delay on energy harvesting networks. According to a specific data demand, the transmitter needs to schedule the transmission of data packets using the available energy such that the average delay experienced by the data is minimal. In [1], the problem of minimizing the transmission completion time is considered. Reference [1] and the subsequent literature showed that, due to the concavity of the rate-power relationship, the transmit power must be kept constant between energy harvesting and data arrival events, and the transmitter must schedule data transmissions using longest possible stretches of constant power, subject to energy and data causality. While [1] minimizes the time by which all of the data packets are transmitted, different data packets experience different delays, and the average delay of the system is not minimized. In particular, when the earlier-arriving data packets are transmitted slowly, the later-arriving data packets experience not only the delay in their own transmissions, but a portion of the delay experienced by the earlier-arriving packets, as they have to wait extra time in the data queue while those packets are being transmitted. This compounds the delays that the later-arriving data packets experience. The delay minimization problem was considered previously in [67] for a non energy harvesting system.

We consider the problem of average delay minimization in an energy harvesting system in Chapter 7. First, we consider a single-user channel where the transmitter is equipped with a finite-sized battery and a finite-sized data buffer. We show that, unlike the previous literature, the transmission power should not be kept constant between energy harvesting and data arrival events. We let the power (and therefore the rate) vary even during the transmission of a single packet. We show that the optimal packet scheduling is such that the transmit power starts with a high value and decreases linearly over time possibly reaching zero before the arrival of the next energy or data packet into the system. The high initial transmit power values ensure that earlier bits are transmitted faster, decreasing their own delay and also the delays of the later-arriving data packets. Using a Lagrangian framework, we develop a recursive solution that finds the optimal transmit power over time by determining the optimal Lagrange multipliers.

Next, we consider a two-user energy harvesting broadcast channel where the transmitter is equipped with an infinite-sized battery, and data packets intended for both users are available before the transmission starts. In this system, there is a tradeoff between the delays experienced by both users; as more resources (power) is allocated to a user, its delay decreases while the delay of the other user increases. We consider the minimization of the *sum* delay in the system. We formulate the problem using a Lagrangian framework, and express the optimal solution in terms of Lagrange multipliers. We develop an iterative solution that solves the optimum Lagrange multipliers by enforcing the KKT optimality conditions. Similar to the single-user setting, we show that the optimal transmission power decreases between energy harvests, and may possibly hit zero before the next energy harvest, yielding communication *gaps*, where no data is transmitted. During active communication, data may be sent to both users, or only to the stronger, or only to the weaker user, depending on the energy harvesting profile. We contrast our work with [7] which developed an algorithm that minimized the transmission completion time, i.e., a time by which all data is delivered to users. To that end, [7] studies the throughput maximization problem, and shows that, for general priorities, there exists a *cut-off power* level such that only the total power above this level is used to serve the weaker user. In particular, for sum throughput maximization, this cut-off is infinity, and all power is allocated to packets sent to the stronger user. In contrast, in our *sum delay minimization* problem, the weaker user always gets a share of the transmitted power, as otherwise, its delay becomes unbounded, and the sum delay will not be minimized. In our work, we show that there exists a *cut-off time*, beyond which data is sent only to the weaker user.

In Chapter 8, we study the metric of the *age of information* in an energy harvesting two-hop network. We consider a source node that is collecting measurements from a physical phenomenon and sends updates to a destination through the help of a relay. Both the source and the relay depend on energy harvested from nature to communicate. Updates need to be sent in a *timely* manner; namely, such that the total *age of information* is minimized by a given deadline. The age of information is defined as the time elapsed since the freshest update has reached the destination. Minimizing the age of information metric has been studied mostly in a queuing-theoretic framework; [68] studies a source-destination link under random and deterministic service times. This is extended to multiple sources in [69]. References [70–72] consider variations of the single source system, such as randomly arriving updates, update management and control, and nonlinear age metrics. [73] introduces penalty functions to assess age dissatisfaction; and [74] shows that last-come-first-serve policies are optimal in multi-hop networks.

Our work in Chapter 8 is most closely related to [75, 76], where age minimization in single-user energy harvesting systems is considered; the difference of these works from energy harvesting literature in [1–36] is that the objective is age of information as opposed to throughput or transmission completion time, and the difference of them from age minimization literature in [68–74] is that sending updates incurs energy expenditure where energy becomes available intermittently. [75] considers random service time (time for the update to take effect) and [76] considers zero service time; in our work here, we consider a fixed but non-zero service time.

We consider an energy harvesting two-hop network where a source is sending information updates to a destination through a half-duplex relay. The source and the relay use fixed communication rates. Thus, different from [75, 76], they both incur fixed non-zero amounts of transmission delays to deliver their data. Our setting is offline, and the objective is to minimize the total age of information received by the destination within a given communication session time, subject to energy causality constraints at the source and relay nodes, and data causality constraints at the relay node. We first solve the single-hop version of this problem where the source communicates directly to the destination, with non-zero update transmission delays, extending the offline results in [76]; we observe that introducing non-zero update transmission delays is equivalent to having minimum inter-update time constraints. We then solve the two-hop problem; we first show that it is not optimal for the source to send a new update before the relay finishes forwarding the previous ones, i.e., the relay's data buffer should not contain more than one update packet waiting for service, otherwise earlier arriving packets become stale. Then, we show that the optimal source transmission times are *just in time* for the relay to forward the updates, i.e., it is not optimal to let an update wait in the relay's data buffer after being received; it must be directly forwarded. This contrasts the results in [12, 13] that study throughput maximization in energy harvesting relay networks. In there, throughput-optimal policies are *separable* in the sense that the source transmits the most amount of data to the relay regardless of the relay's energy harvesting profile. In our case, the age-optimal policy is not separable; it treats the source and the relay nodes as one combined node that is communicating to the destination. Hence, our single-hop results serve as a building block to find the solution of the two-hop problem.

In Chapter 9, we draw conclusions for this dissertation.

CHAPTER 2

Optimal Policies for Wireless Networks with Energy Harvesting Transmitters and Receivers: Effects of Decoding Costs

2.1 Introduction

In this chapter, we consider the effects of decoding costs in energy harvesting communication systems. In our setting, receivers, in addition to transmitters, rely solely on energy harvested from nature, and need to spend some energy in order to decode their intended packets. We model the decoding energy as an increasing convex function of the rate of the incoming data. In this setting, in addition to the traditional *energy causality* constraints at the transmitters, we have the *decoding causality* constraints at the receivers, where energy spent by the receiver for decoding cannot exceed its harvested energy. We first consider the point-to-point single-user problem where the goal is to maximize the total throughput by a given deadline subject to both energy and decoding causality constraints. We show that decoding costs at the receiver can be represented as *generalized data arrivals* at the transmitter, and thereby moving all system constraints to the transmitter side. Then, we consider several multi-user settings. We start with a two-hop network where the relay and the destination have decoding costs, and show that *separable* policies, where the transmitter's throughput is maximized irrespective of the relay's transmission energy profile, are optimal. Next, we consider the multiple access channel and the broadcast channel where the transmitters and the receivers harvest energy from nature, and characterize the maximum departure region. In all multi-user settings considered, we decompose our problems into inner and outer problems. We solve the inner problems by exploiting the structure of the particular model, and solve the outer problems by water-filling algorithms.

2.2 Single-User Channel

As shown in Fig. 2.1, we have a transmitter and a receiver, both relying on energy harvested from nature. The time is slotted, and at the beginning of time slot $i \in \{1, \dots, N\}$, energies arrive at a given node ready to be used in the same slot or saved in a battery to be used in future slots. Let $\{E_i\}_{i=1}^N$ and $\{\bar{E}_i\}_{i=1}^N$ denote the energies harvested at each slot for the transmitter and the receiver, respectively, and let $\{p_i\}_{i=1}^N$ denote the transmitter's powers.

Without loss of generality, we assume that the time slot duration is normalized to one time unit. The physical layer is a Gaussian channel with zero-mean unit-variance noise. The objective is to maximize the total amount of data received *and decoded* by the receiver by the deadline N. Our setting is *offline* in the sense that all energy amounts are known prior to transmission.

The receiver must be able to decode the kth packet by the end of the kth

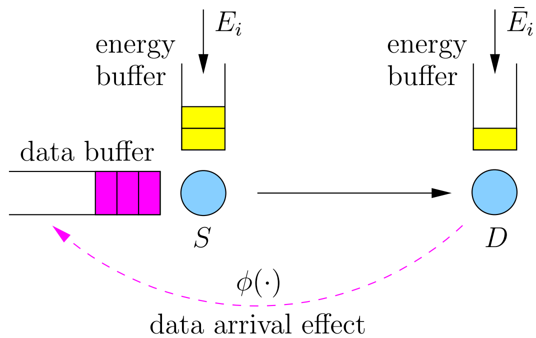


Figure 2.1: Single-user channel with an energy harvesting transmitter and an energy harvesting receiver.

slot. A transmitter transmitting at power p_i in the ith time slot will send at a rate $g(p_i) \triangleq \frac{1}{2}\log_2(1+p_i)$, for which the receiver will spend $\phi(g(p_i))$ amount of power to decode, where ϕ is generally an increasing convex function [37, 38, 42–45]. In the sequel, we will also focus on the specific cases of linear and exponential functions, where $\phi(r) = ar + b$, with $a, b \geq 0$, and $\phi(r) = c2^{dr} + e$, with $c, d \geq 0$ and $c + e \geq 0$. Continuing with a general convex increasing function ϕ, we have the following decoding causality constraints for the receiver:

$$\sum_{i=1}^{k} \phi(g(p_i)) \leq \sum_{i=1}^{k} \bar{E}_i, \quad k = 1, \dots, N \tag{2.1}$$

Therefore, the overall problem is formulated as:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{N} g(p_i) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_i \leq \sum_{i=1}^{k} E_i, \quad \forall k \\ & \sum_{i=1}^{k} \phi(g(p_i)) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \end{aligned} \tag{2.2}$$

where $\mathbf{p}$ denotes the vector of powers. Note that the problem above in general is not a convex optimization problem as (2.1) in general is a non-convex constraint since ϕ is a convex function while g is a concave function [77]. Applying the change of variables $g(p_i) = r_i$, and defining $f \triangleq g^{-1}$ (note that f is a convex function), we have

$$\begin{aligned} \max_{\mathbf{r}} \quad & \sum_{i=1}^{N} r_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} f(r_i) \leq \sum_{i=1}^{k} E_i, \quad \forall k \\ & \sum_{i=1}^{k} \phi(r_i) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \end{aligned} \tag{2.3}$$

which is now a convex optimization problem [77].

We note that the constraints in (2.3), i.e., $\sum_{i=1}^{k} \phi(r_i) \leq \sum_{i=1}^{k} \bar{E}_i$, place upper bounds on the rates of the transmitter by every slot k. This resembles the problem addressed in [1] with data packet arrivals during the communication session. In fact, when $\phi(r) = r$ and $\bar{E}_i = b_i$, where b_i is the amount of data arriving in slot i, these are exactly the data arrival constraints in [1]. A general convex ϕ generalizes this data arrival constraint. We characterize the solution of (2.3) in the following three lemmas and the theorem. The proofs rely on the convexity of f and ϕ generalizing the proof ideas in [1].

Lemma 2.1 *The optimal $\{r_i^*\}$ is monotonically increasing.*

Proof: Assume that there exists a time slot k such that $r_k^* > r_{k+1}^*$, and consider a new policy obtained by replacing both r_k^* and r_{k+1}^* by $\hat{r}_k = \hat{r}_{k+1} \triangleq \frac{r_k^*+r_{k+1}^*}{2}$, and observe that from the convexity of f and ϕ, we have

$$f(\hat{r}_k) + f(\hat{r}_{k+1}) \leq f(r_k^*) + f(r_{k+1}^*) \tag{2.4}$$

$$\phi(\hat{r}_k) + \phi(\hat{r}_{k+1}) \leq \phi(r_k^*) + \phi(r_{k+1}^*) \tag{2.5}$$

In addition, since both f and ϕ are monotonically increasing, we have $f\left(\hat{r}_k\right) \leq f\left(r_k^*\right)$, and $\phi\left(\hat{r}_k\right) \leq \phi\left(r_k^*\right)$. Therefore, the new policy is feasible, and can only save some energy either at the transmitter or at the receiver. This saved energy can be used to increase the rates in the upcoming time slots. Thus, the original policy cannot be optimal. ■

Lemma 2.2 *In the optimal policy, whenever the rate changes in a time slot, at least one of the following events occur: 1) the transmitter consumes all of its harvested energy in transmission, or 2) the receiver consumes all of its harvested energy in decoding, up to that time slot.*

Proof: Assume not, i.e., $r_k^* < r_{k+1}^*$ but both the transmitter and the receiver did not consume all their energies in the kth time slot. Then, we can always increase r_k^* and decrease r_{k+1}^* without conflicting the energy causality or the decoding causality constraints. By the convexity of f and ϕ, this modification would save some energy that can be used to increase the rates in the upcoming time slots. Therefore, the original policy cannot be optimal. ■

Lemma 2.3 *In the optimal policy, by the end of the transmission period, at least* *one of the following events occur: 1) the transmitter's total power consumption in transmission is equal to its total harvested energy, or 2) the receiver's total power consumption in decoding is equal to its total harvested energy.*

Proof: Assume that both conditions are not met. Then, we can increase the rate in the last time slot until either the transmitter, or the receiver, consumes all of its energy. This is always feasible and strictly increases the rate. ■

Theorem 2.1 *Let $\psi \triangleq \phi^{-1}$. A policy is optimal iff it satisfies the following*

$$r_n = \min\left\{ g\left(\frac{\sum_{j=1}^{i_n} E_j - \sum_{j=1}^{i_{n-1}} f(r_j)}{i_n - i_{n-1}} \right), \psi\left(\frac{\sum_{j=1}^{i_n} \bar{E}_j - \sum_{j=1}^{i_{n-1}} \phi(r_j)}{i_n - i_{n-1}} \right) \right\} \tag{2.6}$$

where

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{ g\left(\frac{\sum_{j=1}^{i} E_j - \sum_{j=1}^{i_{n-1}} f(r_j)}{i - i_{n-1}} \right), \psi\left(\frac{\sum_{j=1}^{i} \bar{E}_j - \sum_{j=1}^{i_{n-1}} \phi(r_j)}{i - i_{n-1}} \right) \right\} \tag{2.7}$$

with $i_0 = 0$, and $n = 1, \ldots, N$.

Proof: First, we prove that the optimal policy satisfies (2.6) and (2.7). We show this by contradiction. Let us assume that the optimal policy, that satisfies the necessary lemmas above, is not given by (2.6) and (2.7) and achieves a higher throughput. In particular, let us assume that it coincides with the policy given by (2.6) and (2.7) for all rates $\{r_i\}_{i=1}^{n-1}$ but has a different value for r_n. Let us denote the points of rate increase of this policy by $\{i_k\}$. Thus, there must exist a time index $i' > i_{n-1}$ such that

$$r_n > \min\left\{ g\left(\frac{\sum_{j=1}^{i'} E_j - \sum_{j=1}^{i_{n-1}} f(r_j)}{i' - i_{n-1}} \right), \psi\left(\frac{\sum_{j=1}^{i'} \bar{E}_j - \sum_{j=1}^{i_{n-1}} \phi(r_j)}{i' - i_{n-1}} \right) \right\} \tag{2.8}$$

and let us consider two different cases.

Assume that $i' < i_n$. If the transmitter's energy is the bottleneck at i', then r_n cannot be supported by the transmitter. On the other hand, if the receiver's energy is the bottleneck at i', then r_n cannot be supported by the receiver. Hence, r_n is not feasible in both cases. Now, assume that $i' > i_n$. Then, there will exist a duration $\subseteq [i_n + 1, i']$ where the rate has to decrease in order to satisfy feasibility. This violates the monotonicity property, and hence cannot be optimal.

Second, let us show sufficiency. We show this again by contradiction. Let us assume that the policy that satisfies (2.6) and (2.7) is not optimal. In particular, let us assume that there exists another policy $\{r_i'\}$ that coincides with it for all rates $\{r_i\}_{i=1}^{n-1}$ but has a different value for r_n. Since this new policy should have higher throughput, we have $r_n' > r_n$. Now, assume $i_n' > i_n$. Then, clearly r_n' is not feasible in the duration $[i_{n-1} + 1, i_n]$. On the other hand, if $i_n' < i_n$, then by the monotonicity property, all upcoming rates $\{r_i'\}$ for $i > i_n'$ can only be larger than r_n', which are all larger than r_n. This makes the new policy infeasible by the end of slot i_n since r_n consumes all feasible energy according to (2.6) and (2.7). Thus, the original policy is optimal. ■

Theorem 2.1 shows that decoding costs at the receiver are similar in effect to having a single-user channel with data arrivals during transmission and no decoding

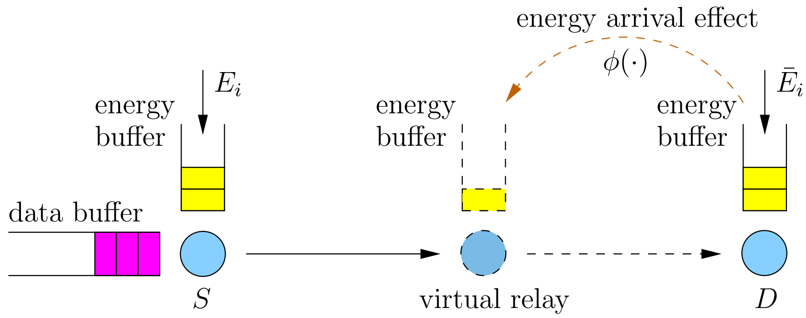


Figure 2.2: Decoding costs viewed as a virtual relay.

costs. This stems from the fact that the transmitter has to adapt its powers (and rates) in order to meet the decoding requirements at the receiver. Therefore, the receiver's harvested energies and the function ϕ control the amount of data the transmitter can send by any given point in time.

Alternatively, we can slightly change the single-user problem (2.3) by adding an extra variable $\bar{r}_i$ as follows

$$\begin{aligned}
\max_{\mathbf{r},\bar{\mathbf{r}}} \quad & \sum_{i=1}^{N} \bar{r}_i \\
\text{s.t.} \quad & \sum_{i=1}^{k} f(r_i) \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} \phi(\bar{r}_i) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \\
& \bar{r}_i \leq r_i, \quad \forall i
\end{aligned} \tag{2.9}$$

This gives the same solution as we will always have $\bar{r}_i^* = r_i^*$ satisfied for all i. Therefore, as shown in Fig. 2.2, we can view the single-user setting with an energy harvesting receiver, as a two-hop setting with a *virtual relay* between the transmitter and the receiver, with a non-energy harvesting receiver. To this end, we separate the decoding costs of the receiver, which are subject to energy harvesting constraints, as a relay which is subject to energy harvesting constraints in its transmissions, and consider the receiver as fully powered [12–17]. The receiver will only receive data if the relay has sufficient energy to forward it. In addition, this energy harvesting virtual relay has no data buffer, thus, its incoming data rate equals its outgoing data rate. The rate through this relay is controlled by $\bar{E}_i$ and ϕ. Thus, the decoding function ϕ puts a *generalized energy arrival effect* to this virtual relay, in a similar way that it puts a *generalized data arrival effect* to the transmitter through Theorem 2.1, as shown in Fig. 2.1.

It is worth mentioning that if we consider the special case where the receiver has no battery to store its energy, this will lead to the following decoding causality constraint

$$\phi(g(p_i)) \leq \bar{E}_i, \quad i = 1, \ldots, N \tag{2.10}$$

which, in view of the generalized data arrival interpretation, can be modeled as a time-varying upper bound on the transmitter's power in each slot

$$p_i^{\max} \triangleq f\left(\psi\left(\bar{E}_i\right)\right) \tag{2.11}$$

where $\psi(\bar{E}_i)$ is the maximum transmission rate of a packet that $\bar{E}_i$ can handle at the decoder, and $p_i^{\max}$ denotes its corresponding maximum transmit power. This problem has been considered in the general framework of [78], and in [6] for the spe-

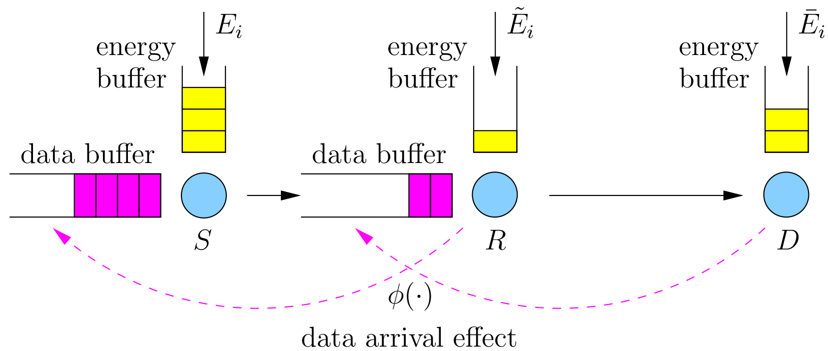


Figure 2.3: Two-hop energy harvesting system with both relay and destination decoding costs.

cial case of a constant maximum power constraint. One solution for this problem is to apply a backward water-filling algorithm that starts from the last slot backwards, where at each slot directional water-filling [3] is applied only on slots whose maximum power constraint is not satisfied with equality. This might cause some wastage of water if the maximum power constraints are tighter than the transmitter's energy causality constraints, which depends primarily on how the function ϕ relates the transmitter's and the receiver's energies.

2.3 Two-Hop Network

We now consider a two-hop network consisting of a single source-destination pair communicating through a relay, as depicted in Fig. 2.3. The relay is full duplex, and it uses a decode-and-forward protocol. The relay has a data buffer to receive its incoming packets from the source. At the beginning of slot i, energies in the amounts of E_i, $\tilde{E}_i$, and $\bar{E}_i$ arrive at the source, relay, and destination, respectively. Unused energies can be saved in their respective batteries.

Let r_i and $\tilde{r}_i$ be the rates of the source and the relay, respectively, in slot

i. Our goal is to maximize the total amount of data received *and decoded* at the destination by the deadline N. We impose decoding costs on both the relay and the destination. The problem is formulated as:

$$
\begin{aligned}
\max_{\mathbf{r},\tilde{\mathbf{r}}} \quad & \sum_{i=1}^{N} \tilde{r}_i \\
\text{s.t.} \quad & \sum_{i=1}^{k} f(r_i) \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} \phi(r_i) + f(\tilde{r}_i) \leq \sum_{i=1}^{k} \tilde{E}_i, \quad \forall k \\
& \sum_{i=1}^{k} \tilde{r}_i \leq \sum_{i=1}^{k} r_i, \quad \forall k \\
& \sum_{i=1}^{k} \phi(\tilde{r}_i) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k
\end{aligned} \tag{2.12}
$$

where the first constraint in (2.12) is the source transmission energy causality constraint, the second one is the relay decode-and-forward causality constraint, the third one is the data causality constraint at the relay, and the last one is the destination decoding causality constraint.

We first note that that if the relay did not have a data buffer, the source and the relay rates will need to be equal, i.e., $\tilde{r}_i = r_i$ for all i. In this case, the problem reduces to be a problem only in terms of the source rates, and could be solved by straightforward generalization of the single-user result in Theorem 2.1 considering three constraints instead of two. In a sense, this would be equivalent to taking the effects of decode-and-forward causality at the relay and decoding causality at the receiver back to the source as two different generalized data arrival effects. This can be further extended to multi-hop networks with relays having no data buffers by taking their constraint effects all the way back to the source.

In our setting, having a data buffer at the relay imposes non-obvious relationships among the source and the relay rates. To tackle this issue, we decompose the problem into inner and outer problems. In the inner problem, we solve for the source and relay rates after fixing a decoding power strategy for the relay node. By that we mean choosing the amounts of powers, $\{\delta_i\}_{i=1}^N$, the relay dedicates to decoding its incoming source packets. These amounts need to be feasible in the sense that $\sum_{i=1}^k \delta_i \leq \sum_{i=1}^k \tilde{E}_i$, $\forall k$. This decomposes the decode-and-forward causality constraint into the following two constraints:

$$\sum_{i=1}^{k} \phi\left(r_i\right) \leq \sum_{i=1}^{k} \delta_i, \qquad \sum_{i=1}^{k} f\left(\tilde{r}_i\right) \leq \sum_{i=1}^{k} \tilde{E}_i - \delta_i, \qquad \forall k \tag{2.13}$$

In the next lemmas and theorem, we characterize the solution of the inner problem. The proofs of the lemmas are extensions of the ones presented in [13] to the case of generalized data arrivals.

Lemma 2.4 *There exists an optimal increasing source rate policy for the inner problem.*

Proof: Assume that there exists a time slot k where $r_k > r_{k+1}$. We have two cases to consider. First, assume $\tilde{r}_k > \tilde{r}_{k+1}$. Let us define a new policy by replacing the kth and $k+1$st source and relay rates by $r' \triangleq \frac{r_k + r_{k+1}}{2}$, and $\tilde{r}' \triangleq \frac{\tilde{r}_k + \tilde{r}_{k+1}}{2}$, respectively. By the convexity of f and ϕ, and linearity of the data causality constraint, the new policy is feasible, and can only save some energy at the source or the relay. This energy can be used in later slots to achieve higher rates.

Now, assume $\tilde{r}_k \leq \tilde{r}_{k+1}$. We argue that the data arrival causality constraint is satisfied with strict inequality at time slot k. For if it were equality, we need to have $\tilde{r}_k \geq r_k$ and $\tilde{r}_{k+1} \leq r_{k+1}$, which leads to $r_k \leq \tilde{r}_k \leq \tilde{r}_{k+1} \leq r_{k+1}$, an obvious contradiction. Now, we can find a small enough $\epsilon > 0$, such that defining a new policy by replacing the kth and $k+1$st source rates by $r_k - \epsilon$ and $r_{k+1} + \epsilon$, respectively, we do not affect the relay rates. By the convexity of f and ϕ, the new policy is feasible, and can only save some energy at the source. This energy can be used in later slots to send more data to the relay, and hence, possibly increasing the relay rates, and the end-to-end throughput. ■

Lemma 2.5 *The optimal increasing source rate policy for the inner problem $\{r_i^*\}$ is given by the single-user problem solution in (2.6) and (2.7), where the transmitter's and the receiver's energies are given by $\{E_i\}$ and $\{\delta_i\}$, respectively.*

Proof: Let us denote the single-user solution by $\{r_i'\}$. Assume for contradiction that it is not optimal for the inner problem. In particular, let $\{r_i^*\}$ and $\{r_i'\}$ be equal for $i = 1, \ldots, k-1$, and differ on the kth slot. We again have two cases to consider. First, assume $r_k^* > r_k'$. In this case, since by Lemma 2.4, $\{r_i^*\}$ is increasing, by similar arguments as in the proof of Theorem 2.1, the policy $\{r_i^*\}$ will eventually not satisfy the source's energy causality or the relay's decoding causality constraints, at some time slot $j \geq k$. Hence, it cannot be optimal.

Now, assume $r_k^* < r_k'$. We argue that this shrinks the feasible set of the relay's rates. We show this by induction. By assumption of this case, it is true at time slot k, that we have $\sum_{i=1}^{k} r_i^* < \sum_{i=1}^{k} r_i'$. Now, assume it is true that for some time slot $j > k$ we have $\sum_{i=1}^{j} r_i^* < \sum_{i=1}^{j} r_i'$, and consider the $j+1$st time slot. If $r_{j+1}^* > r_{j+1}'$, then we are back to the previous case where this cannot be feasible eventually. Therefore, the feasible set of the relay's rates shrinks at time slot $j+1$, and hence, shrinks all over $k, \ldots, N$. Thus, this case cannot be optimal either. ■

Lemma 2.5 states that the optimal source policy is separable [12, 13] in the sense that the source maximizes its throughput to the relay irrespective of how the relay spends its transmission energy. This stems from the fact that the relay has an infinite data buffer to store its incoming source packets. Therefore, once we fix a decoding power strategy at the relay, we get separability. The following theorem, which is an extended version of Theorem 2.1, gives the optimal relay rates for the inner problem. The proof is similar to that of Theorem 2.1 and is omitted for brevity.

Theorem 2.2 *Given the optimal source rates $\{r_i^*\}$, the optimal relay rates for the inner problem is given by*

$$\tilde{r}_n^* = \min\left\{ g\left(\frac{\sum_{j=1}^{i_n} \tilde{E}_j - \delta_j - \sum_{j=1}^{i_{n-1}} f(\tilde{r}_j^*)}{i_n - i_{n-1}}\right), \right. \\ \left. \psi\left(\frac{\sum_{j=1}^{i_n} \bar{E}_j - \sum_{j=1}^{i_{n-1}} \phi(\tilde{r}_j^*)}{i_n - i_{n-1}}\right), \frac{\sum_{j=1}^{i_n} r_j^* - \sum_{j=1}^{i_{n-1}} r_j^*}{i_n - i_{n-1}} \right\} \tag{2.14}$$

where i_n is the $\arg\min$ *of the expression in (2.14) as in (2.6)-(2.7), and $i_0 = 0$.*

Denoting the solution of the inner problem by $R(\boldsymbol{\delta})$, we now find the optimal relay decoding strategy $\{\delta_i^*\}$ by solving the following outer problem:

$$\begin{aligned} \max_{\boldsymbol{\delta}} \quad & R(\boldsymbol{\delta}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \delta_i \leq \sum_{i=1}^{k} \tilde{E}_i, \quad \forall k \end{aligned} \tag{2.15}$$

We have the following lemma regarding the outer problem.

Lemma 2.6 *$R(\boldsymbol{\delta})$ is a concave function.*

Proof: Consider two decoding power strategies $\boldsymbol{\delta}_1$, $\boldsymbol{\delta}_2$, and let $\{\mathbf{r}_1, \tilde{\mathbf{r}}_1\}$, $\{\mathbf{r}_2, \tilde{\mathbf{r}}_2\}$ be their corresponding source and relay optimal inner problem rates, respectively. Let $\boldsymbol{\delta}_\theta \triangleq \theta\boldsymbol{\delta}_1 + (1-\theta)\boldsymbol{\delta}_2$, for some $0 \leq \theta \leq 1$, and consider the rate policy defined by $\mathbf{r}_\theta \triangleq \theta\mathbf{r}_1 + (1-\theta)\mathbf{r}_2$, and $\tilde{\mathbf{r}}_\theta \triangleq \theta\tilde{\mathbf{r}}_1 + (1-\theta)\tilde{\mathbf{r}}_2$, for the source, and the relay, respectively. By the convexity of f and ϕ, the policy $\{\mathbf{r}_\theta, \tilde{\mathbf{r}}_\theta\}$ is feasible for the decoding strategy $\boldsymbol{\delta}_\theta$. Therefore, we have

$$R(\boldsymbol{\delta}_\theta) \geq \sum_{i=1}^{N} \tilde{r}_{\theta i} = \theta R(\boldsymbol{\delta}_1) + (1-\theta) R(\boldsymbol{\delta}_2) \tag{2.16}$$

proving the concavity of $R(\boldsymbol{\delta})$. ■

Therefore, the outer problem is a convex optimization problem [77]. We propose a water-filling algorithm to solve the outer problem [20]. We first note that $R(\boldsymbol{\delta})$ does not possess any monotonicity properties in the feasible region. For instance, $R(\tilde{\mathbf{E}}) = R(\mathbf{0}) = 0$, while $R(\boldsymbol{\delta})$ is strictly positive for some $\boldsymbol{\delta}$ in between. Thus, at the optimal relay decoding power strategy, not all the relay's decoding energy will be exhausted. To this end, we add an extra $N+1$st slot where we can possibly discard some energy. We start by filling up each slot by its corresponding energy/water level and we leave the extra $N+1$st slot initially empty. Meters are put in between bins to measure the amount of water passing. We let water flow to the right only if this increases the objective function. After each iteration, water can be called back if this increases the objective function. All the amount of water that is in the extra slot is eventually discarded, but may be called back also during the iterations. Since with each water flow the objective function monotonically increases, problem feasibility is maintained throughout the process, and due to the convexity of the problem, the algorithm converges to the optimal solution.

2.4 Multiple Access Channel

We now consider a two-user Gaussian MAC as shown in Fig. 2.4. The two transmitters harvest energy in amounts $\{E_{1i}\}_{i=1}^{N}$ and $\{E_{2i}\}_{i=1}^{N}$, respectively, and the receiver harvests energy in amounts $\{\bar{E}_i\}_{i=1}^{N}$. The receiver noise is with zero-mean and unit-variance. The capacity region for this channel is given by [49]:

$$\begin{aligned} r_1 &\leq g(p_1) \\ r_2 &\leq g(p_2) \\ r_1 + r_2 &\leq g(p_1 + p_2) \end{aligned} \tag{2.17}$$

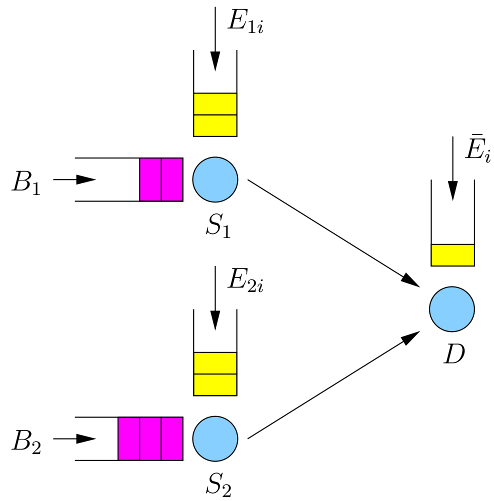


Figure 2.4: Two-user MAC with energy harvesting transmitters and receiver.

where p_1 and p_2 are the powers used by the first and the second transmitter, respectively.

In addition to the usual energy harvesting causality constraints on the transmitters [5], we impose a receiver decoding cost. We note that there can be different ways to impose this constraint depending on how the receiver employs the decoding procedure. In the next two sub-sections, we consider two kinds of decoding procedures, namely, simultaneous decoding, and successive decoding [46, 49]. Changing the decoding model affects the optimal power allocation for both users so as to adapt to how the receiver spends its power.

2.4.1 Simultaneous Decoding

In this case, the two transmitters can only send at rates whose sum can be decoded at the receiver. A power control policy $\{p_{1i}, p_{2i}\}_{i=1}^N$ is feasible if the following are satisfied:

$$\sum_{i=1}^{k} p_{1i} \le \sum_{i=1}^{k} E_{1i}, \quad \forall k$$
$$\sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} E_{2i}, \quad \forall k$$
$$\sum_{i=1}^{k} \phi\left(g\left(p_{1i}+p_{2i}\right)\right) \le \sum_{i=1}^{k} \bar{E}_{i}, \quad \forall k \tag{2.18}$$

From here on, we assume a specific structure for the decoding function ϕ for mathematical tractability and ease of presentation. In particular, we assume that it is exponential with parameters $c = 1$, $d = 2$ and $e = -1$, i.e., $\phi(r) = g^{-1}(r) = 2^{2r} - 1$. Let B_j denote the total departed bits from the jth user by time slot N. Our aim is to characterize the *maximum departure region*, $\mathcal{D}(N)$, which is the region of (B_1, B_2) the transmitters can depart by time slot N, through a feasible policy. The following lemmas characterize this region [5].

Lemma 2.7 *The maximum departure region, $\mathcal{D}(N)$, is the union of all (B_1, B_2), over all feasible policies $\{p_{1i}, p_{2i}\}_{i=1}^{N}$, where for any fixed power policy, (B_1, B_2) satisfy*

$$B_1 \le \sum_{i=1}^{N} g(p_{1i})$$
$$B_2 \le \sum_{i=1}^{N} g(p_{2i})$$
$$B_1 + B_2 \le \sum_{i=1}^{N} g(p_{1i}+p_{2i}) \tag{2.19}$$

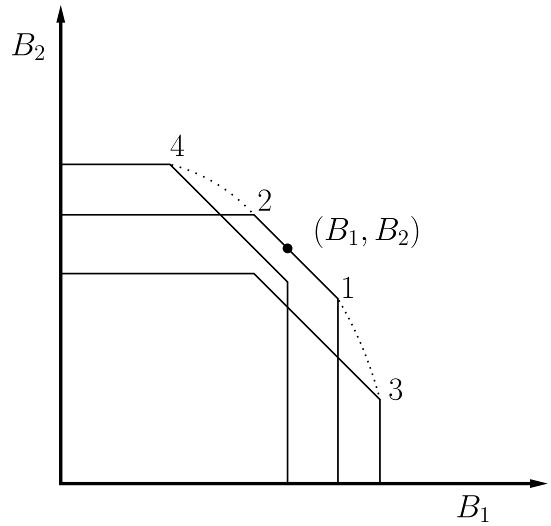


Figure 2.5: Departure region of a two-user MAC.

Lemma 2.8 *$\mathcal{D}(N)$ is a convex region.*

Each point on the boundary of $\mathcal{D}(N)$, see Fig. 2.5, can be characterized by solving a weighted sum rate maximization problem subject to feasibility conditions (2.18). Let μ_1 and μ_2 be the non-negative weights for the first and the second user rates, respectively. Assuming without loss of generality that $\mu_1 > \mu_2$, and defining $\mu \triangleq \frac{\mu_2}{\mu_1 - \mu_2}$, we then need to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p}_1, \mathbf{p}_2} \quad & \sum_{i=1}^{N} g(p_{1i}) + \mu \sum_{i=1}^{N} g(p_{1i} + p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\ & \sum_{i=1}^{k} p_{1i} + p_{2i} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \end{aligned} \tag{2.20}$$

We note that the above problem resembles the one formulated in [20] for a diamond channel with energy cooperation. First, we state a necessary condition of optimality for the above problem.

Lemma 2.9 *In the optimal solution for (2.20), by the end of the transmission period, at least one of the following occur: 1) both transmitters consume all of their harvested energies in transmission, or 2) the receiver consumes all of its harvested energy in decoding.*

Proof: Assume without loss of generality that transmitter 1 does not consume all of its energies in transmission, and that the receiver also does not consume all of its energies in decoding. Then, we can always increase the value of p_{1N} until either transmitter 1 or the receiver consume their energies. This strictly increases the objective function. ■

We decompose the optimization problem (2.20) into two nested problems. First, we solve for $\mathbf{p}_2$ in terms of $\mathbf{p}_1$, and then solve for $\mathbf{p}_1$. Let us define the following inner problem:

$$\begin{aligned} G(\mathbf{p}_1) \triangleq \max_{\mathbf{p}_2} \quad & \sum_{i=1}^{N} g(p_{1i} + p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} Q_i, \quad \forall k \end{aligned} \tag{2.21}$$

where the modified energy levels Q_i are defined as follows:

$$Q_i = M_i - M_{i-1},$$

$$M_i = \min\left\{\sum_{j=1}^{i} E_{2j}, \sum_{j=1}^{i} \bar{E}_j - p_{1j}\right\}, \quad M_0 = 0 \tag{2.22}$$

Then, we have the following lemma.

Lemma 2.10 *$G(\mathbf{p}_1)$ is a decreasing concave function in $\mathbf{p}_1$.*

Proof: G is a decreasing function of $\mathbf{p}_1$ since the feasible set shrinks with $\mathbf{p}_1$. To show concavity, let us choose two points $\mathbf{p}_1^{(1)}$ and $\mathbf{p}_1^{(2)}$, and take their convex combination $\mathbf{p}_1^{\theta} = \theta\mathbf{p}_1^{(1)} + (1-\theta)\mathbf{p}_1^{(2)}$ for some $0 \le \theta \le 1$. Let $\mathbf{p}_2^{(1)}$ and $\mathbf{p}_2^{(2)}$ denote the solutions of the inner problem (2.21) at $\mathbf{p}_1^{(1)}$ and $\mathbf{p}_1^{(2)}$, respectively. Now, let $\mathbf{p}_2^{\theta} \triangleq \theta\mathbf{p}_2^{(1)} + (1-\theta)\mathbf{p}_2^{(2)}$, and observe that, from the linearity of the constraint set, $\mathbf{p}_2^{\theta}$ is feasible with respect to $\mathbf{p}_1^{\theta}$. Therefore, we have

$$\begin{aligned}
G\left(\mathbf{p}_1^{\theta}\right) &\ge \sum_{i=1}^{N} g\left(p_{1i}^{\theta} + p_{2i}^{\theta}\right) \\
&\ge \sum_{i=1}^{N} \theta g\left(p_{1i}^{(1)} + p_{2i}^{(1)}\right) + (1-\theta) g\left(p_{1i}^{(2)} + p_{2i}^{(2)}\right) \\
&= \theta G\left(\mathbf{p}_1^{(1)}\right) + (1-\theta) G\left(\mathbf{p}_1^{(2)}\right)
\end{aligned} \tag{2.23}$$

where the second inequality follows from the concavity of g. ■

We observe that the inner problem (2.21) is a single-user energy harvesting maximization problem with fading, whose solution is via directional water-filling of $\{Q_i\}_{i=1}^N$ over the inverse of the fading levels $\{1 + p_{1i}\}_{i=1}^N$ as presented in [3]. Next, we solve the outer problem given by:

$$\begin{aligned}
\max_{\mathbf{p}_1} \quad & \mu G\left(\mathbf{p}_1\right) + \sum_{i=1}^{N} g(p_{1i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} T_i, \quad \forall k
\end{aligned} \tag{2.24}$$

where we define the water levels $T_i = L_i - L_{i-1}$, with $L_i = \min\left\{\sum_{j=1}^{i} E_{1j}, \sum_{j=1}^{i} \bar{E}_j\right\}$, and $L_0 = 0$. The minimum is added to ensure the feasibility of the inner problem. Note that, by Lemma 2.10, the outer problem is a convex optimization problem [77]. We first note that at the optimal policy, first user's modified energies $\{T_i\}$ need not be fully utilized by the end of transmission. This is because the objective function is not increasing in $\mathbf{p}_1$. To this end, we use the iterative water-filling algorithm for the outer problem proposed in Section 2.3 to solve this outer problem. Since the problem is convex, iterations converge to the optimal solution.

Note that the above formulation obtains the dotted points in the curved portion of the departure region in Fig. 2.5. Specific points in the departure region, e.g., points 1 and 3 in Fig. 2.5, can be found by specific schemes [79], by solving the problem for the cases $\mu_1 = \mu_2$ and $\mu_1\mu_2 = 0$.

2.4.2 Successive Cancelation Decoding

We now let the receiver employ successive decoding, where it aims at decoding the corner points, and then uses time sharing if necessary to achieve the desired rate pair [46, 49]. For instance, if the system is operating at its lower corner point, then the receiver first decodes the message of the second user, by treating the first user's signal as noise, then decodes the message of the first user, after subtracting the second user's signal from its received signal. For $\mu_1 > \mu_2$, we are always at a lower corner point at every time slot, and therefore the weighted sum rate maximization problem can be formulated as:

$$\begin{aligned}
\max_{\mathbf{p}_1,\mathbf{p}_2} \quad & \mu_1 \sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g\left(\frac{p_{2i}}{1+p_{1i}}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{1i} + \frac{p_{2i}}{1+p_{1i}} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k
\end{aligned} \tag{2.25}$$

where the last inequality comes from the fact that the receiver is decoding the second user's message first by treating the first user's signal as noise, and thereby spends $\phi\left(g\left(\frac{p_{2i}}{1+p_{1i}}\right)\right)$ amount of energy to decode this message, and then spends $\phi\left(g\left(p_{1i}\right)\right)$ amount of energy to decode the first user's message after subtracting the second user's signal.

Observe that the last constraint, i.e., the decoding causality constraint, is non-convex. Therefore, one might need to invoke the time-sharing principle in order to fully characterize the boundary of the maximum departure region. In terms of the rates the problem can be written as:

$$\begin{aligned}
\max_{\mathbf{r}_1,\mathbf{r}_2} \quad & \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \\
\text{s.t.} \quad & \sum_{i=1}^{k} 2^{2r_{1i}} - 1 \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} 2^{2r_{1i}} \left(2^{2r_{2i}} - 1\right) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{2i}} - 2 \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k
\end{aligned} \tag{2.26}$$

which is a non-convex problem due to the second user's energy causality constraint. In fact, the above problem is a signomial program, a generalized form of a geometric program, where posynomials can have negative coefficients [77]. Next, we use the idea of successive convex approximation [47] to provide an algorithm that converges to a local optimal solution.

By applying the change of variables $x_{ji} \triangleq 2^{2r_{ji}} - 1$, $j = 1, 2$, and some algebraic manipulations:

$$\begin{aligned}
\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{t}_1,\mathbf{t}_2} \quad & \sum_{i=1}^{N} t_{1i}^{-\mu_1} t_{2i}^{-\mu_2} \\
\text{s.t.} \quad & \sum_{i=1}^{k} x_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} (1 + x_{1i})\, x_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} x_{1i} + x_{2i} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \\
& t_{1i} \leq 1 + x_{1i}, \quad \forall i
\end{aligned}$$

$$t_{2i} \leq 1 + x_{2i}, \quad \forall i \tag{2.27}$$

Now, the problem looks very similar to a geometric program except for the last two sets of constraints. These constraints are written in the form of a monomial less than a posynomial, which will not allow us to write the problem in convex form by the usual geometric programming transformations [77]. We will follow an approach introduced in [48] in order to iteratively approximate the posynomials on the right hand side by monomials, and thereby reaching a geometric program that can be efficiently solved [77]. Approximations should be chosen carefully such that iterations converge to a local optimum solution of the original problem [47]. Towards that, we use the arithmetic-geometric mean inequality to write:

$$1 + x \geq \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{x}{1-\alpha}\right)^{1-\alpha} \triangleq u(x;\alpha) \tag{2.28}$$

which holds for $0 \leq \alpha \leq 1$. In particular, equality holds at a point $x_k \geq 0$ if we choose $\alpha = \frac{1}{1+x_k}$. Therefore, the monomial function $u(x;\alpha_k)$ approximates the posynomial function $1 + x$ at $x = x_k$. Substituting this approximation, we obtain that at the $k+1$st iteration, we need to solve the following geometric program:

$$\begin{aligned}
\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{t}_1,\mathbf{t}_2} \quad & \sum_{i=1}^{N} t_{1i}^{-\mu_1} t_{2i}^{-\mu_2} \\
\text{s.t.} \quad & \sum_{i=1}^{k} x_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} (1 + x_{1i})\, x_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k
\end{aligned}$$

$$
\begin{aligned}
&\sum_{i=1}^{k} x_{1i} + x_{2i} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k \\
&\frac{t_{1i}}{u\left(x_{1i}; \alpha_{1i}^{(k)}\right)} \leq 1, \quad \forall i \\
&\frac{t_{2i}}{u\left(x_{2i}; \alpha_{2i}^{(k)}\right)} \leq 1, \quad \forall i
\end{aligned} \tag{2.29}
$$

where $\alpha_{ji}^{(k)} \triangleq \frac{1}{1+x_{ji}^{(k)}}$, $j = 1, 2$, and $x_{ji}^{(k)}$ is the solution of the kth iteration. We pick an initial feasible point $\left(\mathbf{x}_1^{(0)}, \mathbf{x}_2^{(0)}\right)$ and run the iterations. The choice of the approximating monomial function u satisfies the conditions of convergence stated in [47], and therefore, the iterative solution of problem (2.29) converges to a point $(\mathbf{x}_1^*, \mathbf{x}_2^*)$ that is local optimal for problem (2.25). Finally, we get the original power allocations by substituting $p_{1i}^* = x_{1i}^*$, and $p_{2i}^* = (x_{1i}^* + 1)\, x_{2i}^*$.

2.5 Broadcast Channel

We now consider a two-user Gaussian BC with energy harvesting transmitter and receivers as shown in Fig. 2.6. Energies arrive in amounts E_i, $\bar{E}_{1i}$, and $\bar{E}_{2i}$, at the transmitter, and the receivers 1 and 2, respectively. By superposition coding [49], the weaker user is required to decode its message while treating the stronger user's interference as noise. While the stronger user is required to decode both messages successively by first decoding the weaker user's message, and then subtracting it to decode its own. The receiver noises have variances 1 and $\sigma^2 > 1$.

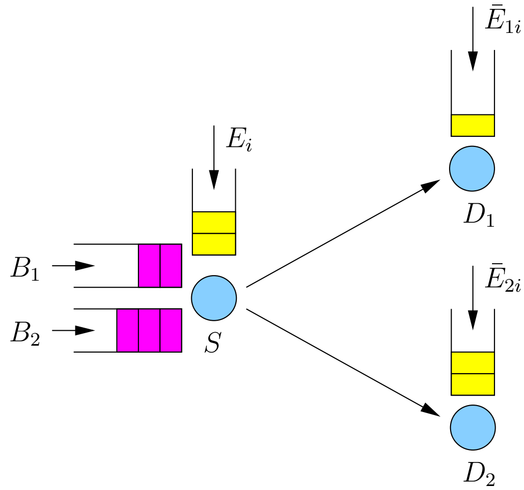


Figure 2.6: Two-user BC with energy harvesting transmitter and receivers.

Under a total transmit power P, the capacity region of the Gaussian BC is [49]:

$$r_1 \leq \frac{1}{2}\log_2\left(1+\alpha P\right), \quad r_2 \leq \frac{1}{2}\log_2\left(1+\frac{(1-\alpha)P}{\alpha P+\sigma^2}\right) \tag{2.30}$$

working on the boundary of the capacity region we have:

$$P = \left(\sigma^2 - 1\right)2^{2r_2} + 2^{2(r_1+r_2)} - \sigma^2 \triangleq F\left(r_1, r_2\right) \tag{2.31}$$

where $F(r_1, r_2)$ is the minimum power needed by the transmitter to achieve rates r_1 and r_2. Note that F is an increasing convex function of both rates.

As in the MAC case, the goal here is to characterize the maximum departure region:

$$\begin{aligned} \max_{\mathbf{r_1},\mathbf{r_2}} \quad & \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \\ \text{s.t.} \quad & \sum_{i=1}^{k} F\left(r_{1i}, r_{2i}\right) \leq \sum_{i=1}^{k} E_i, \quad \forall k \end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^{k} \phi(r_{1i} + r_{2i}) \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k \\
&\sum_{i=1}^{k} \phi(r_{2i}) \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k
\end{aligned} \tag{2.32}$$

where the first constraint in (2.32) is the source transmission energy causality constraint, and second and third constraints are the decoding causality constraints at the stronger and weaker receivers, respectively. Here also, we take the decoding cost function ϕ to be $\phi(r) = 2^{2r} - 1$.

By virtue of superposition coding, we see that, in the optimization problem in (2.32), the decoding causality constraint of the stronger user is a function of both rates intended for the two users, as it is required to decode both messages. While the decoding causality constraint for the weaker user is a function of its own rate only. By the convexity of F and ϕ, the maximum departure region is convex, and thus the weighted sum rate maximization in (2.32) is sufficient to characterize its boundary [7]. In addition, the optimization problem in (2.32) is convex [77].

We note that a related problem has been considered in [10], where the authors characterized transmission completion time minimization policies for a BC setting with data arrivals during transmission. There, the solution is found by sequentially solving an equivalent energy consumption minimization problem until convergence. Their solution is primarily dependent on Newton's method [77]. Some structural insights are also presented about the optimal solution. In our setting, we consider the case with receiver side decoding costs, and generalize the data arrivals concept by considering the convex function ϕ. In addition, our formulation imposes further interactions between the strong and the weak user's data, by allowing a constraint (strong user's) that is put on the sum of both rates, instead of on individual rates.

We characterize the solution of the problem according to the relation between μ_1 and μ_2 as follows. If $\mu_1 \geq \mu_2$, then due to the degradedness of the second user, it is optimal to put all power into the first user's message. This way, the problem reduces to a single-user problem:

$$\begin{aligned} \max_{\mathbf{r_1}} \quad & \sum_{i=1}^{N} r_{1i} \\ \text{s.t.} \quad & \sum_{i=1}^{k} 2^{2r_{1i}} - 1 \leq \sum_{i=1}^{k} W_i, \quad \forall k \end{aligned} \tag{2.33}$$

where the modified energy levels $\{W_i\}$ are defined as follows:

$$\begin{aligned} W_i &= L_i - L_{i-1}, \\ L_i &= \min\left\{\sum_{j=1}^{i} E_j, \sum_{j=1}^{i} \bar{E}_{1j}\right\}, \quad L_0 = 0 \end{aligned} \tag{2.34}$$

On the other hand, if $\mu_1 < \mu_2$, then we need to investigate the necessary KKT optimality conditions [77]. We write the Lagrangian for the problem (2.32) as follows:

$$\begin{aligned} \mathcal{L} = & -\mu_1 \sum_{i=1}^{N} r_{1i} - \mu_2 \sum_{i=1}^{N} r_{2i} \\ & + \sum_{i=1}^{N} \lambda_i \left(\sum_{j=1}^{i} \left(\sigma^2 - 1\right) 2^{2r_{2j}} + 2^{2(r_{1j}+r_{2j})} - \sigma^2 - E_j \right) \\ & + \sum_{i=1}^{N} \nu_{1i} \left(\sum_{j=1}^{i} 2^{2(r_{1j}+r_{2j})} - 1 - \bar{E}_{1j} \right) \end{aligned}$$

$$+\sum_{i=1}^{N}\nu_{2i}\left(\sum_{j=1}^{i}2^{2r_{2j}}-1-\bar{E}_{2j}\right)$$
$$-\sum_{i=1}^{N}\eta_{1i}r_{1i}-\sum_{i=1}^{N}\eta_{2i}r_{2i} \tag{2.35}$$

Taking the derivative with respect to r_{1i} and r_{2i} and equating to zero, we obtain:

$$2^{2(r_{1i}+r_{2i})} = \frac{\mu_1+\eta_{1i}}{\sum_{j=i}^{N}\lambda_j+\nu_{1j}} \tag{2.36}$$
$$2^{2r_{2i}} = \frac{\mu_2-\mu_1+\eta_{2i}-\eta_{1i}}{\sum_{j=i}^{N}(\sigma^2-1)\lambda_j+\nu_{2j}} \tag{2.37}$$

along with the complementary slackness conditions:

$$\lambda_i\left(\sum_{j=1}^{i}\left(\sigma^2-1\right)2^{2r_{2j}}+2^{2(r_{1j}+r_{2j})}-\sigma^2-E_j\right)=0,\quad \forall i$$
$$\nu_{1i}\left(\sum_{j=1}^{i}2^{2(r_{1j}+r_{2j})}-1-\bar{E}_{1j}\right)=0,\quad \forall i$$
$$\nu_{2i}\left(\sum_{j=1}^{i}2^{2r_{2j}}-1-\bar{E}_{2j}\right)=0,\quad \forall i$$
$$\eta_{1i}r_{1i}=0,\quad \eta_{2i}r_{2i}=0,\quad \forall i \tag{2.38}$$

From here, we state the following lemmas

Lemma 2.11 *The sum rate $\{r^*_{1i}+r^*_{2i}\}$ is monotonically increasing.*

Proof: We prove this by contradiction. Assume that there exists some time slot k such that $r_{1k}+r_{2k}>r_{1(k+1)}+r_{2(k+1)}$. From (2.36), since the denominator cannot increase, the numerator has to decrease for the sum rate to decrease, i.e., $\eta_{1k}>$

$\eta_{1(k+1)} \geq 0$. From complementary slackness, we must have $r_{1k} = 0$. Therefore, in order for the sum rate to decrease we must have $r_{2k} > r_{2(k+1)}$, which in turn leads to $\eta_{2k} = 0$.

From (2.37), we know that for the weak user's rate to decrease, the numerator has to decrease, i.e., we must have $\eta_{2(k+1)} - \eta_{1(k+1)} < \eta_{2k} - \eta_{1k}$. Since $\eta_{2k} = 0$, this is equivalent to having $\eta_{2(k+1)} < \eta_{1(k+1)} - \eta_{1k}$. However, we know from above that $\eta_{1k} > \eta_{1(k+1)}$, i.e., $\eta_{2(k+1)} < 0$, an obvious contradiction by non-negativity of the Lagrange multipliers. ■

Lemma 2.12 *The weak user's rate $\{r^*_{2i}\}$ is monotonically increasing.*

Proof: We also prove this by contradiction. Assume that there exists some time slot k such that $r_{2k} > r_{2(k+1)}$. From (2.37), since the denominator cannot increase, the numerator has to decrease for the weak user's rate to decrease, i.e., $\eta_{2(k+1)} - \eta_{1(k+1)} < \eta_{2k} - \eta_{1k}$. Let us consider two different cases. First, assume $\eta_{1k} \geq \eta_{1(k+1)}$. Therefore, we must have $\eta_{2k} > \eta_{2(k+1)} + (\eta_{1k} - \eta_{1(k+1)}) \geq 0$, and thus, by complementary slackness, $r_{2k} = 0$, and hence, $r_{2(k+1)}$ cannot be less since it cannot drop below zero. Now, assume $\eta_{1k} < \eta_{1(k+1)}$. In this case, by complementary slackness, $r_{1(k+1)} = 0$. By Lemma 2.11, we have $r_{1k} + r_{2k} \leq r_{2(k+1)}$, i.e., $r_{2(k+1)} \geq r_{2k}$, which is a contradiction. ■

With the change of variables: $p_{ti} \triangleq 2^{2(r_{1i}+r_{2i})} - 1$, and $p_{2i} \triangleq 2^{2r_{2i}} - 1$, (2.32) becomes:

$$\max_{\mathbf{p}_t, \mathbf{p}_2} \quad \mu_1 \sum_{i=1}^{N} g\left(p_{ti}\right) + \left(\mu_2 - \mu_1\right) \sum_{i=1}^{N} g\left(p_{2i}\right)$$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{k} (\sigma^2 - 1) p_{2i} + p_{ti} \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k \\
& p_{ti} \geq p_{2i}, \quad \forall i
\end{aligned} \tag{2.39}
$$

We now decompose the above problem into an inner and an outer problem and iterate between them until convergence. First, we fix the value of $\mathbf{p}_2$, and solve the following inner problem:

$$
\begin{aligned}
H(\mathbf{p}_2) \triangleq \max_{\mathbf{p}_t} \quad & \sum_{i=1}^{N} g(p_{ti}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} V_i, \quad \forall k \\
& p_{ti} \geq p_{2i}, \quad \forall i
\end{aligned} \tag{2.40}
$$

where the modified energy levels V_i are defined as follows

$$
\begin{aligned}
& V_i = B_i - B_{i-1}, \\
& B_i = \min \left\{ \sum_{j=1}^{i} \bar{E}_{1j}, \sum_{j=1}^{i} E_j - (\sigma^2 - 1) p_{2j} \right\}, \quad B_0 = 0
\end{aligned} \tag{2.41}
$$

We have the following lemma for this inner problem whose proof is similar to that of Lemma 2.10.

Lemma 2.13 $H(\mathbf{p}_2)$ *is a decreasing concave function in* $\mathbf{p}_2$.

We note that the $\mathbf{p}_2$ vector serves as a *minimum power constraint* to the inner problem. Let us write the Lagrangian for the inner problem:

$$\mathcal{L} = -\sum_{i=1}^{N} g(p_{ti}) + \sum_{j=1}^{N} \lambda_j \left(\sum_{i=1}^{j} p_{ti} - \sum_{i=1}^{j} V_i \right) - \sum_{i=1}^{N} \mu_i \left(p_{ti} - p_{2i} \right) \tag{2.42}$$

Taking the derivative with respect to p_{ti} and equating to zero, we obtain:

$$p_{ti} = \frac{1}{\sum_{j=i}^{N} \lambda_j - \mu_i} - 1 \tag{2.43}$$

First, let us examine the necessary conditions for the optimal power to increase, i.e., $p_{ti} < p_{t(i+1)}$. This occurs iff $\lambda_i + \mu_{i+1} > \mu_i \geq 0$. Thus, we must either have $\lambda_i > 0$ which means that, by the complementary slackness, we have to consume all the available energy by the end of the ith slot. Or, we have $\mu_{i+1} > 0$ which means that $p_{t(i+1)} = p_{2(i+1)}$. Next, let us examine the necessary conditions for the optimal power to decrease, i.e., $p_{ti} > p_{t(i+1)}$. This occurs iff $\mu_i > \lambda_i + \mu_{i+1} \geq 0$, and therefore, we must have $p_{ti} = p_{2i}$.

We note from Lemmas 2.11 and 2.12 that both $\{p_{2i}^*\}$ and $\{p_{ti}^*\}$ are monotonically increasing. Therefore, we only focus on fixing an increasing feasible $\mathbf{p}_2$. This, when combined with the above conditions, leads to the following lemma.

Lemma 2.14 *For a fixed increasing* $\mathbf{p}_2$*, the optimal solution* $\mathbf{p}_t$ *of the inner problem is also increasing.*

Proof: By the KKT conditions stated above, if we have $p_{ti} > p_{t(i+1)}$, then we must have $p_{ti} = p_{2i}$. Thus, we will have $p_{t(i+1)} < p_{ti} = p_{2i} \leq p_{2(i+1)}$, i.e., the minimum

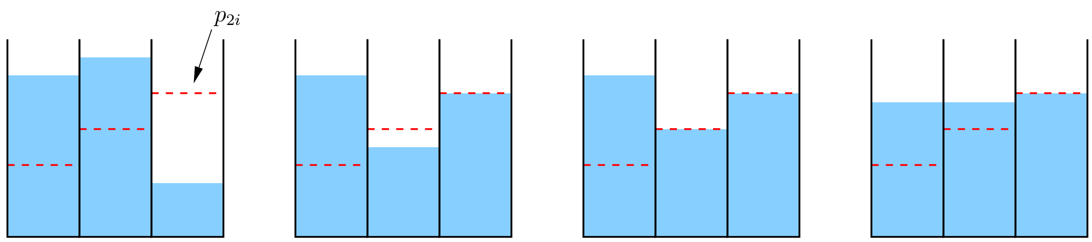
Algorithm 1

Initialize the status of each bin $S_i = V_i$
Mark bins by their minimum power requirements $\{p_{2i}\}_{i=1}^{N}$
Set $k = N$
while $k \geq 1$ **do**
  if $S_k < p_{2k}$ **then**
    Pour water into the kth bin from previous bins, in a backward manner, until equality holds
  else
    Do directional water-filling over the current and upcoming bins $\{k, k+1, \ldots, N\}$
  end if
  Update the status of each bin
  $k \leftarrow k - 1$
end while

power constraint is not satisfied at the $i + 1$st slot. ■

Therefore, choosing an increasing $\mathbf{p}_2$ in the outer problem ensures that the inner problem's solution $\mathbf{p}_t$ is also increasing, and thereby, satisfies the conditions of Lemmas 2.11 and 2.12. We solve the inner problem by Algorithm 1. The algorithm's main idea is to equalize the powers as much as possible via directional water-filling [3] while satisfying the minimum power requirements.

Observe that the algorithm gives a feasible power profile; it examines each slot, and does not move backwards unless the minimum power requirement is satisfied. If there is an excess energy above the minimum, say at slot k, it performs directional water-filling which will occur if $S_k > S_{k+1}$ (let us consider water-filling only over two bins for simplicity). Since the minimum power requirement vector $\mathbf{p}_2$ is increasing, after equalizing the energies the updated status will satisfy $S_k = S_{k+1} > p_{2(k+1)} \geq p_{2k}$, i.e., the minimum power requirement is always satisfied if directional water-filling occurs. Also observe that the algorithm cannot give out a decreasing power



1: Initialization | 2: Filling last bin first | 3: Filling middle bin | 4: Directional water-filling from first bin

Figure 2.7: Numerical example for the BC inner problem.

profile since $\mathbf{p}_2$ is increasing.

According to the KKT conditions, the power increases from slot k to slot $k+1$ only if $p_{t(k+1)} = p_{2(k+1)}$ or the total energy is consumed by slot k. We see that the algorithm satisfies this condition. Power increases only if directional water-filling is not applied at slot k, which means that either some of the water was poured forward in the previous iteration to satisfy $p_{t(k+1)} = p_{2(k+1)}$, or no water was poured which means that all energy is consumed by slot k.

A numerical example for a three-slot system is shown in Fig. 2.7. The minimum power requirements are shown by red dotted lines in each bin. According to the algorithm, we first initialize by pouring all the amounts of water in their corresponding bins. We begin by checking the last bin, and we see that it needs some extra water to satisfy its minimum power requirement. Thus, we pour water forward from the middle bin until the minimum power requirement of the last bin is satisfied with equality. This causes a deficiency in the middle bin, and therefore, we pour water forward from the first bin until the minimum power requirement of the middle bin is satisfied with equality. Since the problem is feasible, the amount of water remaining in the first bin should satisfy its minimum power requirement.

In fact, in this example, there is an excess amount that is therefore used to equalize the water levels of the first two bins via directional water-filling. This ends the algorithm and gives the optimum power profile.

We now find the optimum value of $\mathbf{p}_2$ by solving the following outer problem:

$$\begin{aligned} \max_{\mathbf{p}_2} \quad & \mu H\left(\mathbf{p}_2\right) + \sum_{i=1}^{N} g(p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} K_i, \quad \forall k \end{aligned} \tag{2.44}$$

where $\mu \triangleq \frac{\mu_1}{\mu_2 - \mu_1}$, and the modified water levels K_i are given by:

$$\begin{aligned} K_i &= A_i - A_{i-1}, \\ A_i &= \min\left\{ \sum_{j=1}^{i} \bar{E}_{2j}, \sum_{j=1}^{i} \bar{E}_{1j}, \frac{1}{\sigma^2} \sum_{j=1}^{i} E_j \right\}, \quad A_0 = 0 \end{aligned} \tag{2.45}$$

where the extra terms in the A_i expression are to ensure feasibility of the inner problem. By Lemma 2.13, the outer problem is a convex optimization problem [77]. We solve it by an algorithm similar to that of the two-hop network outer problem, except that we only focus on choosing increasing power vectors $\mathbf{p}_2$ in each iteration. By convexity of the problem, the iterations converge to the optimal solution.

2.6 Numerical Results

In this section, we present numerical results for the considered systems models. We focus on the specific case where $g(x) = \log(1 + x)$, and $\phi = g^{-1}$. Starting

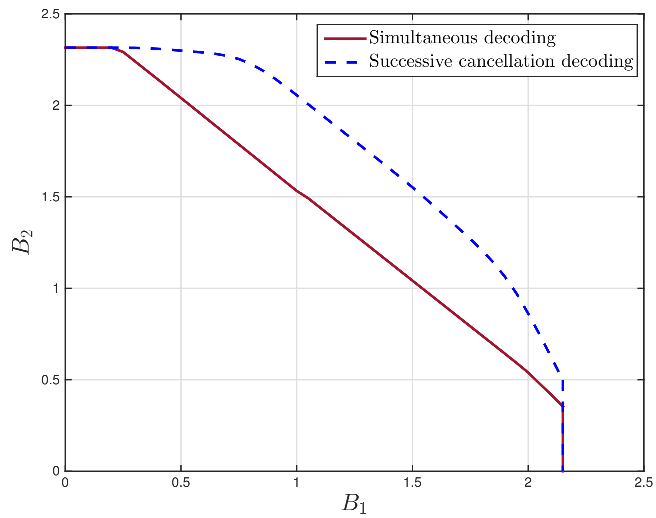


Figure 2.8: Departure regions of a MAC with simultaneous and successive cancellation decoding.

with the single-user channel, we consider a five-slot system with energy amounts of $\mathbf{E} = [2, 2, 1, 2.5, 0.5]$ at the transmitter, and $\bar{\mathbf{E}} = [1, 1, 0.5, 2.5, 3]$ at the receiver. The optimal rates in this case according to Theorem 2.1 are given by $\mathbf{r}^* = [0.6061, 0.6061, 0.6061, 1.2528, 1.3863]$. As we see, the rates are non-decreasing, which is consistent with Lemma 2.1, and they strictly increase only after consuming all the receiver's energies in decoding by the end of the third slot, and again by the end of the fourth one, which is consistent with Lemma 2.2.

In Fig. 2.8, we plot the maximum departure regions for a MAC with simultaneous decoding and successive cancellation decoding. We consider a system of three time slots, during which the nodes harvest the energies: $\mathbf{E}_1 = [0.5, 1, 2]$, $\mathbf{E}_2 = [1, 2, 0.5]$, and $\bar{\mathbf{E}} = [1.5, 2, 0.5]$. We observe that the simultaneous decoding region lies strictly inside the successive decoding region. The latter, given by the geometric programming framework, is only a local optimal solution; one can therefore

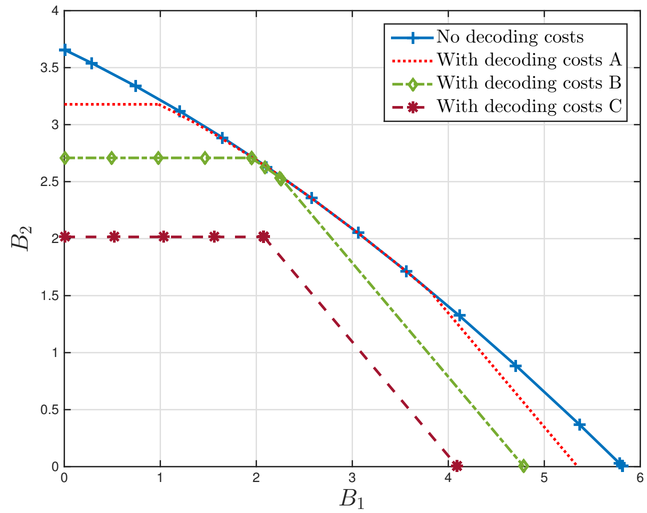


Figure 2.9: Departure regions of a BC with and without decoding costs.

achieve even higher rates if a global optimal solution is attained.

Finally, in Fig. 2.9, we provide some simulation results to illustrate the difference between the departure regions with and without decoding costs for a BC. We consider a system of three time slots, where the energy profile of the transmitter is given by $\mathbf{E} = [5, 6, 7]$. The maximum departure region with no decoding costs is shown in blue. We vary the energy profiles at the receivers to show the effect of the decoding costs on the maximum departure region. We start by setting $\bar{\mathbf{E}}_1 = [4, 5, 6]$, and $\bar{\mathbf{E}}_2 = [1, 2, 3]$, to get region A in red. Then we lower the values to $\bar{\mathbf{E}}_1 = [3, 4, 5]$, and $\bar{\mathbf{E}}_2 = [1, 1.5, 2]$, to get region B in green. Finally, we lower the values again to $\bar{\mathbf{E}}_1 = [2, 3, 4]$, and $\bar{\mathbf{E}}_2 = [0.5, 1, 1.5]$, to get region C in brown. We note that as we lower the energy profiles at the receivers, the decoding causality constraints become more binding, and therefore, the region progressively shrinks.

2.7 Conclusion

In this chapter, we considered decoding costs in energy harvesting communication networks. In our settings, we assumed that receivers, in addition to transmitters, rely on energy harvested from nature. Receivers need to spend a decoding power that is a function of the incoming rate in order to receive their packets. This gave rise to the *decoding causality* constraints: receivers cannot spend energy in decoding prior to harvesting it. We first considered a single-user setting and maximized the throughput by a given deadline. Next, we considered two-hop networks and characterized the end-to-end throughput maximizing policies. Then, we considered two-user MAC and BC settings, with focus on exponential decoding functions, and characterized the maximum departure regions. In most of the models considered, we were able to move the receivers' decoding costs effect back to the transmitters as *generalized data arrivals*; transmitters need to adapt their powers (and rates) not only to their own energies, but to their intended receivers' energies as well. Such adaptation is governed by the characteristics of the decoding function.

Throughout this chapter, we only considered receiver decoding costs in our models without considering transmitter processing costs. On the other hand, other works have considered the processing costs at the transmitter [24–29] without considering decoding costs at the receiver. In their models, the transmitter spends a constant amount of power per unit time whenever it is communicating to account for circuitry processing; while in our model, the receiver spends a decoding power which is a function of the incoming data rate. In Chapter 4, we combine the two approaches to account for both the processing costs at the transmitter and the decoding costs at the receiver in a single setting.

CHAPTER 3

Energy Harvesting Cooperative Multiple Access Channel with Decoding Costs

3.1 Introduction

In this chapter, we consider an energy harvesting cooperative multiple access channel with decoding costs, see Fig. 3.1. In this setting, users cooperate at the physical layer (data cooperation) in order to increase the achievable rates. Data cooperation comes at the expense of decoding costs: each user spends some amount of its harvested energy to decode the message of the other user, before forwarding both messages to the receiver. The decoding power spent is an increasing convex function of the incoming message rate. We characterize the optimal power scheduling policies that achieve the boundary of the maximum departure region subject to energy causality constraints and decoding costs by using a generalized water-filling algorithm.

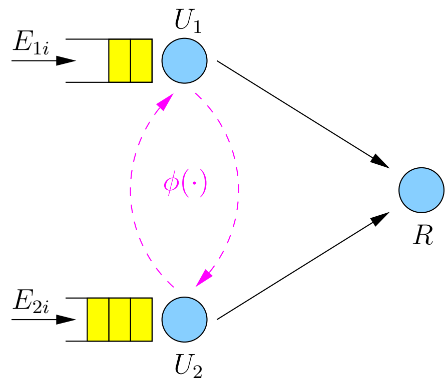


Figure 3.1: Energy harvesting cooperative MAC with decoding costs.

3.2 System Model and Problem Formulation

We consider a time-slotted system, where energies arrive in amounts of E_{1i} and E_{2i} at the first and the second user, respectively, in slot i. The energy arriving at each user can be used for transmission, decoding, or can be saved in a battery to be used in future slots. The users communicate with the receiver over a Gaussian MAC, with a noise variance $\sigma^2 > 1$ at the receiver. They also overhear each other's transmission over stronger links: the channels between the users are assumed to be Gaussian with unit-variance. In order to make use of the overheard information, the messages are transmitted to the receiver using block Markov superposition coding [50]. Users 1 and 2 create common information using powers p_{12} and p_{21}, and convey the created common information to the receiver using powers p_{u1} and p_{u2}. Since user-receiver links are weaker than user-user links, direct transmission is not considered [80].

For a given power policy $(p_{12}, p_{21}, p_{u1}, p_{u2})$, a rate pair (r_1, r_2) belongs to the achievable rate region of the cooperative MAC, denoted by $\mathcal{F}_{CMAC}\left(p_{12}, p_{21}, p_{u1}, p_{u2}\right)$, if [50]

$$r_1 \leq \frac{1}{2}\log\left(1+p_{12}\right) \tag{3.1}$$

$$r_2 \leq \frac{1}{2}\log\left(1+p_{21}\right) \tag{3.2}$$

$$r_1 + r_2 \leq \frac{1}{2}\log\left(\frac{S}{\sigma^2}\right) \tag{3.3}$$

where $S \triangleq \sigma^2 + p_{12} + p_{21} + \left(\sqrt{p_{u1}} + \sqrt{p_{u2}}\right)^2$. Throughout this chapter, we denote $g(p) \triangleq \frac{1}{2}\log(1+p)$.

Our goal is to characterize the maximum departure region [5], $\mathcal{F}_{CMAC}$, subject to energy causality constraints and decoding costs at both users. Since $\mathcal{F}_{CMAC}$ is a convex region, its boundary can be characterized by solving the following weighted sum rate maximization problem for all $\mu_1, \mu_2 > 0$,

$$\begin{aligned}
\max_{\substack{\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_{12},\mathbf{p}_{21}\\ \mathbf{p}_{u1},\mathbf{p}_{u2}}} \quad & \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \\
\text{s.t.} \quad & \left(r_{1i}, r_{2i}\right) \in \mathcal{F}_{CMAC}\left(p_{12i}, p_{21i}, p_{u1i}, p_{u2i}\right), \ \forall i \\
& \sum_{i=1}^{k} p_{12i} + p_{u1i} + \phi(r_{2i}) \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{21i} + p_{u2i} + \phi(r_{1i}) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& r_{1i}, r_{2i}, p_{12i}, p_{21i}, p_{u1i}, p_{u2i} \geq 0
\end{aligned} \tag{3.4}$$

where $\phi(r)$, an increasing convex function in r, is the decoding power (cost) needed to decode a message of rate r. Therefore, each user needs to adapt its powers (and rates) to both its own and the other user's energy arrivals.

3.3 Properties of the Optimal Policy

We first show that in the cooperative MAC, the optimal rate allocation (r_1, r_2) can be expressed directly in terms of powers p_{12} and p_{21} used for common message generation.

Lemma 3.1 *There exists an optimal policy for problem (3.4) where the two inequalities (3.1) and (3.2) hold with equality* $\forall i$.

Proof: Assume that in the optimal policy (3.1) does not hold with equality for some time slot k. Then, we decrease p_{12k} and increase p_{u1k} by the same amount, until (3.1) holds with equality. This either increases S_k, or keeps it constant, hence the third inequality still holds. The new power allocation is energy feasible. Since the rate allocation did not change, the newly obtained policy is optimal as well. Similar arguments follow if the second inequality does not hold with equality. ■

We remark here that in the cooperative MAC with no decoding costs [51], the optimal policy is to send at a rate pair so that (3.3) as well holds with equality, or else the rates can be improved [51, Lemma 2]. However this is not necessarily true in the presence of decoding costs, as increasing one of the user's rate comes at the expense of decreasing the other user's rate, as some of the power used for transmission needs to be shifted to decoding at the cooperative partner.

In the sequel, we focus on the case of exponential decoding costs. Specifically, we set $\phi = a \cdot g^{-1}$, for some decoding power factor $a > 0$ [81]. By Lemma 3.1, the problem can now be written only in terms of the powers as

$$
\begin{aligned}
\max_{\mathbf{p}_{12},\mathbf{p}_{21},\mathbf{p}_{u1},\mathbf{p}_{u2}} \quad & \sum_{i=1}^{N} \mu_1 g(p_{12i}) + \mu_2 g(p_{21i}) \\
\text{s.t.} \quad & g(p_{12i}) + g(p_{21i}) \leq \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right), \quad \forall i \\
& \sum_{i=1}^{k} p_{12i} + p_{u1i} + ap_{21i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{21i} + p_{u2i} + ap_{12i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& p_{12i}, p_{21i}, p_{u1i}, p_{u2i} \geq 0
\end{aligned}
\tag{3.5}
$$

which is not a convex optimization problem due to the first set of constraints. Next, we characterize the (local) optimality conditions for problem (3.5). The Lagrangian is

$$
\begin{aligned}
\mathcal{L} = & -\left(\sum_{i=1}^{N} \mu_1 g(p_{12i}) + \mu_2 g(p_{21i})\right) + \sum_{i=1}^{N} \lambda_i \left(g(p_{12i}) + g(p_{21i}) - \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right)\right) \\
& + \sum_{k=1}^{N} \gamma_{1k} \left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + ap_{21i} - \sum_{i=1}^{k} E_{1i}\right) \\
& + \sum_{k=1}^{N} \gamma_{2k} \left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + ap_{12i} - \sum_{i=1}^{k} E_{2i}\right) \\
& - \left(\sum_{i=1}^{N} \omega_{1i} p_{u1i} + \omega_{2i} p_{u2i} + \eta_{1i} p_{12i} + \eta_{2i} p_{21i}\right)
\end{aligned}
\tag{3.6}
$$

where $\{\lambda_i, \gamma_{1i}, \gamma_{2i}, \eta_{1i}, \eta_{2i}, \omega_{1i}, \omega_{2i}\}$ are non-negative Lagrange multipliers. Differentiating with respect to the powers and equating to zero we get the following KKT conditions

$$\sum_{k=i}^{N} \gamma_{1k} + a\gamma_{2k} = \frac{\mu_1 - \lambda_i}{1 + p_{12i}} + \frac{\lambda_i}{S_i} + \eta_{1i} \tag{3.7}$$

$$\sum_{k=i}^{N} \gamma_{2k} + a\gamma_{1k} = \frac{\mu_2 - \lambda_i}{1 + p_{21i}} + \frac{\lambda_i}{S_i} + \eta_{2i} \tag{3.8}$$

$$\sum_{k=i}^{N} \gamma_{1k} = \left(1 + \sqrt{\frac{p_{u2i}}{p_{u1i}}}\right) \frac{\lambda_i}{S_i} + \omega_{1i} \tag{3.9}$$

$$\sum_{k=i}^{N} \gamma_{2k} = \left(1 + \sqrt{\frac{p_{u1i}}{p_{u2i}}}\right) \frac{\lambda_i}{S_i} + \omega_{2i} \tag{3.10}$$

along with the complementary slackness conditions

$$\lambda_i \left(g(p_{12i}) + g(p_{21i}) - \frac{1}{2} \log \left(\frac{S_i}{\sigma^2} \right) \right) = 0, \quad \forall i \tag{3.11}$$

$$\gamma_{1k} \left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + a p_{21i} - \sum_{i=1}^{k} E_{1i} \right) = 0, \quad \forall k \tag{3.12}$$

$$\gamma_{2k} \left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + a p_{12i} - \sum_{i=1}^{k} E_{2i} \right) = 0, \quad \forall k \tag{3.13}$$

$$\eta_{1i} p_{12i} = 0, \;\; \eta_{2i} p_{21i} = 0, \quad \forall i \tag{3.14}$$

$$\omega_{1i} p_{u1i} = 0, \;\; \omega_{2i} p_{u2i} = 0, \quad \forall i \tag{3.15}$$

Note that, for the derivatives in (3.9) and (3.10) to be well defined, the cooperative powers p_{u1i}, p_{u2i} must be non-zero; otherwise the problem formulation needs to be revisited. Since the case where the users do not send any cooperative codewords occurs very rarely in practice, in this work, we focus only on policies where p_{u1i} and p_{u2i} are positive, i.e., $\omega_{1i} = \omega_{2i} = 0$. We have the following claim regarding the optimal value of λ_i.

Lemma 3.2 *The optimal λ_i satisfies $\lambda_i \leq \max\{\mu_1, \mu_2\}$.*

Proof: First, note that by concavity of the objective function, it is sub-optimal to move all the energy in slot i forward to future slots. This means that either p_{12i} or p_{21i} is strictly positive for any i. By complementary slackness, this means that either $\eta_{1i} = 0$ or $\eta_{2i} = 0$. Without loss of generality, assume $\eta_{1i} = 0$ for some i. Substituting (3.9) and (3.10) into (3.7), we get

$$\left(1 + \sqrt{\frac{p_{u2i}}{p_{u1i}}}\right) \frac{\lambda_i}{S_i} + \left(1 + \sqrt{\frac{p_{u1i}}{p_{u2i}}}\right) \frac{a\lambda_i}{S_i} = \frac{\mu_1 - \lambda_i}{1 + p_{12i}} + \frac{\lambda_i}{S_i} \tag{3.16}$$

Observe that we always have

$$\left(1 + \sqrt{\frac{p_{u2i}}{p_{u1i}}}\right) \frac{\lambda_i}{S_i} \geq \frac{\lambda_i}{S_i} \tag{3.17}$$

and hence, to satisfy (3.16) we need to have

$$0 \leq \left(1 + \sqrt{\frac{p_{u1i}}{p_{u2i}}}\right) \frac{a\lambda_i}{S_i} \leq \frac{\mu_1 - \lambda_i}{1 + p_{12i}} \tag{3.18}$$

which leads to $\lambda_i \leq \mu_1 \leq \max\{\mu_1, \mu_2\}$. ■

Note that if $\lambda_i > \mu_1$ for some i, then we must have $\eta_{1i} > 0$ so that (3.16) is satisfied (after adding η_{1i} to its right hand side). We will use this observation later in the upcoming proofs. The next lemma shows that we can overcome the non-convexity issue of problem (3.5) by using its relation to problem (3.4).

Lemma 3.3 *Any local optimal point for problem (3.5) is also a local optimal point* *for problem (3.4).*

Proof: We prove the lemma by showing that any primal and dual variables satisfying the KKT conditions for problem (3.5) correspond to those satisfying the KKT conditions for problem (3.4). The KKT optimality conditions for (3.4) are

$$\lambda_{1i} + \lambda_{12i} = \mu_1 + \nu_{1i} \tag{3.19}$$

$$\lambda_{2i} + \lambda_{12i} = \mu_2 + \nu_{2i} \tag{3.20}$$

$$\sum_{k=i}^{N} \gamma_{1k} + a\gamma_{2k} = \frac{\lambda_{1i}}{1 + p_{12i}} + \frac{\lambda_{12i}}{S_i} + \eta_{1i} \tag{3.21}$$

$$\sum_{k=i}^{N} \gamma_{2k} + a\gamma_{1k} = \frac{\lambda_{2i}}{1 + p_{21i}} + \frac{\lambda_{12i}}{S_i} + \eta_{2i} \tag{3.22}$$

$$\sum_{k=i}^{N} \gamma_{1k} = \left(1 + \sqrt{\frac{p_{u2i}}{p_{u1i}}}\right) \frac{\lambda_{12i}}{S_i} \tag{3.23}$$

$$\sum_{k=i}^{N} \gamma_{2k} = \left(1 + \sqrt{\frac{p_{u1i}}{p_{u2i}}}\right) \frac{\lambda_{12i}}{S_i} \tag{3.24}$$

along with the complementary slackness conditions

$$\lambda_{1i}\left(r_{1i} - g(p_{12i})\right) = 0, \quad \forall i \tag{3.25}$$

$$\lambda_{2i}\left(r_{2i} - g(p_{21i})\right) = 0, \quad \forall i \tag{3.26}$$

$$\lambda_{12i}\left(r_{1i} + r_{2i} - \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right)\right) = 0, \quad \forall i \tag{3.27}$$

$$\gamma_{1k}\left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + ap_{21i} - \sum_{i=1}^{k} E_{1i}\right) = 0, \quad \forall k \tag{3.28}$$

$$\gamma_{2k}\left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + ap_{12i} - \sum_{i=1}^{k} E_{2i}\right) = 0, \quad \forall k \tag{3.29}$$

$$\eta_{1i}p_{12i} = 0, \;\; \eta_{2i}p_{21i} = 0, \quad \forall i \tag{3.30}$$

$$\nu_{1i} r_{1i} = 0,\ \nu_{2i} r_{2i} = 0, \quad \forall i \tag{3.31}$$

Now, consider a KKT point for problem (3.5), i.e., some feasible primal and dual variables $\{\tilde{p}_{jki}, \tilde{p}_{uji}, \tilde{\gamma}_{ji}, \tilde{\lambda}_i, \tilde{\eta}_{ji}\}$, $j, k \in \{1, 2\}$, $j \neq k$, satisfying (3.7)-(3.14). We then assign the following values for the variables of problem (3.4)

$$p_{12i} = \tilde{p}_{12i},\ p_{21i} = \tilde{p}_{21i},\ p_{u1i} = \tilde{p}_{u1i},\ p_{u2i} = \tilde{p}_{u2i} \tag{3.32}$$

$$r_{1i} = \log\left(1 + \tilde{p}_{12i}\right),\ r_{2i} = \log\left(1 + \tilde{p}_{21i}\right) \tag{3.33}$$

$$\gamma_{1i} = \tilde{\gamma}_{1i},\ \gamma_{2i} = \tilde{\gamma}_{2i} \tag{3.34}$$

$$\lambda_{12i} = \tilde{\lambda}_i,\ \lambda_{1i} = \left(\mu_1 - \tilde{\lambda}_i\right)^+,\ \lambda_{2i} = \left(\mu_2 - \tilde{\lambda}_i\right)^+ \tag{3.35}$$

$$\nu_{1i} = \left(\tilde{\lambda}_i - \mu_1\right)^+,\ \nu_{2i} = \left(\tilde{\lambda}_i - \mu_2\right)^+ \tag{3.36}$$

$$\eta_{1i} = \tilde{\eta}_{1i} + \left(\mu_1 - \tilde{\lambda}_i\right)^-,\ \eta_{2i} = \tilde{\eta}_{2i} + \left(\mu_2 - \tilde{\lambda}_i\right)^- \tag{3.37}$$

where $(\cdot)^+ = \max\{0, \cdot\}$ and $(\cdot)^- = \min\{0, \cdot\}$. Using the observation stated right after Lemma 3.2, we can directly verify that (3.19)-(3.31) are satisfied using the above assignments. ■

We note that problem (3.4) is convex, and thus its KKT conditions are also sufficient for optimality [77]. Therefore, by Lemma 3.3, we can optimally solve problem (3.4) by characterizing the KKT points of problem (3.5), which we focus on in the remainder of this chapter.

A power allocation policy which uses all available energy by the end of the transmission is called an *energy consuming policy*. The next lemma shows that, it is sufficient to restrict our attention to energy consuming policies.

Lemma 3.4 *There exists an optimal policy for problem (3.5) where both users exhaust all their energies, in transmission and decoding, by the end of communication.*

Proof: Let one of the users, say user 1, have some leftover energy at the end of transmission. Then, we can increase p_{u1N} until user 1's energy is exhausted. This is feasible, as it increases the right hand side of (3.3), and does not change the rates, and therefore, is optimal. ■

Note that (3.3) is a constraint on the total data rate. When it holds with equality, the users send at the maximum allowed data rate. We call such policies *data consuming policies.* The next lemma shows that it is sufficient to restrict our attention to policies that are data consuming in the last slot.

Lemma 3.5 *There exists an optimal policy for problem (3.5) that is data consuming in the last time slot.*

Proof: If (3.3) is not tight in slot N, then we can decrease, say, p_{u1N} until the data consumption constraint holds with equality in time slot N. This is energy feasible, and does not change the rates, and therefore, is optimal. ■

3.4 Single Energy Arrival

In this section, we consider the case where each user harvests only one packet of energy. By Lemma 3.4, both users consume all the available energy, i.e., we have

$$p_{12} + ap_{21} + p_{u1} = E_1, \quad p_{21} + ap_{12} + p_{u2} = E_2 \tag{3.38}$$

We now solve the above equations for p_{12} and p_{21} in terms of the cooperative powers p_{u1} and p_{u2}, and substitute back in problem (3.5) for the $N = 1$ case to get the following reduced problem in terms of the cooperative powers[1]

$$\begin{aligned}
\max_{p_{u1},p_{u2}} \quad & \mu_1 g\left(\frac{E_1 - aE_2}{1-a^2} - \frac{p_{u1} - ap_{u2}}{1-a^2}\right) + \mu_2 g\left(\frac{E_2 - aE_1}{1-a^2} - \frac{p_{u2} - ap_{u1}}{1-a^2}\right) \\
\text{s.t.} \quad & g\left(\frac{E_1 - aE_2}{1-a^2} - \frac{p_{u1} - ap_{u2}}{1-a^2}\right) + g\left(\frac{E_2 - aE_1}{1-a^2} - \frac{p_{u2} - ap_{u1}}{1-a^2}\right) \leq \frac{1}{2}\log\left(\frac{S_u}{\sigma^2}\right) \\
& 0 \leq p_{u1} \leq E_1, \quad 0 \leq p_{u2} \leq E_2 \\
& a(E_2 - p_{u2}) \leq E_1 - p_{u1} \leq \frac{E_2 - p_{u2}}{a}
\end{aligned} \tag{3.39}$$

where the last constraint assures the non-negativity of p_{12} and p_{21}, and the term S_u is given by

$$S_u \triangleq \sigma^2 + \frac{E_1 + E_2 + ap_{u1} + ap_{u2} + 2(1+a)\sqrt{p_{u1}p_{u2}}}{1+a} \tag{3.40}$$

We solve the above problem over two stages as follows.

[1]Without loss of generality, we focus on the case $a < 1$ throughout this chapter. Similar analysis follows for the case $a \geq 1$.

Stage 1: First, we solve a relaxed problem by ignoring the data consumption constraint. Note that the relaxed problem is a convex problem. To solve it, we further note that, if the last constraint in problem (3.39) is not binding, i.e., if both p_{12} and p_{21} are strictly positive, then by taking derivative of the objective function with respect to the cooperative powers, the solution of the relaxed problem is found by solving the following two linear equations in (p_{u1}, p_{u2})

$$\left(\frac{1}{a\mu_2} + \frac{a}{\mu_1}\right) p_{u2} - \left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right) p_{u1} = c_1 \tag{3.41}$$

$$\left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right) p_{u2} - \left(\frac{a}{\mu_2} + \frac{1}{a\mu_1}\right) p_{u1} = c_2 \tag{3.42}$$

where the constants c_1 and c_2 are given by

$$c_1 = \frac{1 - a^2 + E_2 - aE_1}{a\mu_2} - \frac{1 - a^2 + E_1 - aE_2}{\mu_1} \tag{3.43}$$

$$c_2 = \frac{1 - a^2 + E_2 - aE_1}{\mu_2} - \frac{1 - a^2 + E_1 - aE_2}{a\mu_1} \tag{3.44}$$

If (3.41)-(3.42) admit a solution, $(\tilde{p}_{u1}, \tilde{p}_{u2})$, not satisfying the last constraint in (3.39), then by the concavity of the objective function, the solution is given by projecting $(\tilde{p}_{u1}, \tilde{p}_{u2})$ onto this last constraint set, which will make one of the constraint's inequalities hold with equality. Substituting this into the objective function, the relaxed problem in this case gets simplified to a one-variable convex optimization problem that can be solved by first derivative analysis over the feasible region. We denote the solution of the relaxed problem by $(\bar{p}_{u1}, \bar{p}_{u2})$.

Stage 2: We now check whether $(\bar{p}_{u1}, \bar{p}_{u2})$ satisfies the data consumption constraint. Denote the left hand side of the constraint by $G\left(\bar{p}_{u1}, \bar{p}_{u2}\right)$ and let $\bar{S}_u = S_u\big|_{(\bar{p}_{u1},\bar{p}_{u2})}$. If the constraint is not satisfied, then we have

$$G\left(\bar{p}_{u1}, \bar{p}_{u2}\right) > \frac{1}{2}\log\left(\frac{\bar{S}_u}{\sigma^2}\right) \tag{3.45}$$

Hence, the goal now is to find the closest point (p^*_{u1}, p^*_{u2}) to $(\bar{p}_{u1}, \bar{p}_{u2})$ such that $G\left(p^*_{u1}, p^*_{u2}\right) = \frac{1}{2}\log\left(S^*_u/\sigma^2\right)$. Towards that end, we note that $\frac{1}{2}\log\left(S_u/\sigma^2\right)$ is increasing in (p_{u1}, p_{u2}), and that $G(E_1, E_2) = 0$. By the concavity of G, the two functions $G(p_{u1}, p_{u2})$ and $\frac{1}{2}\log\left(S_u/\sigma^2\right)$ are guaranteed to intersect at some point $(p^*_{u1}, p^*_{u2}) > (\bar{p}_{u1}, \bar{p}_{u2})$. The optimal (p^*_{u1}, p^*_{u2}) is the pair at which the intersection of the two functions yields the maximum value for the objective function.

This concludes our discussion on the single energy arrival scenario. In the next section, we use this result to extend the analysis to the general multiple energy arrival scenario.

3.5 Multiple Energy Arrivals

We present an iterative generalized water-filling algorithm that optimally solves problem (3.5) for general N. We need to determine the optimal energy distribution among the slots for each user. We first initialize the energy state vectors $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$ and solve for each slot i independently using the results of the previous section with energies S_{1i} and S_{2i}. Next, given the powers in each slot, we determine λ_i by solving (3.16) if $p_{12i} > 0$ (and if $p_{21i} > 0$ we solve a similar equation with appropriate coefficients). Next, we solve equations (3.7)-(3.10) for all the remaining Lagrange multipliers treating $\sum_{k=i}^{N}\gamma_{1k}$ and $\sum_{k=i}^{N}\gamma_{2k}$ as variables of their own, because we are solving for each slot independently. Let us define

$$\kappa_{1i} \triangleq \frac{1}{\sum_{k=i}^{N}\gamma_{1k}+a\gamma_{2k}}, \qquad \kappa_{2i} \triangleq \frac{1}{\sum_{k=i}^{N}\gamma_{2k}+a\gamma_{1k}} \tag{3.46}$$

We can compute $\{\kappa_{1i},\kappa_{2i}\}_{i=1}^{N}$ given the initialization policy. We interpret these terms as *generalized water levels* to be equalized to the extent possible among the slots. We have the following lemma regarding their optimal values.

Lemma 3.6 *The optimal generalized water levels $\{\kappa_{1i}^*,\kappa_{2i}^*\}$ for problem (3.5) are non-decreasing, and increase synchronously. The latter event occurs only if at least one user consumes its energy in transmission and decoding.*

Proof: The first part follows by noting that due to the non-negativity of the Lagrange multipliers $\{\gamma_{1i},\gamma_{2i}\}$, the denominators of the water levels in (3.46) are non-increasing. For the second part, since $a>0$, *both* denominators decrease from slot i to slot $i+1$ iff at least $\gamma_{1i}>0$ or $\gamma_{2i}>0$. This makes both water levels increase synchronously. Finally, by complementary slackness, if we have $\gamma_{ji}>0$, then user j consumes its energy in slot i, $j=1,2$. ■

Next, we check if the obtained water levels satisfy the conditions of the previous lemma. If not, then some energy needs to flow forward until they satisfy these conditions. However, due to the decoding costs, energy transfer from one user affects both water levels, and therefore both users' powers. Hence, we keep record of how much energy is transferred forward at each user by, e.g., putting measuring meters in between the slots of each user [18]. We start by updating slots 1 and 2, followed by slots 2 and 3, and so on. If at a given two slots $(i, i+1)$ we have $\kappa_{1i} > \kappa_{1(i+1)}$ or $\kappa_{2i} > \kappa_{2(i+1)}$ then energy flows from slot i to $i+1$ from either one or both users until the water levels are equalized. We keep iterating until the conditions of Lemma 3.6 are satisfied for all the slots. During the iterations, energy can be drawn back, using the values stored in the meters, if this increases the objective function. Iterations converge to a KKT point of problem (3.5), which is, by Lemma 3.3, a KKT point of problem (3.4), and thereby the optimal solution.

3.6 Numerical Results

In this section, we present some simple numerical examples. We consider a five slot system with energies $\mathbf{E}_1 = [5, 1, 6, 2, 2]$ and $\mathbf{E}_2 = [2, 3, 4, 3, 4]$ at the first and the second user, respectively. The receiver noise variance is set to $\sigma^2 = 1.2$.

We solve the problem with different values of decoding costs and plot $B_j = \sum_{i=1}^{N} r_{ji}$, the number of total departed bits for user j, in Fig. 3.2. For reference, we plot the case $a = 0$ studied in [51] that provides the largest departure region, and also the non-cooperative (direct) MAC departure region studied in [5]. We observe that the departure region shrinks as we increase the decoding cost. With $a = 0.3$, the region is still completely outside the non-cooperative MAC region, showing the advantage of data cooperation. For the case $a = 0.7$, the regions intersect, and not all operating points are better than the non-cooperative MAC. Finally, for a relatively large $a = 2$, the departure region is completely inside the non-cooperative

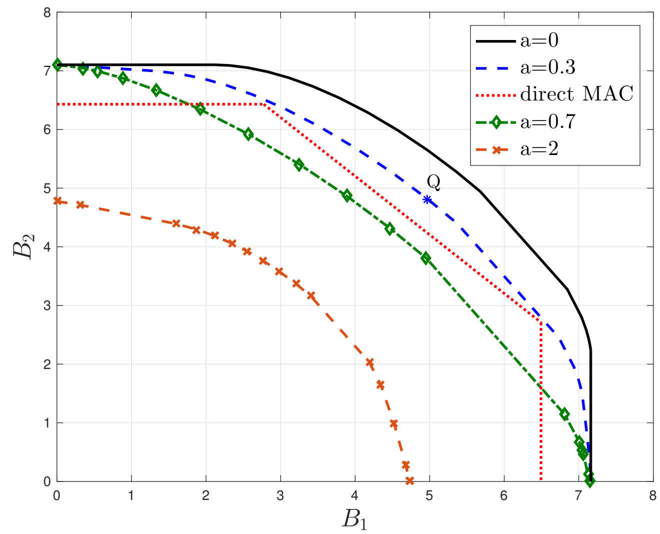


Figure 3.2: Departure regions for different values of the decoding cost parameter.

MAC region, showing that the users achieve higher rates if they do not cooperate due to the high decoding costs they incur. Therefore, the results show that it is not always better to perform data cooperation, but rather it depends on how much energy each user spends to decode the other user's message.

We also compute the optimal generalized water levels for a particular operating point: Q in Fig. 3.2 for the case of $a = 0.3$ with $\mu_1 = \mu_2 = 1$. Iterations converge to: $\boldsymbol{\kappa}_1^* = [4.1, 16.3, 17.5, 17.5, 30.7]$ and $\boldsymbol{\kappa}_2^* = [3.1, 6.6, 7.3, 7.3, 9.2]$. We see that the water levels are non-decreasing, and increase simultaneously, as stated in Lemma 3.6.

3.7 Conclusion

In this chapter, we considered an energy harvesting cooperative multiple access channel (MAC) where users cooperate at the physical layer (data cooperation) in order to increase the achievable rates at the expense of decoding costs; each user spends some amount of its harvested energy to decode the message of the other user, before forwarding both messages to the receiver. We characterized the optimal power scheduling policies that achieve the boundary of the maximum departure region subject to energy causality constraints and decoding costs by using a generalized water-filling algorithm. When considering decoding costs, results show that it is not always better to perform data cooperation, but rather it depends on how much energy each user spends to decode the other user's message.

CHAPTER 4

Energy Harvesting Two-Way Channels with Decoding and Processing Costs

4.1 Introduction

In this chapter, we study the effects of decoding and processing costs in an energy harvesting two-way channel, see Fig. 4.1. We design the optimal offline power scheduling policies that maximize the sum throughput by a given deadline, subject to energy causality constraints, decoding causality constraints, and processing costs at both users. In this system, each user spends energy to transmit data to the other user, and also to decode data coming from the other user; that is, each user divides its harvested energy for transmission and reception. Further, each user incurs a processing cost per unit time as long as it communicates. The power needed for decoding the incoming data is modeled as an increasing convex function of the incoming data rate; and the power needed to be *on*, i.e., the processing cost, is modeled to be a constant per unit time. We solve this problem by first considering the cases with decoding costs only and processing costs only individually. In each case, we solve the single energy arrival scenario, and then use the solution's insights

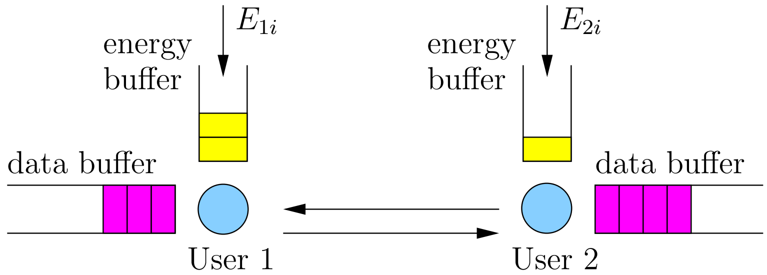


Figure 4.1: Two-way channel with energy harvesting transceivers.

to provide an iterative algorithm that solves the multiple energy arrivals scenario. Then, we consider the general case with *both* decoding and processing costs in a single setting, and solve it for the most general scenario of multiple energy arrivals.

4.2 The Case with Only Decoding Costs

4.2.1 Single Energy Arrival

In this section, we consider the case where both users have a single energy arrival each. Users 1 and 2 have E_1 and E_2 amounts of energy available at the beginning of communication, respectively. Without loss of generality, the communication takes place over a time slot of unit length. The physical layer is Gaussian with unit-variance noise at both users. In the full-duplex Gaussian two-way channel, the sum rate is given by the sum of the single-user rates [49]. Therefore, the rate per user is the single-user Shannon rate of $\frac{1}{2}\log(1+p)$, where p is the transmit power. Throughout this chapter, log is the natural logarithm. A receiver decodes a message of rate r by spending a decoding power $\phi(r)$ that is exponential in the incoming rate, i.e., $\phi(r) = a(e^{br} + c)$ for some $a, b > 0$ and $c \geq -1$. Throughout this chapter, we take $b = 2$ and $c = -1$ for convenience and mathematical tractability. Without loss of generality, any other such exponential decoding power can be handled by appropriately modifying the incoming energy. Therefore, if the first user transmits with power p, the incoming rate is $\frac{1}{2}\log(1+p)$, and the second user spends a power of ap to decode the incoming data. Thus, the throughput maximization problem is

$$\begin{aligned} \max_{p_1,p_2} \quad & \frac{1}{2}\log(1+p_1)+\frac{1}{2}\log(1+p_2) \\ \text{s.t.} \quad & p_1+ap_2 \leq E_1 \\ & p_2+ap_1 \leq E_2 \end{aligned} \tag{4.1}$$

where p_1 and p_2 are the powers of users 1 and 2, respectively. We assume $a \neq 1$, for if $a = 1$, by concavity of the log, the optimal solution will be given by $p_1^* = p_2^* = \min\{E_1, E_2\}/2$. We have the following lemma regarding this problem.

Lemma 4.1 *In the optimal policy, at least one user consumes all of its energy in transmission and decoding. This is the user with the smaller energy.*

Proof: The first part of the lemma follows directly by noting that if neither of the constraints holds with equality, then we can increase the power (and therefore rate) of one of the users until one of the constraints becomes tight. Now assume that $E_1 \leq E_2$, but only the second user consumes all of its energy, i.e., $p_2^* + ap_1^* = E_2 \geq E_1 > p_1^* + ap_2^*$, which further leads to having

$$p_1^* < p_2^*, \quad \text{if } a < 1 \tag{4.2}$$

$$p_1^* > p_2^*, \quad \text{if } a > 1 \tag{4.3}$$

Let us consider the case in (4.2) (similar arguments follow for the case in (4.3)), choose some $\epsilon > 0$, and define the following new policy: $\tilde{p}_1 = p_1^* + \epsilon$, $\tilde{p}_2 = p_2^* - \epsilon$. Since the first user did not consume all of its energy, we can choose ϵ small enough such that the new policy consumes the following amounts of energy

$$\tilde{p}_2 + a\tilde{p}_1 = p_2^* + ap_1^* - (1-a)\epsilon < E_2 \tag{4.4}$$

$$\tilde{p}_1 + a\tilde{p}_2 = p_1^* + ap_2^* + (1-a)\epsilon \leq E_1 \tag{4.5}$$

By concavity of the log, this new policy strictly increases the sum rate, and therefore, the original policy cannot be optimal, i.e., the first user has to consume all of its energy. ■

The above lemma states that, in the presence of decoding costs, one user may not be able to use up all of its energy. This is because each user now needs to adapt its power (and rate) to both its own energy and to the energy of the other user, in order to guarantee decodability. This makes the user with smaller energy be a bottleneck for the system.

Without loss of generality, we continue assuming $E_1 \leq E_2$. Therefore, by Lemma 4.1, we have $p_1^* + ap_2^* = E_1$. Substituting this condition in (4.1), we get the following problem for $a < 1$

$$\begin{aligned} \max_{p_2} \quad & \frac{1}{2}\log\left(1 + E_1 - ap_2\right) + \frac{1}{2}\log\left(1 + p_2\right) \\ \text{s.t.} \quad & 0 \leq p_2 \leq \frac{E_2 - aE_1}{1 - a^2} \end{aligned} \tag{4.6}$$

Alternatively, we get the following problem for $a > 1$

$$\begin{aligned} \max_{p_1} \quad & \frac{1}{2}\log\left(1+p_1\right)+\frac{1}{2}\log\left(1+\frac{E_1-p_1}{a}\right) \\ \text{s.t.} \quad & 0 \leq p_1 \leq \frac{aE_2-E_1}{a^2-1} \end{aligned} \tag{4.7}$$

In both problems, the objective function is concave and the feasible set is an interval. It then follows that the optimal power can be found via equating the derivative of the objective function to 0, and projecting the solution onto the feasible set. For instance, the optimal second user power in problem (4.6) is given by

$$p_2^* = \min\left\{\left[\frac{1+E_1-a}{2a}\right]^+, \frac{E_2-aE_1}{1-a^2}\right\} \tag{4.8}$$

where $[x]^+ = \max(x, 0)$.

4.2.2 Multiple Energy Arrivals

We now consider the case of multiple energy arrivals. Energies arrive at the beginning of time slot i with amounts E_{1i} and E_{2i} at the first and the second user, respectively, ready to be used in the same slot. Unused energies are saved in batteries for later slots. The goal is to maximize the sum throughput by a given deadline N. The problem becomes

$$\max_{\boldsymbol{p}_1,\boldsymbol{p}_2} \quad \sum_{i=1}^{N}\frac{1}{2}\log\left(1+p_{1i}\right)+\frac{1}{2}\log\left(1+p_{2i}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{1i} + ap_{2i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$
$$\sum_{i=1}^{k} p_{2i} + ap_{1i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \tag{4.9}$$

which is a convex optimization problem [77]. The Lagrangian is

$$\mathcal{L} = -\sum_{i=1}^{N} \frac{1}{2} \log\left(1 + p_{1i}\right) - \sum_{i=1}^{N} \frac{1}{2} \log\left(1 + p_{2i}\right) + \sum_{k=1}^{N} \lambda_{1k} \left(\sum_{i=1}^{k} p_{1i} + ap_{2i} - \sum_{i=1}^{k} E_{1i} \right)$$
$$+ \sum_{k=1}^{N} \lambda_{2k} \left(\sum_{i=1}^{k} p_{2i} + ap_{1i} - \sum_{i=1}^{k} E_{2i} \right) \tag{4.10}$$

where $\{\lambda_{1k}\}$ and $\{\lambda_{2k}\}$ are non-negative Lagrange multipliers associated with the energy causality constraints of the first and the second user, respectively. KKT optimality conditions [77] are

$$p_{1i} = \frac{1}{\sum_{k=i}^{N} (\lambda_{1k} + a\lambda_{2k})} - 1, \quad \forall i \tag{4.11}$$
$$p_{2i} = \frac{1}{\sum_{k=i}^{N} (\lambda_{2k} + a\lambda_{1k})} - 1, \quad \forall i \tag{4.12}$$

along with the complementary slackness conditions

$$\lambda_{1k} \left(\sum_{i=1}^{k} p_{1i} + ap_{2i} - \sum_{i=1}^{k} E_{1i} \right) = 0, \quad \forall k \tag{4.13}$$
$$\lambda_{2k} \left(\sum_{i=1}^{k} p_{2i} + ap_{1i} - \sum_{i=1}^{k} E_{2i} \right) = 0, \quad \forall k \tag{4.14}$$

In the following lemmas, we characterize the properties of the optimal solution of this problem.

Lemma 4.2 *In the optimal policy, both users' powers are non-decreasing in time, i.e.,* $p_{1(i+1)} \geq p_{1i}$ *and* $p_{2(i+1)} \geq p_{2i}$, $\forall i$.

Proof: The proof follows from (4.11)-(4.12) since the denominators are non-negative and non-increasing as $\lambda_{1k}, \lambda_{2k} \geq 0$, $\forall k$. ■

Lemma 4.3 *In the optimal policy, the power of user* $j \in \{1, 2\}$ *increases in a time slot only if at least one of the two users consumes all of its available energy in transmission/decoding in the previous time slot.*

Proof: From (4.11)-(4.12), we see that powers can only increase from slot i to slot $i + 1$ if at least λ_{1i} or λ_{2i} is strictly positive, or else powers will stay the same. By complementary slackness conditions in (4.13)-(4.14), we see that the first (resp., second) user's energies must all be consumed by slot i if $\lambda_{1i} > 0$ (resp., $\lambda_{2i} > 0$). ■

Lemma 4.4 *In the optimal policy, powers of both users increase synchronously.*

Proof: Let us assume that we have $p_{1i} < p_{1(i+1)}$. By Lemma 4.3, we must have at least $\lambda_{1i} > 0$ or $\lambda_{2i} > 0$. This in turn makes $p_{2i} < p_{2(i+1)}$ from (4.12). Similarly, if we have $p_{2i} < p_{2(i+1)}$, then we must also have $p_{1i} < p_{1(i+1)}$ from (4.11). This concludes the proof. ■

4.2.2.1 The Case of Two Arrivals

We now solve the case of two energy arrivals at each user explicitly. We will provide an iterative algorithm to solve the general multiple energy arrivals case by utilizing the two-slot solution. In a two-slot setting, it is optimal to have at least one user consume all of its energy in the second slot. It is not clear, however, if this is the case in the first slot. Towards that, we check the feasible energy consumption strategies and choose the one that gives the maximum sum rate. For each strategy, we find the optimal residual energy transferred from the first to the second slot for a given user. We begin by checking a constant-power strategy which, by concavity of the objective function, is optimal if it is feasible [1]. This occurs when neither user consumes all of its energy in the first slot, and hence, by Lemma 4.3, the powers of each user in the two slots are equal, i.e., $p_{11} = p_{12} \triangleq p_1$, and $p_{21} = p_{22} \triangleq p_2$. This leaves us with solving a *single-arrival* problem, as discussed in Section 4.2.1, with the average energy $E_1 = \frac{E_{11}+E_{12}}{2}$ and $E_2 = \frac{E_{21}+E_{22}}{2}$, at the first and the second user, respectively. There can be four more energy consumption strategies to check if the above is infeasible. We highlight one of them in the following analysis. The remaining ones follow similarly.

We consider the strategy in which the first user consumes all of its energy in the first slot, and the second user consumes all of its energy in the second slot. The second user may have some residual energy left from the first slot to be used in the second slot. Denoting this energy residual by r, we have: $p_{11} + ap_{21} = E_{11}$, and $p_{21} + ap_{11} = E_{21} - r$. Solving these two equations for p_{11} and p_{21}, we obtain: $p_{11} = \frac{E_{11}-a(E_{21}-r)}{1-a^2}$, and $p_{21} = \frac{E_{21}-r-aE_{11}}{1-a^2}$. Since the second user consumes all of its energy in the second slot we have: $p_{22} + ap_{12} = E_{22} + r$. Next, we divide the energy consumption in the second slot between the two users as: $p_{12} = \frac{\delta}{a}$ and $p_{22} = E_{22} + r - \delta$, for some $\delta \geq 0$. Finding the optimal sum rate in this strategy is tantamount to solving for the optimal values of r and δ. Thus, problem (4.9) for

$N=2$ in this case can be rewritten as

$$
\begin{aligned}
\max_{r,\delta} \quad & \frac{1}{2}\log\left(1+\frac{E_{11}-a(E_{21}-r)}{1-a^2}\right)+\frac{1}{2}\log\left(1+\frac{E_{21}-r-aE_{11}}{1-a^2}\right) \\
& +\frac{1}{2}\log\left(1+\frac{\delta}{a}\right)+\frac{1}{2}\log\left(1+E_{22}+r-\delta\right) \\
\text{s.t.} \quad & 0\leq\delta\leq E_{22}+r \\
& \left(E_{21}-\frac{E_{11}}{a}\right)^{+}\leq r\leq E_{21}-aE_{11} \\
& \delta\leq\frac{a}{1-a^2}\left(E_{12}-a\left(E_{22}+r\right)\right)
\end{aligned}
\tag{4.15}
$$

which is a convex optimization problem in (r,δ) [77]. Note that for the above problem to be feasible, we need to have: $E_{21}\geq aE_{11}$, and $E_{12}\geq aE_{22}$. Other consumption strategies will have similar necessary conditions.

To solve the above problem, we first assume that the Lagrange multiplier associated with the last constraint is zero, i.e., the constraint is not binding (this is the energy causality constraint of the first user in the second time slot), and obtain a solution. The solution is optimal if it satisfies that constraint with strict inequality. Otherwise, the constraint is binding, and needs to be satisfied with equality. In the latter case, we substitute $\delta=\frac{1}{1-a^2}\left(E_{12}-a\left(E_{22}+r\right)\right)$ in the objective function and solve a problem of only one variable, r, which can be solved by direct first derivative analysis over the feasible region of r. We now characterize the solution after removing that last constraint. We define $r_1\triangleq\left(E_{21}-\frac{E_{11}}{a}\right)^{+}$ and $r_2\triangleq E_{21}-aE_{11}$ for convenience, and introduce the following Lagrangian

$$\mathcal{L} = -\frac{1}{2}\log\left(1 + \frac{E_{11} - a(E_{21} - r)}{1 - a^2}\right) - \frac{1}{2}\log\left(1 + \frac{E_{21} - r - aE_{11}}{1 - a^2}\right) - \frac{1}{2}\log\left(1 + \frac{\delta}{a}\right)$$
$$- \frac{1}{2}\log\left(1 + E_{22} + r - \delta\right) + \lambda_\delta(\delta - E_{22} - r) - \eta_\delta\delta + \lambda_r(r - r_1) + \eta_r(r_2 - r) \tag{4.16}$$

where λ_δ, η_δ, λ_r, and η_r are the non-negative Lagrange multipliers. Taking the derivatives with respect to δ, r, and equating to 0, we get the following

$$\frac{1}{a + \delta} + \eta_\delta = \frac{1}{1 + E_{22} + r - \delta} + \lambda_\delta \tag{4.17}$$

$$\frac{1}{1 + E_{22} + r - \delta} + \frac{a}{1 - a^2 + E_{11} - a(E_{21} - r)} + \eta_r = \frac{1}{1 - a^2 + E_{21} - r - aE_{11}} + \lambda_r \tag{4.18}$$

From (4.17), we solve for δ in terms of r as follows

$$\delta(r) = \begin{cases} 0, & a > 1 + E_{22} + r \\ \frac{1 + E_{22} + r - a}{2}, & 1 - (E_{22} + r) \leq a \leq 1 + E_{22} + r \\ E_{22} + r, & a < 1 - (E_{22} + r) \end{cases} \tag{4.19}$$

Next, we find the optimal value of r. For that, we substitute by $\delta(r)$ in (4.18). Assuming that the middle expression in (4.19) holds, we have

$$\eta_r + f_1(r) = \lambda_r + f_2(r) \tag{4.20}$$

where f_1 and f_2 are given by

$$f_1(r) = \frac{2}{1 + E_{22} + a + r} + \frac{a}{1 - a^2 + E_{11} - aE_{21} + ar} \tag{4.21}$$

$$f_2(r) = \frac{1}{1 - a^2 + E_{21} - aE_{11} - r} \tag{4.22}$$

To solve this, we first assume $\lambda_r = \eta_r = 0$, and equate both sides of (4.20). The existence of a feasible solution of r in this case depends on the extreme values of f_1 and f_2. In particular, since $f_1(r)$ is decreasing in r, while $f_2(r)$ is increasing in r, the solution exists if and only if $f_1(r_2) \leq f_2(r_2)$ and $f_1(r_1) \geq f_2(r_1)$. Note that such solution can be found, for example, by a bisection search. If this condition is not satisfied, then one of the Lagrange multipliers (λ_r, η_r) needs to be strictly positive in order to equate both sides in (4.20). In particular, if $f_1(r_2) > f_2(r_2)$, then we need $\lambda_r > 0$, which implies by complementary slackness that $r = r_2$. On the other hand, if $f_1(r_1) < f_2(r_1)$, then we need $\eta_r > 0$, which implies by complementary slackness that $r = r_1$. After solving for r, we check if it is consistent with the chosen expression of $\delta(r)$ by checking the conditions in (4.19). If not, then we check the other two cases: $\delta(r) = 0$ and $\delta(r) = E_{22} + r$, and re-solve for r. The analysis in these cases follows similarly as above. This concludes the solution of the two-slot case. In the next section, we use the above analysis to find the optimal solution in the general case of multiple energy arrivals.

4.2.2.2 Iterative Solution for the General Case

We solve problem (4.9) iteratively in a two-slot by two-slot manner, starting from the last two slots and going backwards. Once we reach the first two slots, we re-iterate starting from the last two slots, and go backwards again. Iterations stop if the powers do not change after we reach the first two slots. The details are as follows.

We first initialize the energy status of each slot of both users by $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$, where $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are vectors of energy arrivals at user 1 and 2, respectively, and solve each slot independently, as discussed in Section 4.2.1, to get an initial feasible power policy $\left\{\boldsymbol{p}_1^{(0)}, \boldsymbol{p}_2^{(0)}\right\}$. We then start by examining slots $N-1$ and N. We solve the throughput maximization problem for these two slots with energies $\left\{S_{1(N-1)}, S_{1N}\right\}$ and $\left\{S_{2(N-1)}, S_{2N}\right\}$ at the first and second user, respectively, as discussed in Section 4.2.2.1. After we solve this problem, we update the energy status vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, and move back one slot to examine slots $N-2$ and $N-1$. We solve the throughput maximization problem for these two slots using the updated energy status $\left\{S_{1(N-2)}, S_{1(N-1)}\right\}$ and $\left\{S_{2(N-2)}, S_{2(N-1)}\right\}$ at the first and second user, respectively. We update the energy status vector after solving this problem, and continue moving backwards until we solve for slots 1 and 2. After that, we get another feasible power policy $\left\{\boldsymbol{p}_1^{(1)}, \boldsymbol{p}_2^{(1)}\right\}$, where the superscript stands for the iteration index. We then compare this power policy with the initial one. If they are the same, we stop. If not, we perform this process again starting from the last two slots, going backwards, until we get an updated power policy $\left\{\boldsymbol{p}_1^{(2)}, \boldsymbol{p}_2^{(2)}\right\}$.

We stop after the kth iteration if $\boldsymbol{p}_1^{(k-1)} = \boldsymbol{p}_1^k$ and $\boldsymbol{p}_2^{(k-1)} = \boldsymbol{p}_2^k$. Since the sum throughput can only increase with the iterations, and since it is also upper bounded due to the energy constraints, the convergence of the above two-slot iterations is guaranteed.

Next, we check whether the limit point satisfies the KKT optimality conditions. Namely, we solve for the Lagrange multipliers in (4.11) and (4.12). If they are all non-negative, then the KKT conditions are satisfied and, by the convexity of the problem, the limit point is optimal [77]. If not, then the energy status vectors need to be updated. This might be the case for instance if while updating some given two slots, more than necessary amount of energy is transferred forward. While this may be optimal with respect to these two slots, it does not take into consideration the energy arrival vectors in the entire N slots. Therefore, in such cases, we perform another round of iterations where we take some of the energy back if this increases the objective function. Taking energy back without violating causality can be done, e.g., via putting measuring meters in between the slots during the two-slot update phase to record the amount of energy moving forward [18]. Since the problem feasibility is maintained with each update, and by the convexity of the problem, cycling through all the slots infinitely often converges to the optimal policy.

This concludes the discussion of the problem with only decoding costs. In the next section, we discuss the case with only processing costs.

4.3 The Case with Only Processing Costs

4.3.1 Single Energy Arrival

In this section, we study the case where each user has only one energy arrival. In this two-way setting, we incorporate the processing costs into our problem as follows: each user incurs a processing cost when it is *on* for either transmitting or receiving or both. We note that due to the processing costs, it might be optimal for the users to be turned on for only a portion of the time. In this case, the transmission scheme becomes *bursty* [53]. At this point, it is not clear whether it is optimal for the two users to be fully synchronized, i.e., switch on/off simultaneously. For instance, it might be the case that the second user's energy is higher, and therefore it uses the channel for a larger portion of the time $\theta_2 > \theta_1$. In this case, the first user stops transmitting after θ_1 amount of the time, but stays on for an extra $\theta_2 - \theta_1$ amount of time to receive the rest of the second user's data. The same argument could hold for the second user if the first user's energy is larger. Therefore, for the general case of $\theta_1 \neq \theta_2$, each user stays on for a $\max\{\theta_1, \theta_2\}$ amount of time. We formulate the problem as

$$
\begin{aligned}
\max_{\theta_1,\theta_2,p_1,p_2} \quad & \frac{\theta_1}{2}\log(1+p_1) + \frac{\theta_2}{2}\log(1+p_2) \\
\text{s.t.} \quad & \theta_1 p_1 + \max\{\theta_1,\theta_2\}\epsilon_1 \leq E_1 \\
& \theta_2 p_2 + \max\{\theta_1,\theta_2\}\epsilon_2 \leq E_2 \\
& 0 \leq \theta_1, \theta_2 \leq 1
\end{aligned}
\tag{4.23}
$$

where ϵ_j is the processing cost per unit time for user j, $j = 1, 2$.

We have the following two lemmas regarding this problem: Lemma 4.5 states that both users need to use up all of their available energies. Lemma 4.6 states that both users need to be fully synchronized, i.e., they need to turn on for exactly the same duration of time, and turn off together. Hence, whenever a user is turned on, it both sends and receives data.

Lemma 4.5 *In the optimal solution of problem (4.23), both users exhaust their available energies.*

Proof: This follows by directly noting that if one user does not use all its energy, then we can increase its power until it does. This strictly increases the objective function. ■

Lemma 4.6 *In the optimal solution of problem (4.23), we have* $\theta_1^* = \theta_2^*$.

Proof: We show this by contradiction. Assume without loss of generality that it is optimal to have $\theta_1 < \theta_2$. By Lemma 4.5, we have the powers given by

$$p_1 = \frac{E_1 - \theta_2 \epsilon_1}{\theta_1}, \qquad p_2 = \frac{E_2}{\theta_2} - \epsilon_2 \tag{4.24}$$

Therefore, we rewrite problem (4.23) as

$$\begin{aligned} \max_{\theta_1, \theta_2} \quad & \frac{\theta_1}{2} \log\left(1 + \frac{E_1 - \theta_2 \epsilon_1}{\theta_1}\right) + \frac{\theta_2}{2} \log\left(1 + \frac{E_2}{\theta_2} - \epsilon_2\right) \\ \text{s.t.} \quad & 0 \leq \theta_1 \leq \theta_2 \leq \theta_m \end{aligned} \tag{4.25}$$

where $\theta_m \triangleq \min\{1, \frac{E_1}{\epsilon_1}, \frac{E_2}{\epsilon_2}\}$ assures positivity of powers. Next, we note that the first term in the objective function above is monotonically increasing in θ_1, and therefore its value is maximized at the boundary of the feasible set, i.e., at $\theta_1 = \theta_2$, which gives a contradiction. ■

By Lemma 4.6, problem (4.23) now reduces to having only one time variable $\theta \triangleq \theta_1 = \theta_2$

$$\begin{aligned} \max_{\theta, p_1, p_2} \quad & \frac{\theta}{2}\log(1+p_1) + \frac{\theta}{2}\log(1+p_2) \\ \text{s.t.} \quad & \theta(p_1+\epsilon_1) \leq E_1 \\ & \theta(p_2+\epsilon_2) \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{4.26}$$

We will solve (4.26), and its most general multiple energy arrival version, in the rest of this section. We first note that the problem is non-convex. Applying the change of variables: $\bar{p}_1 \triangleq \theta p_1$, $\bar{p}_2 \triangleq \theta p_2$, we get the following equivalent problem

$$\begin{aligned} \max_{\theta, \bar{p}_1, \bar{p}_2} \quad & \frac{\theta}{2}\log\left(1+\frac{\bar{p}_1}{\theta}\right) + \frac{\theta}{2}\log\left(1+\frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & \bar{p}_1+\theta\epsilon_1 \leq E_1 \\ & \bar{p}_2+\theta\epsilon_2 \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{4.27}$$

which is convex, as the objective function is now concave because it is the perspective of a concave function [77], and the constraints are affine in both variables. Using Lemma 4.5, we equate the energy constraints and substitute them back in the objective function to get

$$\max_{0 \le \theta \le \theta_m} \quad \frac{\theta}{2} \log\left(1 + \frac{E_1 - \theta\epsilon_1}{\theta}\right) + \frac{\theta}{2} \log\left(1 + \frac{E_2 - \theta\epsilon_2}{\theta}\right) \tag{4.28}$$

where θ_m is as in Lemma 4.6. Note that the objective function in the above problem is concave since the function $x \log(b + c/x)$ is concave in x, for $x > 0$, and for any real-valued constants b and c. Since the feasible set is an interval, it then follows that the optimal solution is given by projecting stationary points of the objective function onto the feasible set. Differentiating, we obtain the following equation in θ

$$f_1(\theta) \cdot f_2(\theta) = e^{-2} \tag{4.29}$$

where the function $f_j(\theta)$, for $j = 1, 2$, is defined as

$$f_j(\theta) \triangleq \frac{e^{(\epsilon_j - 1)/((E_j/\theta) - (\epsilon_j - 1))}}{(E_j/\theta) - (\epsilon_j - 1)} \tag{4.30}$$

One can show that $f_j(\theta)$ is monotonically increasing in θ, for all θ feasible. Therefore, (4.29) has a unique solution in θ, which we denote by $\bar{\theta}$. Finally, the optimal (burstiness factor) θ^* is given by $\theta^* = \min\left\{\bar{\theta}, 1\right\}$.

We note that the value of θ^* can be strictly less than 1, which leads to *bursty transmission* from the two users. The amount of burstiness depends on the available energies at both users and their processing costs, the relation among which is captured by the functions f_1 and f_2 in (4.29). The two users' energies and processing costs affect each other; one user having relatively low energy or relatively high processing cost can decrease the value of θ^*, i.e., increase the amount of burstiness in the channel. Finally, once the optimal θ^* is found, the optimal powers of the users are found by substituting θ^* in the energy constraints.

4.3.2 Multiple Energy Arrivals

We now extend our results to the case of multiple energy arrivals. During slot i, the two users can be turned on for a θ_i portion of the time. We argue that the users have to be synchronized. For if they were not, then given the optimal energy distribution among the slots, we can synchronize both users in each slot independently, which gives higher throughput, as discussed in the single energy arrival scenario. Then, the problem becomes

$$\begin{aligned} \max_{\boldsymbol{\theta},\boldsymbol{p}_1,\boldsymbol{p}_2} \quad & \sum_{i=1}^{N} \frac{\theta_i}{2}\log(1+p_{1i}) + \frac{\theta_i}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \theta_i(p_{1i}+\epsilon_1) \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} \theta_i(p_{2i}+\epsilon_2) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\ & 0 \leq \theta_i \leq 1, \quad \forall i \end{aligned} \tag{4.31}$$

As we did in the single energy arrival case, we apply the change of variables $\bar{p}_{1i} = \theta_i p_{1i}$ and $\bar{p}_{2i} = \theta_i p_{2i}$, $\forall i$, to get the following equivalent convex optimization problem

$$\begin{aligned}
\max_{\boldsymbol{\theta},\bar{\boldsymbol{p}}_1,\bar{\boldsymbol{p}}_2} \quad & \sum_{i=1}^{N} \frac{\theta_i}{2} \log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{k} \bar{p}_{1i} + \theta_i \epsilon_1 \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} \bar{p}_{2i} + \theta_i \epsilon_2 \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \bar{p}_{1i} \geq 0, \quad \bar{p}_{2i} \geq 0, \quad 0 \leq \theta_i \leq 1, \quad \forall i
\end{aligned} \tag{4.32}$$

The Lagrangian for this problem is

$$\begin{aligned}
\mathcal{L} = & -\left(\sum_{i=1}^{N} \frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right)\right) - \sum_{i=1}^{N}\eta_{1i}\bar{p}_{1i} - \sum_{i=1}^{N}\eta_{2i}\bar{p}_{2i} \\
& + \sum_{j=1}^{N}\lambda_{1j}\left(\sum_{i=1}^{j}\bar{p}_{1i} + \theta_i\epsilon_1 - \sum_{i=1}^{j}E_{1i}\right) + \sum_{j=1}^{N}\lambda_{2j}\left(\sum_{i=1}^{j}\bar{p}_{2i} + \theta_i\epsilon_2 - \sum_{i=1}^{j}E_{2i}\right) \\
& + \sum_{i=1}^{N}\omega_i\left(\theta_i - 1\right) - \sum_{i=1}^{N}\nu_i\theta_i
\end{aligned} \tag{4.33}$$

where λ_{1i}, η_{1i}, λ_{2i}, η_{2i}, ω_i, ν_i are non-negative Lagrange multipliers. Differentiating with respect to $\bar{p}_{1i}$ and $\bar{p}_{2i}$, we obtain the following KKT optimality conditions

$$\frac{\bar{p}_{1i}}{\theta_i} = \left(\frac{1}{\sum_{j=i}^{N}\lambda_{1j}} - 1\right)^+, \qquad \frac{\bar{p}_{2i}}{\theta_i} = \left(\frac{1}{\sum_{j=i}^{N}\lambda_{2j}} - 1\right)^+ \tag{4.34}$$

along with the usual complementary slackness conditions [77]. The following two lemmas characterize the optimal power policy for problem (4.32). The proofs follow as in Lemmas 4.2 and 4.3, and are omitted for brevity.

Lemma 4.7 *In the optimal solution of problem (4.32), powers of both users are non-decreasing over time.*

Lemma 4.8 *In the optimal solution of problem (4.32), if a user's energy is saved from one time slot to the next, then the powers spent by this user in the two slots have to be equal.*

Next, we note that the optimal solution of problem (4.32) is not unique. For instance, assume that one solution of the problem required some energy to be transferred from the ith to the $(i+1)$st slot at both users, and that the optimal values of θ_i and θ_{i+1} are both less than 1. By Lemma 4.8, since we transferred some energy between the two slots, we must have equal powers in both slots. Now, if we transfer an extra amount of energy between the two slots, this allows us to do the following: 1) decrease the value of θ_i and increase that of θ_{i+1}, and 2) change the value of $\bar{p}_{ji}$ and $\bar{p}_{j(i+1)}$, $j = 1, 2$, correspondingly so that we obtain the same values of powers at the two slots as before. This leaves us with the same value for the objective function, as what we did is that we changed the values of the pre-log factors in a feasible manner while keeping the values inside the logs as they were. We can keep doing this until either slot $i+1$ is completely filled, i.e., $\theta_{i+1} = 1$, or all of the energy is transferred from slot i, i.e., $\theta_i = 0$.

We coin this type of policies as *deferred* policies; no new time slots are opened unless all time slots in the future are completely filled, i.e., $0 < \theta_i \leq 1$ iff $\theta_k = 1$, $\forall k = i+1, \ldots, N$. Consequently, $\{\theta_i\}_{i=1}^N$ will be non-decreasing. There can only be one unique optimal deferred policy for problem (4.32). In the sequel, we determine that policy.

4.3.2.1 Optimal Deferred Policy

Finding the optimal deferred policy relies on the fact that, by energy causality, energies can only be used after they have been harvested. To this end, we begin from the last slot, and make sure that it is completely filled, i.e., it has no burstiness, before opening up a previous slot. We apply a modified version of the single energy arrival result iteratively in a backward manner through two main phases: 1) deferring, and 2) refinement. These are illustrated as follows.

We first start by the deferring phase. The goal of this phase is to determine an initial feasible deferred policy. In the refinement phase, the optimality of such policy is investigated. We first initialize the energy status of each slot of both users by $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$, and start from the last slot and move backwards. In the kth slot, we start by examining the use of the kth slot energies in the kth slot only. This is done using the results of the single energy arrival (4.29). If the resulting $\theta_k < 1$, then we transfer some energy from previous slots forward to the kth slot until either it is completely filled, i.e., $\theta_k = 1$, or all previous slots' energies are exhausted. We test the possibility of the former condition by moving all energy from a previous slot $l < k$, and re-solving for θ_k. If the result is unity, then the energies of slot l can for sure fill out slot k. Next, we show how much energy is actually needed to do so.

We have two conditions to satisfy: 1) $\theta_k = 1$, and 2) powers of user j in slots l

and k are equal, $p_{jl} = p_{jk} \triangleq p'_j$, if user j transfers energy from slot l to k (according to Lemma 4.8). Let us denote the burstiness in slot l by θ'. Hence, if both users transfer energy, the optimal policy is found by solving the following problem

$$\begin{aligned}
\max_{\theta', p'_1, p'_2} \quad & \frac{1+\theta'}{2}\log(1+p'_1) + \frac{1+\theta'}{2}\log(1+p'_2) \\
\text{s.t.} \quad & (1+\theta')(p'_1+\epsilon_1) = S_{1l} + S_{1k} \\
& (1+\theta')(p'_2+\epsilon_2) = S_{2l} + S_{2k} \\
& 0 \leq \theta' \leq 1
\end{aligned} \tag{4.35}$$

Following the same analysis as in the single energy arrival case, we solve

$$f_1\left(1+\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{4.36}$$

On the other hand, if only the first user transfers energy, the optimal policy is found by replacing the second constraint in problem (4.35) by $\theta'(p_{2l} + \epsilon_2) = S_{2l}$, where $p_{2k} = S_{2k} - \epsilon_2$ in this case. This gives the following to solve for θ'

$$f_1\left(1+\theta'\right) \cdot f_2\left(\theta'\right) = e^{-2} \tag{4.37}$$

Similarly, if the transfer is done only from the second user we solve

$$f_1\left(\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{4.38}$$

In all the three cases of energy transfer above, the equations to solve have an increasing left hand side, and hence a unique solution. Finally, the optimal policy is the one that gives the maximum sum throughput among the feasible ones. It is worth noting that, by the concavity of the objective function, transferring energy from both users is optimal if feasible, since it equalizes arguments (powers) of a concave objective function [1].

If the initially resulting $\theta_k = 1$ in the kth slot, we do directional water-filling over the future slots, which gives the optimal sum rate [3]. Next, we check if energy should be transferred from a previous slot l from the first, second, or both users, in exactly the same way as above, i.e., by solving (4.36)-(4.38). If energy transfer (from either or both users) is feasible and gives a higher objective function, we do directional water-filling again from slot k over future slots, followed by repeating the above energy transfer checks once more. These inner iterations stop if either no energy transfer occurs, or no directional water-filling occurs. The deferring phase ends after examining the first slot. During this phase, we record how much energy is being moved forward to fill up future slots. Meters are put in between slots for that purpose.

In the refinement phase, the goal is to check whether the currently reached energy distribution is optimal. One reason it might not be optimal is that during the deferring phase, some excess amounts of energy can be transferred from, e.g., slot k forward unnecessarily without taking into account the energies available before slot k. We check the optimality of the deferring phase policy by performing two-slot updates starting from the last two slots going backwards. During the updates,

Algorithm 2 Optimal deferred policy

Phase 1: Deferring

Set $\boldsymbol{S}_1 = \boldsymbol{E}_1$, $\boldsymbol{S}_2 = \boldsymbol{E}_2$, $\boldsymbol{m}_1 = \boldsymbol{m}_2 = \boldsymbol{0}$, and $k = N$
while $k \geq 1$ **do**
  Using energies $\{S_{1k}, S_{2k}\}$, solve for θ_k using (4.29)
  if $\theta_k < 1$ **then**
    repeat
      Transfer all energy from slot $k - l$ to slot k
      Re-solve for θ_k using (4.29)
      if Slot k is completely filled **then**
        Find energy needed to fill it using (4.36)-(4.38)
      else $l \leftarrow \min\{l + 1, k - 1\}$
      end if
    until $\theta_k = 1$, or all previous energies are exhausted
  else
    repeat
      Directional water-filling over slots $\{k, \ldots, N\}$
      Check for energy transfer using (4.36)-(4.38)
    until No water-filling or energy transfer occur
  end if
  Update the energy status values $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$
  Update the meters' values $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$
  $k \leftarrow k - 1$
end while

Phase 2: Refinement

repeat
  for $k = 0 : N - 2$ **do**
    Update the energy status of slots $(N - k - 1, N - k)$ taking energy back if needed
  end for
until Meters' values $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ do not change
$\boldsymbol{p}_1^* = \boldsymbol{S}_1$, and $\boldsymbol{p}_2^* = \boldsymbol{S}_2$.

energy can be drawn back from future slots if this increases the objective function as long as it does not violate causality. This can be done by checking the values stored in the meters in between the slots. See [82] for details on how to update a given two slots. We summarize the steps of finding the optimal solution discussed in this section in Algorithm 2.

4.4 Decoding and Processing Costs Combined

We have thus far considered throughput maximizing policies for two-way channels with either decoding or processing costs. In this section, we study the general setting with *both* decoding and processing costs. In this setup, user j spends a decoding cost whenever it is receiving the other user's message, and in addition to that, it incurs a processing cost per unit time ϵ_j whenever it is operating. We allow user j to transmit for a θ_j portion of the time, and formulate the general problem where θ_1 can be different than θ_2 as follows

$$\begin{aligned} \max_{\boldsymbol{\theta},\boldsymbol{p}} \quad & \sum_{i=1}^{N} \frac{\theta_{1i}}{2} \log\left(1+p_{1i}\right) + \frac{\theta_{2i}}{2} \log\left(1+p_{2i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \theta_{1i} p_{1i} + \theta_{2i} a p_{2i} + \max(\theta_{1i}, \theta_{2i})\epsilon_1 \le \sum_{i=1}^{k} E_{1i}, \ \forall k \\ & \sum_{i=1}^{k} \theta_{2i} p_{2i} + \theta_{1i} a p_{1i} + \max(\theta_{1i}, \theta_{2i})\epsilon_2 \le \sum_{i=1}^{k} E_{2i}, \ \forall k \\ & 0 \le \theta_{1i}, \theta_{2i} \le 1, \quad \forall i \end{aligned} \tag{4.39}$$

Note that the above problem is a generalization of the problems considered in

Sections 4.2 and 4.3. On one hand, if we set $a = 0$, i.e., do not consider decoding costs, we get back to problem (4.31), after applying the synchronization argument to get $\theta_{1i} = \theta_{2i}$, $\forall i$. On the other hand, setting $\epsilon_1 = \epsilon_2 = 0$, i.e., not considering processing costs, and applying the change of variables $\bar{p}_j \triangleq \theta_j p_j$, $j = 1, 2$, we get

$$\begin{aligned}
\max_{\boldsymbol{\theta}, \bar{\boldsymbol{p}}} \quad & \sum_{i=1}^{N} \frac{\theta_{1i}}{2} \log\left(1 + \frac{\bar{p}_{1i}}{\theta_{1i}}\right) + \frac{\theta_{2i}}{2} \log\left(1 + \frac{\bar{p}_{2i}}{\theta_{2i}}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{k} \bar{p}_{1i} + a\bar{p}_{2i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} \bar{p}_{2i} + a\bar{p}_{1i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& 0 \leq \theta_{1i}, \theta_{2i} \leq 1, \quad \forall i
\end{aligned} \tag{4.40}$$

It is direct to see that the objective function is increasing in $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$, and therefore the maximum is attained at $\boldsymbol{\theta}_1^* = \boldsymbol{\theta}_2{}^* = 1$, i.e., we get back to problem (4.9). We solve problem (4.39) in the remainder of this chapter.

4.4.1 Single Energy Arrival

We first consider the case where each user harvests only one energy packet. Note that (4.39) is not a convex optimization problem. We apply the change of variables $\bar{p}_j \triangleq \theta_j p_j$, $j = 1, 2$, to get

$$\begin{aligned}
\max_{\theta_1, \theta_2, \bar{p}_1, \bar{p}_2} \quad & \frac{\theta_1}{2} \log\left(1 + \frac{\bar{p}_1}{\theta_1}\right) + \frac{\theta_2}{2} \log\left(1 + \frac{\bar{p}_2}{\theta_2}\right) \\
\text{s.t.} \quad & \bar{p}_1 + a\bar{p}_2 + \max(\theta_1, \theta_2)\epsilon_1 \leq E_1
\end{aligned}$$

$$
\begin{aligned}
&\bar{p}_2 + a\bar{p}_1 + \max(\theta_1, \theta_2)\epsilon_2 \leq E_2 \\
&0 \leq \theta_1, \theta_2 \leq 1
\end{aligned} \tag{4.41}
$$

which is now a convex optimization problem [77]. Next, we have the following lemma.

Lemma 4.9 *In the optimal solution of problem (4.41), $\theta_1^* = \theta_2^*$.*

Proof: Assume, e.g., $\theta_1^* < \theta_2^*$. Setting $\theta_1 = \theta_2^*$ is always feasible since the feasible set is only affected by the maximum of the θ_1 and θ_2. This strictly increases the objective function since it is monotonically increasing in θ_1. ■

Lemma 4.9 shows that it is optimal for the two users to be fully synchronized; they turn on, exchange information, and then turn off simultaneously, similar to what Lemma 4.6 states in the scenario with no decoding costs. This reduces the problem to the following

$$
\begin{aligned}
\max_{\theta, \bar{p}_1, \bar{p}_2} \quad & \frac{\theta}{2} \log\left(1 + \frac{\bar{p}_1}{\theta}\right) + \frac{\theta}{2} \log\left(1 + \frac{\bar{p}_2}{\theta}\right) \\
\text{s.t.} \quad & \bar{p}_1 + a\bar{p}_2 + \theta\epsilon_1 \leq E_1 \\
& \bar{p}_2 + a\bar{p}_1 + \theta\epsilon_2 \leq E_2 \\
& 0 \leq \theta \leq 1
\end{aligned} \tag{4.42}
$$

We have the following lemma regarding this problem, whose proof is similar to that of Lemma 4.1.

Lemma 4.10 *In the optimal solution of problem (4.42), at least one user consumes all its energy.*

Next, we solve (4.42) for the case $a = 1$. By the previous lemma, we have $\bar{p}_1^* + \bar{p}_2^* = \min\{E_1 - \theta^*\epsilon_1, E_2 - \theta^*\epsilon_2\}$, and by concavity of the objective function, we further have $\bar{p}_1^* = \bar{p}_2^*$. Substituting the powers back in the objective function, we get a reduced problem in only one variable θ

$$\max_{0 \le \theta \le \theta_m} \quad \theta \log\left(1 + \frac{\min\{E_1 - \theta\epsilon_1, E_2 - \theta\epsilon_2\}}{2\theta}\right) \tag{4.43}$$

where $\theta_m \triangleq \min\left\{1, \frac{E_1}{\epsilon_1}, \frac{E_2}{\epsilon_2}\right\}$ assures the positivity of the powers. Note that by monotonicity of the log, and non-negativity of θ, we have

$$\begin{aligned} &\theta \log\left(1 + \frac{\min\{E_1 - \theta\epsilon_1, E_2 - \theta\epsilon_2\}}{2\theta}\right) \\ &\qquad = \min\left\{\theta \log\left(1 + \frac{E_1 - \theta\epsilon_1}{2\theta}\right), \theta \log\left(1 + \frac{E_2 - \theta\epsilon_2}{2\theta}\right)\right\} \end{aligned} \tag{4.44}$$

It is direct to show that each of the terms inside the minimum expression on the right hand side of the above equation is concave in θ, and therefore the minimum of the two is also concave in θ [77]. Hence, problem (4.43) is a convex optimization problem [77]. Let us define $\bar{\theta} \triangleq \frac{E_1 - E_2}{\epsilon_1 - \epsilon_2}$ as the value of θ at which $E_1 - \theta\epsilon_1 = E_2 - \theta\epsilon_2$. We now consider two different cases.

The first case is when $\bar{\theta} \notin [0, \theta_m]$, then the minimum expression in the objective function reduces to only one of its two terms for all θ feasible. Let us assume without loss of generality that it is equal to $E_1 - \theta\epsilon_1$. Hence, taking the derivative of the objective function and setting it to 0, we solve the following for θ

$$\log\left(1 - \frac{\epsilon_1}{2} - \frac{E_1}{2\theta}\right) = \frac{E_1/2\theta}{1 - \epsilon_1/2 + E_1/2\theta} \tag{4.45}$$

The above equation has a unique solution since both sides are monotone in θ; the term on the left is higher than the term on the right as θ approaches 0; and is lower than the term on the right as θ approaches $\frac{E_1}{\epsilon_1}$. We denote this unique solution by $\hat{\theta}$. We note that in this problem, we always have $\theta^* > 0$; we also have $\theta^* = \theta_m$ only if $\theta_m = 1$, or else the throughput is zero. Thus, if $\theta_m < 1$, then $\hat{\theta}$ is always feasible and $\theta^* = \hat{\theta}$. While if $\theta_m = 1$, then $\hat{\theta}$ might not be feasible, and therefore in general we have $\theta^* = \min\{\hat{\theta}, 1\}$. This concludes the first case.

The second case is when $\bar{\theta} \in [0, \theta_m]$. In this case, depending on the sign of $\epsilon_1 - \epsilon_2$, the minimum expression in the objective function is given by one term in the interval $[0, \bar{\theta}]$ (let us assume it to be $E_1 - \theta\epsilon_1$ without loss of generality), and is given by the other term $(E_2 - \theta\epsilon_2)$ in the interval $[\bar{\theta}, \theta_m]$. We solve the problem in this case sequentially as follows: We solve (4.45) for $\hat{\theta}_1$ and compute $\theta_1^* = \min\{\hat{\theta}_1, 1\}$. If θ_1^* is less than $\bar{\theta}$ then, by concavity of the objective function, it is the optimal solution. Else, if $\theta_1^* \geq \bar{\theta}$, we solve the following equation

$$\log\left(1 - \frac{\epsilon_2}{2} - \frac{E_2}{2\theta}\right) = \frac{E_2/2\theta}{1 - \epsilon_2/2 + E_2/2\theta} \tag{4.46}$$

for $\hat{\theta}_2$ and compute $\theta_2^* = \min\{\hat{\theta}_2, 1\}$, which will now be no less than $\bar{\theta}$, and is equal to the optimal solution. We finally note that $\theta^* = \bar{\theta}$ iff $\theta_1^* = \theta_2^* = \bar{\theta}$. This concludes the second case.

Next, we discuss the case $a < 1$ (similar arguments follow for the case $a > 1$, and are omitted for brevity). We have the following lemma in this case, whose proof is similar to that of Lemma 4.1.

Lemma 4.11 *If the energies and processing costs are such that $E_1 - \theta\epsilon_1$ is less (resp., larger) than $E_2 - \theta\epsilon_2$ for all θ feasible, then the first (resp., second) user consumes all its energy.*

We solve the problem by assuming the situation of the above lemma is true, i.e., one user is energy tight for all θ feasible. If this is not the case, then as we did in the $a = 1$ case above, we solve the problem twice assuming one user is tight at each time, and check which is feasible (or equivalently pick the solution with higher sum throughput). Thus, without loss of generality, we assume the first user consumes all its energy, i.e., we have $\bar{p}_1 = E_1 - \theta\epsilon_1 - a\bar{p}_2$. Substituting this in problem (4.42), we get the following

$$\begin{aligned} \max_{\theta, \bar{p}_2} \quad & \frac{\theta}{2}\log\left(1 + \frac{E_1 - \theta\epsilon_1 - a\bar{p}_2}{\theta}\right) + \frac{\theta}{2}\log\left(1 + \frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & 0 \leq \bar{p}_2 \leq \frac{E_1 - \theta\epsilon_1}{a} \\ & \bar{p}_2 \leq \frac{E_2 - aE_1 - \theta(\epsilon_2 - a\epsilon_1)}{1 - a^2} \\ & 0 \leq \theta \leq \theta_m \end{aligned} \tag{4.47}$$

where the upper bound in the first constraint assures the non-negativity of the first user's power. We note that if $\bar{p}_2^* \in \{0, \frac{E_1-\theta^*\epsilon_1}{a}\}$, i.e., if either of the two users is not transmitting, the problem reduces to the following in terms of only one variable θ

$$\max_{0\le\theta\le\theta_m} \quad \frac{\theta}{2}\log\left(1+\frac{E_1-\theta\epsilon_1}{\theta}\right) \tag{4.48}$$

which can be solved in a similar manner as we solved problem (4.43). On the other hand, if the third constraint is tight, i.e., if the second user also consumes all its energy, the problem becomes

$$\max_{\tilde{\theta}_l\le\theta\le\tilde{\theta}_m} \quad \frac{\theta}{2}\log\left(1+\frac{E_2-aE_1-\theta(\epsilon_2-a\epsilon_1)}{(1-a^2)\theta}\right)+\frac{\theta}{2}\log\left(1+\frac{E_1-aE_2-\theta(\epsilon_1-a\epsilon_2)}{(1-a^2)\theta}\right) \tag{4.49}$$

where $\tilde{\theta}_l$ and $\tilde{\theta}_m$ are such that $E_1-aE_2\ge\theta(\epsilon_1-a\epsilon_2)$ and $E_2-aE_1\ge\theta(\epsilon_2-a\epsilon_1)$, i.e., to assure non-negativity of powers. Note that the objective function in the above problem is concave. Hence, following a Lagrangian approach [77], we solve the following for θ

$$\tilde{f}_1(\theta)\cdot\tilde{f}_2(\theta)=e^{-2} \tag{4.50}$$

where $\tilde{f}_j(\theta)$, $j=1,2$ is defined as

$$\tilde{f}_j(\theta)\triangleq\frac{e^{(\tilde{\epsilon}_j-1)/((\tilde{E}_j/\theta)-(\tilde{\epsilon}_j-1)}}{(\tilde{E}_j/\theta)-(\tilde{\epsilon}_j-1)} \tag{4.51}$$

with $\tilde{E}_j \triangleq E_j - aE_k$ and $\tilde{\epsilon}_j \triangleq \epsilon_j - a\epsilon_k$, $j \neq k$. We note that the above equation is similar to (4.29), in the case with only processing costs. It can be shown by simple first derivative analysis that $\tilde{f}_1$ and $\tilde{f}_2$ are both increasing in θ, and therefore (4.50) has a unique solution. Let us denote such solution by $\tilde{\theta}$. Finally, by concavity of the objective function, the optimal θ^* in this case is given by projecting $\tilde{\theta}$ onto the feasible set $\{\theta : \tilde{\theta}_l \leq \theta \leq \tilde{\theta}_m\}$ [77].

Now that we know how to solve problem (4.47) when either of the first two constraints is tight, we proceed to solve the problem in general as follows. We first solve the problem assuming $\bar{p}_2^*$ is an interior point, i.e., neither of the first two constraints is tight. If the solution in this case is feasible, then it is optimal. Else, by concavity of the objective function, we project the solution onto the feasible set $\left\{\bar{p}_2 : 0 \leq \bar{p}_2 \leq \min\left\{\frac{E_1-\theta\epsilon_1}{a}, \frac{E_2-aE_1-\theta(\epsilon_2-a\epsilon_1)}{1-a^2}\right\}\right\}$. In case $\bar{p}_2$ is given by the upper limit in this feasible set, we solve the problem twice assuming the minimum expression is given by one of its terms in each, and pick the one with higher throughput.

Finally, it remains to present the interior point solution. We introduce the following Lagrangian for the problem in this case

$$\mathcal{L} = -\frac{\theta}{2}\log\left(1 + \frac{E_1 - \theta\epsilon_1 - a\bar{p}_2}{\theta}\right) - \frac{\theta}{2}\log\left(1 + \frac{\bar{p}_2}{\theta}\right) + \omega(\theta - \theta_m) \tag{4.52}$$

Taking the derivative with respect to $\bar{p}_2$ and θ and equating to 0, we get the following

$$a\left(1 + \frac{\bar{p}_2}{\theta}\right) = 1 - \epsilon_1 + \frac{E_1 - a\bar{p}_2}{\theta} \tag{4.53}$$

$$\log\left(1+\frac{\bar{p}_2}{\theta}\right)+\log\left(1-\epsilon_1+\frac{E_1-a\bar{p}_2}{\theta}\right)=\frac{\bar{p}_2/\theta}{1+\bar{p}_2/\theta}+\frac{(E_1-a\bar{p}_2)/\theta}{1-\epsilon_1+(E_1-a\bar{p}_2)/\theta}+\omega \tag{4.54}$$

substituting the first equation in the second, and denoting $y \triangleq 1+\bar{p}_2/\theta$, we further get

$$\log(y)=1-\frac{1}{2}\log(a)-\frac{\frac{1}{2}\left(1+(1-\epsilon_1)/a\right)}{y}+\omega/2 \tag{4.55}$$

which has a unique solution, y^*, for $y \geq 1$. If $\omega^* > 0$, then by complementary slackness, $\theta^* = \theta_m$, and $\bar{p}_2^*$ is found by substituting in (4.53), else if $\omega^* = 0$, then θ^* is found by substituting y^* also in (4.53). By that, we conclude our analysis of the single arrival case.

4.4.2 Multiple Energy Arrivals

In this section, we study the multiple energy arrival problem. Following the same synchronization argument as in Section 4.3.2, problem (4.39) reduces to

$$\begin{aligned}
\max_{\boldsymbol{\theta},\bar{\boldsymbol{p}}_1,\bar{\boldsymbol{p}}_2} \quad & \sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)+\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right)\\
\text{s.t.} \quad & \sum_{i=1}^{k}\bar{p}_{1i}+a\bar{p}_{2i}+\theta_i\epsilon_1 \leq \sum_{i=1}^{k}E_{1i}, \quad \forall k\\
& \sum_{i=1}^{k}\bar{p}_{2i}+a\bar{p}_{1i}+\theta_i\epsilon_2 \leq \sum_{i=1}^{k}E_{2i}, \quad \forall k\\
& 0 \leq \theta_i \leq 1, \quad \forall i
\end{aligned} \tag{4.56}$$

which is a convex optimization problem [77]. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\sum_{i=1}^{N} \frac{\theta_i}{2} \log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right) - \sum_{i=1}^{N} \frac{\theta_i}{2} \log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) + \sum_{i=1}^{N} \omega_i(\theta_i - 1) - \sum_{i=1}^{N} \eta_i \theta_i \\ &+ \sum_{k=1}^{N} \lambda_{1k} \left(\sum_{i=1}^{k} \bar{p}_{1i} + a\bar{p}_{2i} + \theta_i \epsilon_1 - \sum_{i=1}^{k} E_{1i} \right) \\ &+ \sum_{k=1}^{N} \lambda_{2k} \left(\sum_{i=1}^{k} \bar{p}_{2i} + a\bar{p}_{1i} + \theta_i \epsilon_2 - \sum_{i=1}^{k} E_{2i} \right) \end{aligned} \tag{4.57}$$

Taking the derivative with respect to $\bar{p}_{1i}$ and $\bar{p}_{2i}$ and equating to 0 we get

$$\frac{\bar{p}_{1i}}{\theta_i} = \left(\frac{1}{\sum_{k=i}^{N} \lambda_{1k} + a\lambda_{2k}} - 1 \right)^{+} \tag{4.58}$$

$$\frac{\bar{p}_{2i}}{\theta_i} = \left(\frac{1}{\sum_{k=i}^{N} \lambda_{2k} + a\lambda_{1k}} - 1 \right)^{+} \tag{4.59}$$

along with the complementary slackness conditions [77]. Therefore, we have the following lemma for this problem. The proof follows using similar arguments as in Lemmas 4.2, 4.3, and 4.4.

Lemma 4.12 *In the optimal policy of problem (4.56), the powers of both users are non-decreasing; increase only if at least one user consumes all energy; and increase synchronously.*

We note that, as discussed in Section 4.3.2, the optimal policy for problem (4.56) is not unique. Using similar arguments, any optimal policy can be transferred into a (unique) deferred policy. Hence, in the remainder of this chapter, we find the optimal deferred policy for problem (4.56). We present an algorithm that is a combination of the ideas used in Sections 4.2 and 4.3 as follows.

We start by a deferring phase similar to the one discussed in Section 4.3.2.1. We highlight the main differences in the following. First, to determine how much energy is needed to be transferred to fill a given slot k from a previous slot l, we assume that both users transfer energy, and similar to problem (4.35), we solve the following single energy arrival problem

$$\begin{aligned}
\max_{\theta,\bar{p}_1,\bar{p}_2} \quad & \frac{1+\theta}{2}\log\left(1+\frac{\bar{p}_1}{1+\theta}\right)+\frac{1+\theta}{2}\log\left(1+\frac{\bar{p}_2}{1+\theta}\right) \\
\text{s.t.} \quad & \bar{p}_1+a\bar{p}_2+(1+\theta)\epsilon_1 \leq S_{1l}+S_{1k} \\
& \bar{p}_2+a\bar{p}_1+(1+\theta)\epsilon_2 \leq S_{2l}+S_{2k} \\
& 0 \leq \theta \leq 1
\end{aligned} \tag{4.60}$$

After solving this problem, we set $\theta_{k-1}=\theta^*$, and $p_{j(k-1)}=p_{jk}=(1+\theta^*)\bar{p}_j^*$, $j=1,2$. The resulting policy is optimal if feasible since it equalizes powers [1]. If not, then we need to check the other ways of transfer, namely, transferring from the first user only, or from the second user only. We also need to assume an energy consumption strategy in slot k, i.e., which user consumes all its energy. We solve for all possible strategies, and pick the one with maximum sum throughput among the feasible ones. We highlight the solution of one energy consumption strategy in the following discussion. The rest follows similarly.

We discuss the strategy of transferring energy only from the second user in slot l, and that the second user consumes all its energy in slot k. Towards that end, we first fix $\theta_l = \theta$, and then, as discussed in Section 4.2.2, we solve the following equivalent problem in (r, δ)

$$
\begin{aligned}
\max_{r,\delta} \quad & \frac{1}{2}\log\left(1+\frac{S_{1l}-aS_{2l}-\theta(\epsilon_1-a\epsilon_2)+ar}{\theta(1-a^2)}\right)+\frac{1}{2}\log\left(1+\frac{\delta}{a}\right) \\
& +\frac{1}{2}\log\left(1+\frac{S_{2l}-aS_{1l}-\theta(\epsilon_2-a\epsilon_1)-r}{\theta(1-a^2)}\right)+\frac{1}{2}\log\left(1+S_{2k}+r-\delta\right) \\
\text{s.t.} \quad & 0 \le \delta \le S_{2k}-\epsilon_2+r \\
& r \ge \left(\frac{aS_{2l}-S_{1l}+\theta(\epsilon_1-a\epsilon_2)}{a}\right)^+ \\
& r \le \min\left\{S_{2l}, S_{2l}-aS_{1l}-\theta(\epsilon_2-a\epsilon_1)\right\} \\
& \delta \le \frac{a}{1-a^2}\left(S_{1k}-aS_{2k}-(\epsilon_1-a\epsilon_2)-ar\right)
\end{aligned} \tag{4.61}
$$

We note that the above problem is exactly the same as problem (4.15) if we set $\theta = 1$, and $\epsilon_1 = \epsilon_2 = 0$. With processing costs, the problem can be solved similarly. We solve the above problem for all given θ and do a one dimensional line search to find the optimal θ_l^*.

By the end of the deferring phase above, there will exist a time slot k^*, after which all time slots are completely filled, and before which all time slots are empty, i.e., we will have $\theta_l = 1$, $\forall l > k^*$; $\theta_l = 0$, $\forall l < k^*$; and $\theta_{k^*} \le 1$. We can now focus on the non-empty time slots $k^*, \dots, N$. Each will have a certain energy distribution $\{S_{ji}\}_{i=k^*}^N$, $j = 1, 2$, from the deferring phase. We also record the amount of energy transferred to future slots in meters as we did in Section 4.2.2. Next, we check if such energy distributions need improvement. We note that if $\theta_{k^*} = 1$, then the problem becomes a decoding cost problem that can be solved iteratively as discussed in Section 4.2.2 with equivalent energies: $\{S_{ji} - \epsilon_j\}_{i=k^*}^{N}$, $j = 1, 2$. If $\theta_{k^*} < 1$, however, then as we reach slots $\{k^*, k^*+1\}$ in the two-slot updates, we update the distributions by finding the best energy transfer strategy, i.e., transfer from only one or both users, as discussed in problems (4.60) and (4.61). Iterations converge to the optimal solution.

4.5 Numerical Results

4.5.1 Deterministic Arrivals

In this section we present numerical examples to further illustrate our results. We begin by the building blocks of the proposed algorithms; two-slot systems. We start with the case with only decoding costs and consider a system with energies $\boldsymbol{E}_1 = [0.5, 3.5]$ and $\boldsymbol{E}_2 = [1, 1.5]$. The decoding power factor is equal to $a = 0.5$. We first solve for each slot independently using the single arrival result to get $\boldsymbol{p}_1 = [0, 1]$ and $\boldsymbol{p}_2 = [0.33, 1.33]$. Then, we find the optimal solution as discussed in Section 4.2.2.1. First, we check the constant-power strategy, where neither user consumes its energy in the first slot, and solve a single arrival problem with average energy arrivals $\bar{E}_1 = 2$ and $\bar{E}_2 = 1.25$ to get $\bar{p}_1 = 1.75$ and $\bar{p}_2 = 0.375$, which are found infeasible. Thus, we move to check the second consumption strategy: the first user consumes all energy in the first slot while the second user consumes all energy in the second slot, i.e., we solve problem (4.15). We first remove the last constraint, and take $\delta(r) = \frac{1+E_{22}+r-a}{2}$, the middle term of (4.19), and solve for r using (4.20). This gives $r = 0.55$, which satisfies the middle constraint in (4.19), thus the assumed $\delta(r)$ is

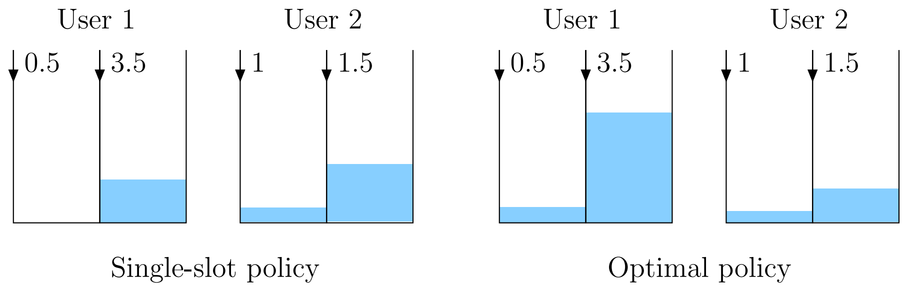


Figure 4.2: Two-slot system with only decoding costs.

correct, and gives $\delta = 1.27$. Finally, we check the relaxed (last) constraint of (4.15); we find that it is satisfied with strict inequality. Therefore, $(r^* = 0.55, \delta^* = 1.27)$ is the optimal solution for this consumption strategy. The corresponding powers are given by $\boldsymbol{p}_1 = [0.36, 2.55]$ and $\boldsymbol{p}_2 = [0.26, 0.77]$. Next, we check the other strategies. Among the feasible ones, we find that the maximum throughput is given by that of the second strategy above, and is therefore the optimal solution of this two-slot system. In Fig. 4.2, we show the single-slot solution on the left and the optimal solution on the right of the figure. The height of the water in blue represents the power level of a user in a given slot. We note that the first user's optimal power in the first slot is larger than the corresponding single-slot power allocation. That is because the second user's optimal power is smaller than the single-slot power allocation, which gives more room for the first user to transmit. This shows how decoding costs closely couple the performance of the two users.

Next, we consider the case with only processing costs, with energies $\boldsymbol{E}_1 = [0.5, 1]$ and $\boldsymbol{E}_2 = [1, 1]$, and processing costs $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.4$. In Fig. 4.3, we present one feasible, and two optimal, power policies. The height of the water levels in blue represents the actual transmit powers $\{p_{1i}, p_{2i}\}$, while the width represents

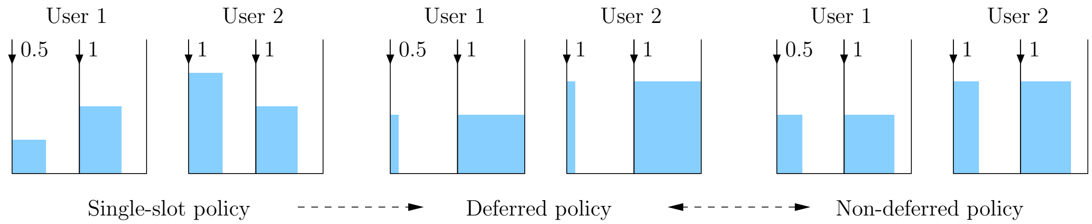


Figure 4.3: Optimal deferred policy in a two-slot system with only processing costs.

the burstiness $\{\theta_i\}$, for $i = 1, 2$. On the left, we solve for each slot independently using the single arrival result. This gives a non-deferred policy with $\boldsymbol{\theta} = [0.47, 0.65]$, $\boldsymbol{p}_1 = [0.57, 1.04]$, $\boldsymbol{p}_2 = [1.75, 1.14]$, and a sum throughput equal to 0.541. We then transfer all the energy from the 1st to the 2nd slot and re-solve for θ_2 using (4.29). The result is $\theta_2 = 1$, which means that the 1st slot's energies are capable of totally filling the 2nd slot. We therefore compute the exact amount needed to do so by setting $\theta_2 = 1$ and solving for $\theta_1 = \theta'$ assuming both users transfer energy, i.e., using (4.36). This gives $\theta_1 = 0.122$, $\boldsymbol{p}_1^* = [0.84, 0.84]$, $\boldsymbol{p}_2^* = [1.39, 1.39]$, and a sum throughput equal to 1.656. This transfer strategy is found feasible, and hence optimal. We show the optimal deferred policy at the middle of Fig. 4.3. Finally, on the right of Fig. 4.3, we show another optimal, yet non-deferred, power policy. This is simply done by shifting some of the water back, in a feasible manner, from slot 2 to slot 1. Namely, we increase the value of θ_1 to 0.35 and decrease that of θ_2 to 0.772, with the same transmit powers. This is a feasible non-deferred policy, and gives the same objective function of 1.656. This shows the non-uniqueness of the solution of problem (4.32).

We now solve a more involved four-slot system with energies $\boldsymbol{E}_1 = [0.9, 0.1, 3, 0.8]$ and $\boldsymbol{E}_2 = [0.8, 1.5, 2, 2]$. Here we consider both decoding and processing costs with

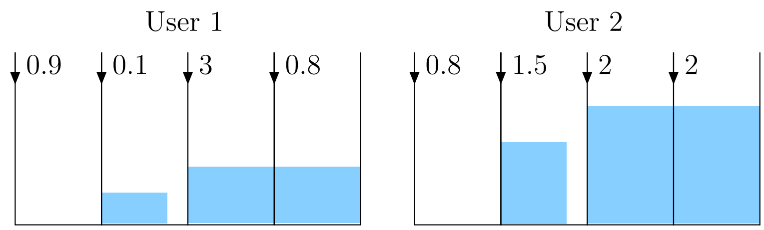


Figure 4.4: Optimal policy in a four-slot system with both decoding and processing costs.

parameters $a = 0.7$, $\epsilon_1 = 0.3$, and $\epsilon_2 = 0.6$. We begin by the initialization step; filling up later slots first in a backward manner. This leaves us with an energy distribution of $\boldsymbol{S}_1 = [0, 1, 1.7788, 2.021]$ and $\boldsymbol{S}_2 = [0, 0.936, 3.236, 2.128]$ at the first and the second user, respectively. We then begin the two-slot updates to check whether the given distributions need improvement. With the possibility of drawing back energy as feasible as imposed by the meters put between slots, our algorithm converges to the optimal solution in 8 iterations. The optimal powers are given by $\boldsymbol{p}_1^* = [0, 0.3585, 0.65, 0.65]$, $\boldsymbol{p}_2^* = [0, 0.9407, 1.357, 1.357]$, and the deferred burstiness is given by $\boldsymbol{\theta}^* = [0, 0.76, 1, 1]$. We see that the optimal powers are non-decreasing, and increase synchronously, as stated in Lemma 4.12, and that $\{\theta_i^*\}$ is non-decreasing, which is an attribute of a deferred policy. The optimal policy is shown in Fig. 4.4. Next, we remove the decoding costs and solve the same problem with only processing costs as discussed in Section 4.3.2. We reach the optimal deferred policy after 5 iterations, which is given by $\boldsymbol{p}_1^* = [0.67, 0.67, 1.6, 1.6]$, $\boldsymbol{p}_2^* = [1.47, 1.47, 1.47, 1.47]$, and $\boldsymbol{\theta}^* = [0.033, 1, 1, 1]$. We notice that the first time slot is utilized in this case, when the decoding costs are removed. Finally, we remove the processing costs and solve the same problem with only decoding costs as discussed in Section 4.2.2. After 7 iterations, we get the optimal $\boldsymbol{p}_1^* = [0.1, 0.1, 0.8, 0.8]$

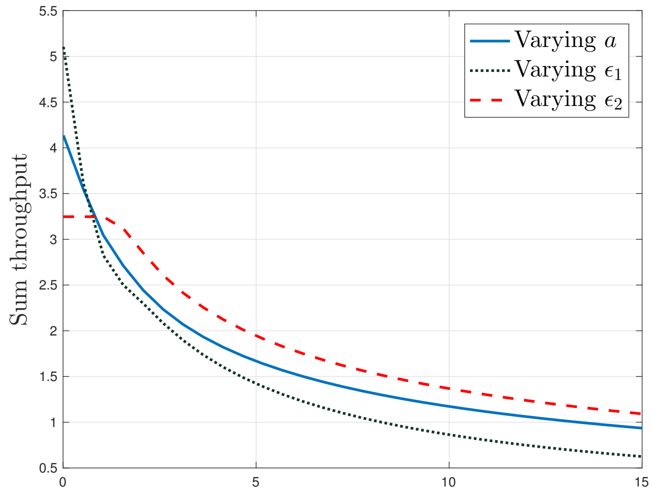


Figure 4.5: Effect of processing and decoding costs on the sum rate in a five-slot system.

and $\boldsymbol{p}_2^* = [0.57, 0.57, 1.57, 1.57]$.

In Fig. 4.5, we show the effect of decoding and processing costs on the sum rate. We consider a five-slot system with $\boldsymbol{E}_1 = [2, 3, 1, 1, 5]$ and $\boldsymbol{E}_2 = [4, 2, 2, 3, 3]$. Initially we set $a = 0.7$, $\epsilon_1 = 0.8$, and $\epsilon_2 = 0.5$. We then vary one parameter and fix the rest, and observe how it affects the sum rate. As expected, adding costs decreases the achievable throughput as we see from the figure. We also note that the sum rate is almost constant for initial small values of ϵ_2. That is due to the fact that the second user's processing costs are not the bottleneck to the system in this range. In fact, the first user is the bottleneck in this range. This shows how the two users are strongly coupled in this two-way setting with decoding and processing costs.

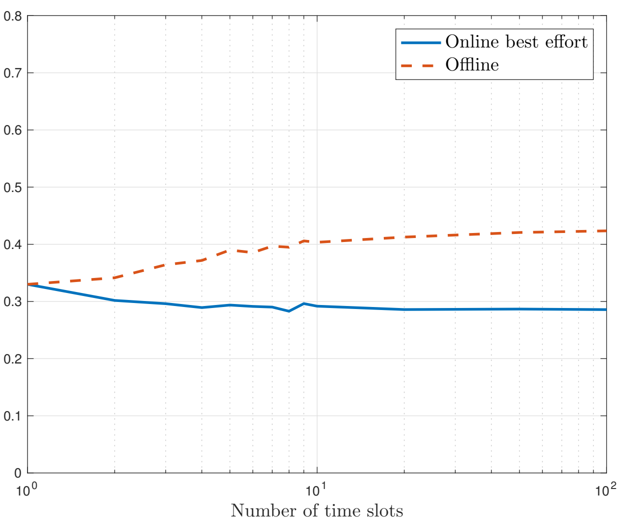


Figure 4.6: Comparison of an online best effort scheme and the optimal offline scheme.

4.5.2 Stochastic Arrivals

We now discuss online scenarios where energy is known causally after being harvested, while only its statistics is known a priori. We present a best effort online scheme to compare with our optimal offline solution. Namely, we assume that the energy harvesting process is i.i.d. with mean μ, and that in time slot i, the jth user energy consumption is bounded by $\min\{b_{ji}, \mu\}$, where b_{ji} is the battery state of user j in slot i, capturing the energy arrival at slot i, E_{ji}, and the residual from previous slots, if any. This scheme decouples the multiple arrival problem into N single arrival problems that can be solved as discussed in Section 4.4.1, without violating the causal knowledge of the energy arrival information. In Fig. 4.6, we plot the average throughput of this online policy for different time slots, and compare it with the optimal offline policy discussed in Section 4.4.2. Energies follow a uniform distribution on $[0,3]$, processing costs are $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.5$, and the decoding cost factor is $a = 0.7$. We run the simulations multiple times for every time slot and take the average, and then plot the sum rate divided by the number of time slots. We see from the figure that as the number of time slots increases, the gap between the online and the offline throughputs increases, and then converges to a constant value. This is due to the fact that in this best effort policy the problem is decoupled as discussed above, and the optimal energy distribution among the slots is no longer achieved, and therefore, the loss of optimality increases with the increase in the number of slots. However, as N grows large, and since we are using i.i.d. arrivals, the best effort policy's loss with respect to the optimal offline one converges to a constant value.

4.6 Conclusion

In this chapter, we designed throughput-optimal offline power scheduling policies in an energy harvesting two-way channel where users incur decoding and processing costs. Each user spends a decoding power that is an exponential function of the incoming rate, and in addition, incurs a constant processing power as long is it is communicating. We first studied the case with only decoding costs, followed by that with only processing costs. We then formulated the general problem with both decoding and processing costs in a single setting, and provided an iterative algorithm to find the optimal power policy in this case using insights from the solutions of the case with only decoding and only processing costs.

CHAPTER 5

Online Fixed Fraction Policies in Energy Harvesting Communication Systems

5.1 Introduction

In this chapter, we consider power scheduling policies for single-user energy harvesting communication systems, where the goal is to characterize *online policies* that maximize the long term average utility, for general concave and monotonically increasing utility functions. The transmitter relies on energy harvested from nature to send its messages to the receiver, and is equipped with a finite-sized battery to store its harvested energy, see Fig. 5.1. Energy packets are i.i.d. over time slots, and are revealed causally to the transmitter. Only the average energy arrival rate is known a priori. We first characterize the optimal solution for the case of Bernoulli arrivals. Then, for general i.i.d. arrivals, we first show that *fixed fraction policies*, in which a fixed fraction of the battery state is consumed in each time slot, are within a constant *multiplicative gap* from the optimal solution for all energy arrivals and battery sizes. We then derive a set of sufficient conditions on the utility function to guarantee that fixed fraction policies are within a constant *additive gap* as well

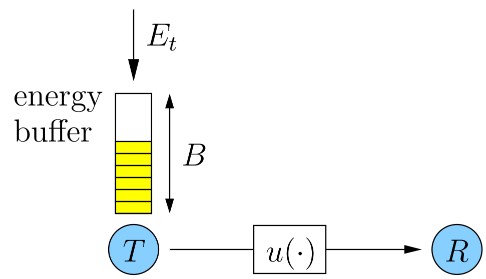


Figure 5.1: Single-user energy harvesting channel with general utility function.

from the optimal solution. We then consider a specific scenario where a sensor node collects samples from a Gaussian source and sends them to a destination node over a Gaussian channel. The goal is to minimize the long term average distortion of the source samples received at the destination. We study two problems: the first is when sampling is cost-free, and the second is when there is a sampling cost incurred whenever samples are collected. We show that fixed fraction policies achieve a long term average distortion that lies within a constant *additive gap* from the optimal solution for all energy arrivals and battery sizes. For the problem with sampling costs, the transmission policy is bursty; the sensor may collect samples and transmit for only a portion of the time.

5.2 General Utility Functions

We consider a single-user channel where the transmitter relies on energy harvested from nature to send its messages to the receiver. Energy arrives (is harvested) in packets of amount E_t at the beginning of time slot t. Without loss of generality, a slot duration is normalized to one time unit. Energy packets follow an i.i.d. distribution with a given mean. Our setting is *online:* the amounts of energy are known causally in time, i.e., after being harvested. Only the mean of the energy arrivals is known a priori. Energy is saved in a battery of finite size B.

Let u be a differentiable, concave, and monotonically increasing function representing a general utility (reward) function, with $u(0) = 0$ and $u(x) > 0$ for $x > 0$, and let g_t denote the transmission power used in time slot t. By allocating power g_t in time slot t, the transmitter achieves $u\left(g_t\right)$ instantaneous reward. Denoting $\mathcal{E}^t \triangleq \{E_1, E_2, \ldots, E_t\}$, a feasible online policy $\boldsymbol{g}$ is a sequence of mappings $\{g_t : \mathcal{E}^t \to \mathbb{R}_+\}$ satisfying

$$0 \leq g_t \leq b_t \triangleq \min\{b_{t-1} - g_{t-1} + E_t, B\}, \quad \forall t \tag{5.1}$$

with $b_1 \triangleq B$ without loss of generality (using similar arguments as in [54, Appendix B]). We denote the above feasible set in (5.1) by $\mathcal{F}$. Given a feasible policy $\boldsymbol{g}$, we define the n-horizon average reward as

$$\mathcal{U}_n(\boldsymbol{g}) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} u\left(g_t\right)\right] \tag{5.2}$$

Our goal is to design online power scheduling policies that maximize the long term average reward subject to (online) energy causality constraints. That is, to characterize

$$\rho^* \triangleq \max_{\boldsymbol{g}\in\mathcal{F}} \lim_{n\to\infty} \mathcal{U}_n(\boldsymbol{g}) \tag{5.3}$$

We note that problem (5.3) can be solved by dynamic programming techniques since the underlying system evolves as a Markov decision process. However, the optimal solution using dynamic programming is usually computationally demanding with few structural insights. Therefore, in the sequel, we aim at finding relatively simple online power control policies that are provably within a constant additive and multiplicative gap from the optimal solution for all energy arrivals and battery sizes.

We assume that $E_t \leq B$ $\forall t$ a.s., since any excess energy above the battery capacity cannot be saved or used. Let $\mu = \mathbb{E}[E_t]$, where $\mathbb{E}[\cdot]$ is the expectation operator, and define

$$q \triangleq \frac{\mathbb{E}[E_t]}{B} \tag{5.4}$$

Then, we have $0 \leq q \leq 1$ since $E_t \leq B$ a.s. We define the power control policy as follows [54]

$$\tilde{g}_t = qb_t \tag{5.5}$$

That is, in each time slot, the transmitter uses a fixed fraction of its available energy in the battery. Such policies were first introduced in [54], and coined *fixed fraction policies* (FFP). Clearly such policies are always feasible since $q \leq 1$. Let $\rho(\tilde{\boldsymbol{g}})$ be the long term average utility under the FFP $\{\tilde{g}_t\}$. Next, we find the *optimal* solution of problem (5.3) under the specific case of Bernoulli energy arrivals. After that, we discuss how the FFP performs under general i.i.d. energy arrivals.

5.2.1 Bernoulli Energy Arrivals

Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process with mean μ as follows

$$\hat{E}_t = \begin{cases} B, & \text{w.p. } p \\ 0, & \text{w.p. } 1-p \end{cases} \tag{5.6}$$

Note that under such specific energy arrival setting, whenever an energy packet arrives, it completely fills the battery, and resets the system. This constitutes a *renewal*. Then, by [83, Theorem 3.6.1] (see also [54]), the following holds for any power control policy $\boldsymbol{g}$

$$\begin{aligned} \lim_{n\to\infty} \hat{\mathcal{U}}_n(\boldsymbol{g}) &= \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} u\left(g_t\right)\right] \\ &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} u\left(g_t\right)\right] \end{aligned} \tag{5.7}$$

where $\hat{\mathcal{U}}_n(\boldsymbol{g})$ is the n-horizon average utility under Bernoulli arrivals, and L is a random variable denoting the inter-arrival time between energy arrivals, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$.

Using the FFP defined in (5.5) in (5.7) gives a lower bound on the long term average utility. Note that by (5.6), the fraction q in (5.4) is now equal to p. Also, the battery state decays exponentially in between energy arrivals, and the FFP is

$$\tilde{g}_t = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{5.8}$$

for all time slots t, where the second equality follows since $pB = \mu$. Using (5.7), problem (5.3) in this case reduces to

$$\begin{aligned} \max_{\boldsymbol{g}} \quad & \sum_{t=1}^{\infty} p(1-p)^{t-1} u\left(g_t\right) \\ \text{s.t.} \quad & \sum_{t=1}^{\infty} g_t \leq B, \quad g_t \geq 0, \quad \forall t \end{aligned} \tag{5.9}$$

which is a convex optimization problem. The Lagrangian is,

$$\mathcal{L} = -\sum_{t=1}^{\infty} p(1-p)^{t-1} u\left(g_t\right) + \lambda \left(\sum_{t=1}^{\infty} g_t - B \right) - \sum_{t=1}^{\infty} \eta_t g_t \tag{5.10}$$

where λ and $\{\eta_t\}$ are Lagrange multipliers. Taking derivative with respect to g_t and equating to 0 we get

$$u'\left(g_t\right) = \frac{\lambda - \eta_t}{p(1-p)^{t-1}} \tag{5.11}$$

Since u is concave, u' is monotonically decreasing and $v \triangleq (u')^{-1}$ exists, and is also monotonically decreasing. By complementary slackness, we have $\eta_t = 0$ for $g_t > 0$, and the optimal power in this case is given by

$$g_t = v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) \tag{5.12}$$

and it now remains to find the optimal λ. We note by monotonicity of v, $\{g_t\}$ is non-increasing, and it holds that

$$g_t = v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) > 0 \Leftrightarrow \lambda < p(1-p)^{t-1}u'(0) \tag{5.13}$$

Hence, if $u'(0)$ is infinite, then (5.13) is satisfied $\forall t$, and the optimal power allocation sequence is an infinite sequence. In this case, we solve the following equation for the optimal λ

$$\sum_{t=1}^{\infty} v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) = B \tag{5.14}$$

which has a unique solution by monotonicity of v.

On the other hand, for finite $u'(0)$, there exists a time slot N, after which the second inequality in (5.13) is violated since λ is a constant and $p(1-p)^{t-1}$ is decreasing. In this case the optimal power allocation sequence is only positive for a finite number of time slots $1 \leq t \leq N$. We note that N is the smallest integer such that

$$\lambda \geq p(1-p)^{N}u'(0) \tag{5.15}$$

Thus, to find the optimal N (and λ), we first assume N is equal to some integer $\{2, 3, 4, \dots\}$, and solve the following equation for λ

$$\sum_{t=1}^{N} v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) = B \tag{5.16}$$

We then check if (5.15) is satisfied for that choice of N and λ. If it is, we stop. If not, we increase the value of N and repeat. This way, we reach a KKT point, which is sufficient for optimality by convexity of the problem [77]. We note that for $u(x) = \frac{1}{2}\log(1+x)$ whose $u'(0)$ is finite, [54] called N, $\tilde{N}$. We generalize their analysis for any concave increasing function u. This concludes the discussion of the optimal solution in the case of Bernoulli energy arrivals.

5.2.2 General i.i.d. Energy Arrivals

We now consider the case of a general i.i.d. energy arrival process. We first have the following two results.

Lemma 5.1 *The optimal solution of problem (5.3) satisfies*

$$\rho^* \leq u(\mu) \tag{5.17}$$

Proof: Following [54] and [59], we first remove the battery capacity constraint setting $B = \infty$. This way, the feasible set $\mathcal{F}$ in (5.1) becomes

$$\sum_{t=1}^{n} g_t \leq \sum_{t=1}^{n} E_t, \quad \forall n \tag{5.18}$$

Then, we remove the expectation and consider the offline setting of problem (5.3), i.e., when energy arrivals are known a priori. Since the energy arrivals are i.i.d., the strong law of large numbers indicates that $\lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} E_t = \mu$ a.s., i.e., for every $\delta > 0$, there exists n large enough such that $\frac{1}{n}\sum_{t=1}^{n} E_t \leq \mu + \delta$ a.s., which implies by (5.18) that the feasible set, for such (δ, n) pair, is given by

$$\frac{1}{n}\sum_{t=1}^{n} g_t \leq \mu + \delta \quad \text{a.s.} \tag{5.19}$$

Now fix such (δ, n) pair. The objective function is given by

$$\frac{1}{n}\sum_{t=1}^{n} u(g_t) \tag{5.20}$$

Since u is concave, the optimal power allocation minimizing the objective function is $g_t = \mu + \delta$, $1 \leq t \leq n$ [77] (see also [1]). Whence, the optimal offline solution is given by $u(\mu+\delta)$. We then have $\rho^* \leq u(\mu+\delta)$. Since this is true $\forall\delta > 0$, we can take δ down to 0 by taking n infinitely large. ■

Theorem 5.1 *The achieved long term average utility under the FFP in (5.5) satisfies*

$$\frac{1}{2} \leq \frac{\rho\left(\tilde{\boldsymbol{g}}\right)}{u\left(\mu\right)} \leq 1 \tag{5.21}$$

Proof: We first derive a lower bound on the long term average utility for Bernoulli energy arrivals under the FFP as follows

$$\begin{aligned}\lim_{n\to\infty} \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) &\overset{(a)}{=} p\sum_{i=1}^{\infty} p(1-p)^{i-1}\sum_{t=1}^{i} u\left(\tilde{g}_t\right) \\ &= \sum_{t=1}^{\infty} p(1-p)^{t-1} u\left((1-p)^{t-1}\mu\right)\end{aligned} \tag{5.22}$$

$$\stackrel{(b)}{\geq} \sum_{t=1}^{\infty} p(1-p)^{2(t-1)} u\left(\mu\right)$$
$$= \frac{1}{2-p} u(\mu) \geq \frac{1}{2} u(\mu) \tag{5.23}$$

where (a) follows by (5.7), (b) follows by concavity of u [77], and the last inequality follows since $0 \leq p \leq 1$. Next, we use the above result for Bernoulli arrivals to bound the long term average utility for general i.i.d. arrivals under the FFP in the following lemma; the proof follows by concavity and monotonicity of u, along the same lines of [54, Section VII-C], and is omitted for brevity.

Lemma 5.2 *Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as in (5.6) with parameter q as in (5.4) and mean $qB = \mu$. Then, the long term average utility under the FFP for any general i.i.d. energy arrivals, $\rho(\tilde{\boldsymbol{g}})$, satisfies*

$$\rho(\tilde{\boldsymbol{g}}) \geq \lim_{n \to \infty} \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) \tag{5.24}$$

Using Lemma 5.1, (5.23), and Lemma 5.2, we have

$$\frac{1}{2} u(\mu) \leq \rho(\tilde{\boldsymbol{g}}) \leq \rho^* \leq u(\mu) \tag{5.25}$$

■

We note that the results in Lemma 5.1 and Theorem 5.1 indicate that the FFP in (5.5) achieves a long term average utility that is within a constant multiplicative gap from the optimal solution that is equal to $\frac{1}{2}$. This result is proved in [54] for

$u(x) = \frac{1}{2}\log(1+x)$. Here, we are generalizing it to work for any concave increasing function u with $u(0) = 0$.

Next, we state the additive gap results. We first define

$$h_\theta(x) \triangleq u(\theta x) - u(x) \tag{5.26}$$

for some $0 \leq \theta \leq 1$, and define the following two classes of utility functions.

Definition 5.1 (Utility Classes) *A utility function u belongs to class (A) if $h_\theta(x)$ does not converge to 0 as $x \to \infty$, and belongs to class (B) if $\lim_{x\to\infty} h_\theta(x) = 0$.*

Now let us define the following function for $0 < \theta < 1$

$$h(\theta) \triangleq \inf_x h_\theta(x) \tag{5.27}$$

whenever the infimum exists. Note that the infimum exists for class (B) utility functions since $h_\theta(x) < 0$ for $x > 0$ by monotonicity of u, and $h_\theta(0) = 0$. We state some properties of the function h in the next lemma.

Lemma 5.3 *$h(\theta)$ is non-positive, concave, and non-decreasing in θ.*

Proof: Since u is increasing and $0 \leq \theta \leq 1$, then $h_\theta(x) < 0$ for all x, and hence the infimum is non-positive. Concavity follows by the concavity of u and the fact that the infimum of concave functions is also concave [77]. Finally, h is non-decreasing since u is monotonically increasing. ■

The next two theorems summarize the additive gap results for utility functions in classes (A) and (B) in Definition 5.1.

Theorem 5.2 *If $h(\theta)$ exists, and if*

$$r \triangleq (1-q)\lim_{t\to\infty}\frac{1-\lim_{x\to\bar{x}_{t+1}} u\left((1-q)^{t+1}x\right)/u(x)}{1-\lim_{x\to\bar{x}_t} u\left((1-q)^{t}x\right)/u(x)} < 1 \tag{5.28}$$

where $\bar{x}_t \in \arg\inf_x h_{(1-q)^t}(x)$; then the achieved long term average utility under the FFP in (5.5) satisfies

$$u(\mu) + \alpha \le \rho(\tilde{\boldsymbol{g}}) \le u(\mu) \tag{5.29}$$

where $\alpha \triangleq \sum_{t=0}^{\infty} q(1-q)^t h\left((1-q)^t\right)$ is finite.

Proof: By Lemma 5.1 and Lemma 5.2, it is sufficient to study the lower bound in the case of Bernoulli arrivals. By (5.22) we have

$$\begin{aligned}
\lim_{n\to\infty}\hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) &= \sum_{t=1}^{\infty} p(1-p)^{t-1}u\left((1-p)^{t-1}\mu\right) \\
&\overset{(c)}{\ge} \sum_{t=1}^{\infty} p(1-p)^{t-1}\left(u(\mu) + h\left((1-p)^{t-1}\right)\right) \\
&= u(\mu) + \sum_{t=0}^{\infty} p(1-p)^t h\left((1-p)^t\right) \\
&\triangleq u(\mu) + \alpha
\end{aligned} \tag{5.30}$$

where (c) follows since $h(\theta)$ exists, and is by definition no larger than $h_\theta(x)$, $\forall x, \theta$. Now to check whether α is finite, we apply the ratio test to check the convergence of the series $\sum_{t=0}^{\infty}(1-p)^t h\left((1-p)^t\right)$. That is, we compute

$$r \triangleq \lim_{t\to\infty}\left|\frac{(1-p)^{t+1}h\left((1-p)^{t+1}\right)}{(1-p)^t h\left((1-p)^t\right)}\right|$$
$$= (1-p)\lim_{t\to\infty}\frac{\inf_x 1-u\left((1-p)^{t+1}x\right)/u(x)}{\inf_x 1-u\left((1-p)^t x\right)/u(x)} \tag{5.31}$$

where the second equality follows by definition of h. Next, we replace $\inf_x$ by $\lim_{x\to\bar{x}_t}$ since $\bar{x}_t \in \arg\inf h_{(1-p)^t}(x)$, and take the limit inside (after the 1). Finally, if $r<1$ then α is finite; if $r>1$ then $\alpha=-\infty$; and if $r=1$ then the test is inconclusive and one has to compute $\lim_{T\to\infty}\sum_{t=0}^{T}p(1-p)^t h\left((1-p)^t\right)$ to get the value of α. ■

Theorem 5.3 *For class* (B) *utility functions, the achieved long term average utility under the FFP in (5.5) satisfies*

$$\lim_{\mu\to\infty}\rho\left(\tilde{\boldsymbol{g}}\right)=\rho^* \tag{5.32}$$

Proof: For utility functions of class (B), we have $\lim_{x\to\infty}u(\theta x)-u(x)=0$. Thus, $\forall\epsilon>0$ there exists $\bar{\mu}$ large enough such that

$$u\left((1-p)^{t-1}\mu\right)>u\left(\mu\right)-\epsilon,\quad \forall\mu\geq\bar{\mu} \tag{5.33}$$

whence, for Bernoulli energy arrivals we have

$$\lim_{n\to\infty}\hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}})=\sum_{t=1}^{\infty}p(1-p)^{t-1}u\left((1-p)^{t-1}\mu\right)$$
$$\geq u\left(\mu\right)-\epsilon,\quad \forall\mu\geq\bar{\mu} \tag{5.34}$$

It then follows by Lemma 5.1 and Lemma 5.2 that

$$\rho^* \geq \rho(\tilde{\boldsymbol{g}}) \geq u(\mu) - \epsilon \geq \rho^* - \epsilon, \quad \forall \mu \geq \bar{\mu} \tag{5.35}$$

and we can take ϵ down to 0 by taking μ infinitely large. ■

We note that the results in Lemma 5.1 and Theorem 5.2 indicate that the FFP in (5.5) achieves a long term average utility, under some sufficient conditions, that is within a constant additive gap from the optimal solution that is equal to $|\sum_{t=0}^{\infty} q(1-q)^t h\left((1-q)^t\right)|$. One can further make this gap independent of q by minimizing it over $0 \leq q \leq 1$. We discuss examples of the above results in Section 5.4, where we also comment on FFP performance under utility functions that do not satisfy the sufficient conditions in Theorem 5.2.

5.3 Specific Scenario: Distortion Minimization

We now focus on a specific scenario of a sensor node collecting i.i.d. Gaussian source samples, with zero-mean and variance σ_s^2, over a sequence of time slots. Samples are compressed and sent over an additive white Gaussian noise channel, with variance σ_c^2, to an intended destination. We consider a strict delay scenario where samples need to be sent during the same time slot in which they are collected. With a mean squared error distortion criterion, the average distortion of the source samples in time slot t, D_t, is given by [49]

$$D_t = \sigma_s^2 \exp(-2r_t) \tag{5.36}$$

where r_t denotes the sampling rate at time slot t.

The sensor uses energy harvested from nature to send its samples over the channel, with minimal distortion, and consumes energy in sampling and transmission. Depending on the physical settings, sampling energy can be a significant system aspect and needs to be taken into consideration [35]. We formulate two different problems for that matter: one without, and the other with sampling costs as follows.

We first consider the case of no sampling cost, where energy is consumed only in transmission. By allocating power g_t at time slot t to the Gaussian channel, the sensor achieves an instantaneous communication rate of [49]

$$r_t = \frac{1}{2} \log\left(1 + g_t/\sigma_c^2\right) \tag{5.37}$$

Given a feasible policy $\boldsymbol{g}$, and using (5.36) and (5.37), we define the n-horizon average distortion as follows

$$\mathcal{D}_n(\boldsymbol{g}) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{\sigma_s^2}{1 + g_t/\sigma_c^2}\right] \tag{5.38}$$

Our goal is to minimize the long term average distortion, subject to (online) energy causality constraints. That is, to characterize the following

$$d^* \triangleq \min_{\boldsymbol{g}\in\mathcal{F}} \lim_{n\to\infty} \mathcal{D}_n(\boldsymbol{g}) \tag{5.39}$$

where $\mathcal{F}$ is as defined in (5.1).

We note that the distortion function $\frac{\sigma_s^2}{1+x/\sigma_c^2}$ is convex and decreasing in x. Hence, the function $\bar{u}(x) \triangleq -\frac{\sigma_s^2}{1+x/\sigma_c^2}+\sigma_s^2$ is concave and increasing in x with $\bar{u}(0) = 0$. One can therefore apply the results of Section 5.2 to problem (5.39) after changing the minimization to maximization and the distortion function to the function $\bar{u}$ above. We will, however, proceed with the minimization problem as is for two main reasons. First, this will allow us to use a different analysis approach to find an additive gap that is relatively easier to compute than computing the term α in Theorem 5.2. Second, we will use this approach later when we consider the case with sampling costs since, as we will discuss, we cannot directly use the analysis in Section 5.2 to solve the problem in the case with sampling costs.

Now let us consider the case where sampling the source incurs an energy cost ϵ per unit time, that is a constant independent of the sampling rate. Due to the sampling cost, collecting all the source samples might not be optimal. Hence, we allow the sensor to be *on* during a $\theta_t \leq 1$ portion of time slot t, and turn off for the remainder of the time slot. The expected distortion achieved in time slot t under this setting is now given by

$$D_t^\epsilon = (1-\theta_t)\sigma_s^2 + \theta_t\sigma_s^2 \exp\left(-2r_t\right) \tag{5.40}$$

and the feasible set $\mathcal{F}^\epsilon$ is now given by the sequence of mappings $\{(\theta_t, g_t) : \ \mathcal{E}^t \to [0,1] \times \mathbb{R}_+\}$ satisfying

$$\theta_t(\epsilon + g_t) \leq b_t \triangleq \min\{b_{t-1} - \theta_{t-1}(\epsilon + g_{t-1}) + E_t, B\}, \ \forall t \tag{5.41}$$

with $b_1 \triangleq B$; compare the feasible set in (5.41) with cost to the feasible set in (5.1) with no additional cost. We note that the problem with sampling costs is formulated slightly different in [35]. In our formulation, the expected distortion is interpreted by time sharing between not transmitting (and hence achieving σ_s^2) and transmitting with rate r_t (and hence achieving $\sigma_s^2 \exp(-2r_t)$). Given a feasible policy $(\boldsymbol{\theta}, \boldsymbol{g})$, and using (5.37) and (5.40), we define the n-horizon average distortion with sampling costs as

$$\mathcal{D}_n^{\epsilon}\left(\boldsymbol{\theta}, \boldsymbol{g}\right) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n}(1-\theta_t)\sigma_s^2 + \theta_t \frac{\sigma_s^2}{1+g_t/\sigma_c^2}\right] \tag{5.42}$$

whence our goal is to characterize

$$d_\epsilon^* \triangleq \min_{(\boldsymbol{\theta}, \boldsymbol{g}) \in \mathcal{F}^\epsilon} \lim_{n\to\infty} \mathcal{D}_n^{\epsilon}\left(\boldsymbol{\theta}, \boldsymbol{g}\right) \tag{5.43}$$

Observe that in the case of sampling costs we optimize over two sequences of variables $\{\theta_t\}$ and $\{g_t\}$. Hence, the analysis in Section 5.2 cannot be directly applied to this case, unlike the case with no sampling costs. Thus, we proceed with a different approach to find an additive gap for this case. We note that online FFP analysis for two variables has been considered previously in [59, 60].

5.3.1 Bernoulli Energy Arrivals

In this section, we discuss the *optimal* solution of problems (5.39) and (5.43) under a Bernoulli energy arrival process as defined in (5.6). We first note that problem

(5.39) can be solved using the same analysis in Section 5.2.1 after replacing the objective function by $\bar{u}(x) \triangleq -\frac{\sigma_s^2}{1+x/\sigma_c^2} + \sigma_s^2$. Hence, in this section we only focus on the case of sampling costs in problem (5.43). Following the analysis in Section 5.2.1, and applying the change of variables $\bar{g}_t \triangleq \theta_t g_t$, problem (5.43) can be rewritten as

$$\begin{aligned} \min_{\boldsymbol{\theta},\boldsymbol{g}} \quad & \sum_{t=1}^{\infty} p(1-p)^{t-1} \left((1-\theta_t)\sigma_s^2 + \theta_t \frac{\sigma_s^2}{1+\frac{\bar{g}_t}{\theta_t \sigma_c^2}} \right) \\ \text{s.t.} \quad & \sum_{t=1}^{\infty} \bar{g}_t + \theta_t \epsilon \leq B \\ & \bar{g}_t \geq 0,\ 0 \leq \theta_t \leq 1, \quad \forall t \end{aligned} \tag{5.44}$$

which is a convex optimization problem since $\frac{\theta}{1+x/\theta}$ is the perspective function of the convex function $\frac{1}{1+x}$ and is therefore jointly convex in (θ, x) [77]. We introduce the following Lagrangian for this problem

$$\begin{aligned} \mathcal{L} = & \sum_{t=1}^{\infty} p(1-p)^{t-1} \left((1-\theta_t)\sigma_s^2 + \theta_t \frac{\sigma_s^2}{1+\frac{\bar{g}_t}{\theta_t \sigma_c^2}} \right) + \lambda \left(\sum_{t=1}^{\infty} \bar{g}_t + \theta_t \epsilon - B \right) - \sum_{t=1}^{\infty} \eta_t \bar{g}_t \\ & - \sum_{t=1}^{\infty} \gamma_t \theta_t + \sum_{t=1}^{\infty} \omega_t (\theta_t - 1) \end{aligned} \tag{5.45}$$

where λ, $\{\eta_t\}$, $\{\gamma_t\}$, and $\{\omega_t\}$ are non-negative Lagrange multipliers. Taking derivative with respect to $\bar{g}_t$ and equating to 0 we get

$$\frac{\sigma_s^2 p(1-p)^{t-1}}{\sigma_c^2 \left(1+\bar{g}_t/\theta_t \sigma_c^2\right)^2} = \lambda - \eta_t \tag{5.46}$$

which can be rewritten as follows using complementary slackness

$$\frac{\bar{g}_t}{\theta_t} = \sigma_c^2 \left(\sqrt{\frac{\sigma_s^2 p(1-p)^{t-1}}{\sigma_c^2 \lambda}} - 1 \right)^+ \tag{5.47}$$

where $(x)^+ = \max\{x, 0\}$. This shows that the optimal power g_t is monotonically decreasing over time, and that there exists a time slot N after which there is no transmission and all powers are 0. Now let us take the derivative of the Lagrangian with respect to θ_t, equate it to 0, and use (5.46) to get

$$\frac{\bar{g}_t}{\theta_t} = \sigma_c^2 \sqrt{\frac{\lambda\epsilon - \gamma_t + \omega_t}{\sigma_c^2(\lambda - \eta_t)}} \tag{5.48}$$

We now have the following result.

Lemma 5.4 *In problem (5.44), let N be the last time slot of transmission according to (5.47). Then, the optimal $\{\theta_t^*\}$ satisfies: $\theta_t^* = 1$ for $t < N$; $0 < \theta_N^* \leq 1$; $\theta_t^* = 0$ for $t > N$.*

Proof: First, we note that $\bar{g}_t = 0$ if and only if $\theta_t = 0$. Clearly $\theta_t = 0$ implies $\bar{g}_t = \theta_t g_t = 0$. To see the other direction, assume $\bar{g}_t = 0$ for some time slot t. Then, the achieved distortion in this time slot is given by σ_s^2 regardless of the value of θ_t. Therefore, setting $\theta_t = 0$ saves ϵ energy per unit time in this time slot that can be used in another time slot i to increase its transmission energy $\bar{g}_i$ and achieve lower distortion. Hence, after time slot N, we see that $\bar{g}_t = 0$ according to (5.47), and hence $\theta_t^* = 0$ for $t > N$.

Next, let us assume that $0 < \theta_j^* < 1$ for some time slot j. By the above argument we have $\bar{g}_j > 0$. By complementary slackness, we also have $\omega_j = \gamma_j = 0$. Hence, by (5.48) we have $\bar{g}_j/\theta_j = \sigma_c^2\sqrt{\epsilon/\sigma_c^2}$. Thus, whenever the transmission is bursty, the transmission power is constant. This constant can be equal to (5.47) at only one time slot since transmission power is decreasing. Moreover, after time slot j, the power can only decrease by increasing the value of γ_t in (5.48), which means by complementary slackness that $\theta_t = 0$ for $t > j$, which further implies that $\bar{g}_t = 0$ for $t > j$. Therefore, $j = N$.

Finally, for $t < N$, the power increases going backwards only by increasing the value of ω_t in (5.48), which means by complementary slackness that $\theta_t^* = 1$ for $t < N$. ■

The previous lemma shows that transmission can be bursty, i.e., $0 < \theta_t < 1$, only in the last time slot of transmission, N. This is similar to the optimal policy under Bernoulli energy arrivals found in [59] in the case of single-user channels with processing costs. We now proceed to find the optimal solution as follows. Note that the problem reduces to finding the optimal N, λ, and θ_N. We fix N and $\theta_N \in (0, 1]$, and solve the following equation for λ

$$\sum_{t=1}^{N-1} \sigma_c^2 \left(\sqrt{\frac{\sigma_s^2 p(1-p)^{t-1}}{\sigma_c^2 \lambda}} - 1 \right) + (N-1)\epsilon + \theta_N \left(\sigma_c^2 \left(\sqrt{\frac{\sigma_s^2 p(1-p)^{N-1}}{\sigma_c^2 \lambda}} - 1 \right) + \epsilon \right) = B \tag{5.49}$$

which has a unique solution since the left hand side is monotonically decreasing in λ. After that, we check if the solution satisfies the following two inequalities

$$\frac{\sigma_s^2 p(1-p)^{N-1}}{\sigma_c^2} > \lambda \tag{5.50}$$

$$\sigma_s^2 p(1-p)^{t-1} + (\sigma_c^2 - \epsilon)\lambda - 2\sigma_s^2\sqrt{\lambda p(1-p)^{t-1}} \geq 0, \quad t \leq N \tag{5.51}$$

where the first inequality ensures the positivity of powers, and the second one ensures the existence of non-negative Lagrange multipliers $\{\omega_t\}_{t=1}^N$. If the two inequalities are satisfied, then this KKT point is the optimal solution by the convexity of the problem. Otherwise, we perform a one-dimensional search over $\theta_N \in (0, 1]$ if any of the two inequalities is violated. If they cannot be simultaneously satisfied for all choices of θ_N, we change the value of N and repeat. The convergence to the optimal solution is guaranteed. This concludes the discussion of the optimal solution under Bernoulli energy arrivals.

5.3.2 General i.i.d. Energy Arrivals

We now discuss how the FFP performs in problems (5.39) and (5.43) under general i.i.d. energy arrivals. For problem (5.39), we define the power control policy as follows [54]

$$\tilde{g}_t = qb_t \tag{5.52}$$

and for problem (5.43), we define it as

$$\tilde{\theta}_t(\epsilon + \tilde{g}_t) = qb_t \tag{5.53}$$

That is, for either problem, in each time slot, the sensor uses a fixed fraction of its available energy in the battery. We note that using (5.54) in problem (5.43) decouples the problem into multiple single-slot problems where the energy consumption in time slot t is qb_t. In the sequel, we show that solving that single-slot problem for $\left(\tilde{\theta}_t, \tilde{g}_t\right)$ gives

$$\tilde{\theta}_t = \min\left\{\frac{qb_t}{\epsilon + \sqrt{\epsilon\sigma_c^2}}, 1\right\}, \qquad \tilde{g}_t = \max\left\{qb_t - \epsilon, \sqrt{\epsilon\sigma_c^2}\right\} \tag{5.54}$$

Observe that in the above assignment, for a single energy arrival, either the transmission power or the *on* time decreases over slots in a fractional manner, i.e., while one decreases the other one is fixed. Let $d\left(\tilde{\boldsymbol{g}}\right)$ and $d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right)$ denote the long term average distortion under $\{\tilde{g}_t\}$ in (5.52) and $\{(\tilde{\theta}_t, \tilde{g}_t)\}$ in (5.54), respectively. We now characterize the performance of FFP in the case of general i.i.d. arrivals in the following two theorems.

Theorem 5.4 *For all i.i.d. energy arrivals with mean μ, the optimal solution of problem (5.39) satisfies*

$$d^* \geq f(\mu) \triangleq \frac{\sigma_s^2}{1 + \mu/\sigma_c^2} \tag{5.55}$$

and the FFP in (5.52) satisfies

$$f(\mu) \le d\left(\tilde{\boldsymbol{g}}\right) \le f(\mu) + \frac{1}{2}\sigma_s^2 \tag{5.56}$$

for all values of μ and σ_c^2.

Proof: *Lower Bounding d^*:* First, we derive the lower bound in (5.55) by means of the offline solution along the same lines as in the proof of Lemma 5.1. Applying the same (δ, n) argument using the strong law of large numbers, the objective function is given by

$$\frac{1}{n}\sum_{t=1}^{n} \frac{\sigma_s^2}{1 + g_t/\sigma_c^2} = \frac{1}{n}\sum_{t=1}^{n} f(g_t) \tag{5.57}$$

It is direct to see that f is convex. Therefore, the optimal power allocation minimizing the objective function is $g_t = \mu + \delta$, $1 \le t \le n$ [77] (see also [1]). Whence, the optimal offline solution is given by $f(\mu+\delta)$. We then have $d^* \ge f(\mu+\delta)$. Since this is true $\forall\delta > 0$, we can take δ down to 0 by taking n infinitely large. Therefore, (5.55) holds.

Upper Bounding d^*: Bernoulli Energy Arrivals: Next, we derive an upper on d^*. Towards that, we first the study a special energy harvesting i.i.d. process: the Bernoulli process. Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as defined in (5.6). Under such specific energy arrival setting, whenever an energy packet arrives, it completely fills the battery, and resets the system. This constitutes a *renewal*. Then, by [83, Theorem 3.6.1] (see also [54]), the following holds for any power control policy $\boldsymbol{g}$

$$\begin{aligned}\lim_{n\to\infty} \hat{\mathcal{D}}_n(\boldsymbol{g}) &= \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{\sigma_s^2}{1+g_t/\sigma_c^2}\right] \\ &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} \frac{\sigma_s^2}{1+g_t/\sigma_c^2}\right] \end{aligned} \tag{5.58}$$

where $\hat{\mathcal{D}}_n(\boldsymbol{g})$ is the n-horizon average distortion under Bernoulli arrivals, and L is a random variable denoting the inter-arrival time between energy arrivals, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$.

Now, substituting by the FFP (5.52) gives an upper bound on d^*. Note that by (5.6), the fraction q in (5.4) is now equal to p. Also note that in between energy arrivals, the battery state decays exponentially, and the FFP in (5.52) gives

$$\tilde{g}_t = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{5.59}$$

for all time slots t, where the second equality follows since $pB = \mu$. Therefore, using (5.58) and (5.59), we bound the distortion under the FFP as follows

$$\begin{aligned}\lim_{n\to\infty} \hat{\mathcal{D}}_n(\tilde{\boldsymbol{g}}) &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} \frac{\sigma_s^2}{1+(1-p)^{t-1}\mu/\sigma_c^2}\right] \\ &\overset{(a)}{\leq} \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} \frac{\sigma_s^2}{1+\mu/\sigma_c^2} + \left(1-(1-p)^{t-1}\right)\sigma_s^2\right] \\ &= f(\mu) + \sigma_s^2\left(1 - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}(1-p)^{t-1}\right]\right) \\ &= f(\mu) + \sigma_s^2\frac{p(1-p)}{1-(1-p)^2}\end{aligned}$$

$$\stackrel{(b)}{\leq} f(\mu) + \frac{\sigma_s^2}{2} \tag{5.60}$$

where (a) follows since $\frac{1}{1+\lambda x} \leq \frac{1}{1+x} + (1-\lambda)$ for $0 \leq \lambda \leq 1$ and $x \geq 0$; and (b) follows since $\frac{p(1-p)}{1-(1-p)^2}$ has a maximum value of $1/2$ for $0 \leq p \leq 1$. Next, we use the above result for Bernoulli arrivals to bound the distortion for general i.i.d. arrivals under the FFP in the following lemma; the proof follows by convexity and monotonicity of f, along the same lines of [54, Section VII-C], and is omitted for brevity.

Lemma 5.5 *Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as in (5.6) with parameter q as in (5.4) and mean $qB = \mu$. Then, the long term average distortion under the FFP for any general i.i.d. energy arrivals, $d(\tilde{\boldsymbol{g}})$, satisfies*

$$d(\tilde{\boldsymbol{g}}) \leq \lim_{n\to\infty} \hat{\mathcal{D}}_n(\tilde{\boldsymbol{g}}) \tag{5.61}$$

Using (5.55), (5.60), and Lemma 5.5, we have

$$f(\mu) \leq d^* \leq d(\tilde{\boldsymbol{g}}) \leq f(\mu) + \frac{\sigma_s^2}{2} \tag{5.62}$$

■

Theorem 5.5 *For all i.i.d. energy arrivals with mean μ, the optimal solution of problem (5.43) satisfies*

$$d_\epsilon^* \geq f_\epsilon(\mu) \triangleq \min_{\theta,\bar{g}} \quad (1-\theta)\sigma_s^2 + \theta \frac{\sigma_s^2}{1+\frac{\bar{g}}{\theta\sigma_c^2}}$$

$$\text{s.t.} \quad \theta\epsilon + \bar{g} \leq \mu, \quad 0 \leq \theta \leq 1 \tag{5.63}$$

and the FFP in (5.54) satisfies

$$f_\epsilon(\mu) \leq d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \leq f_\epsilon(\mu) + \frac{1}{2}\sigma_s^2 \tag{5.64}$$

for all values of ϵ*,* μ*, and* σ_c^2*.*

Proof: *Lower Bounding* d_ϵ^**:* First, we derive the lower bound in (5.63) by means of the offline solution as done in the proof of Theorem 5.4. We first apply the change of variables: $\bar{g}_t \triangleq \theta_t g_t \; \forall t$. The feasible set $\mathcal{F}^\epsilon$ now becomes

$$\sum_{t=1}^{n} \theta_t\epsilon + \bar{g}_t \leq \sum_{t=1}^{n} E_t, \quad \forall n; \quad 0 \leq \theta_t \leq 1, \quad \forall t \tag{5.65}$$

Applying the same (δ, n) argument using the strong law of large numbers, as in the proofs of Theorem 5.4 and Lemma 5.1, the objective function is given by

$$\frac{1}{n}\sum_{t=1}^{n}(1-\theta_t)\sigma_s^2 + \frac{\theta_t\sigma_s^2}{1+\frac{\bar{g}_t}{\theta_t\sigma_c^2}} \triangleq \frac{1}{n}\sum_{t=1}^{n} H\left(\theta_t, \bar{g}_t\right) \tag{5.66}$$

It is direct to see that H is jointly convex in $(\theta_t, \bar{g}_t)$ since the second added term is the perspective of the convex function $f(\bar{g}_t)$ [77]. Therefore, the optimal power allocation minimizing the objective function is $\theta_t\epsilon + \bar{g}_t = \mu + \delta, \; 1 \leq t \leq n$ [77] (see also [1]). We denote this optimal offline solution by $f_\epsilon(\mu+\delta)$ as defined in (5.63). We then have $d_\epsilon^* \geq f_\epsilon(\mu+\delta)$; we take δ down to 0 by taking n infinitely large.

Therefore, (5.63) holds.

Upper Bounding d_ϵ^*: Bernoulli Energy Arrivals: Next, we derive an upper bound on d_ϵ^*. Following the same steps as in the proof of Theorem 5.4, we first consider Bernoulli energy arrivals as in (5.6). In this case we have

$$\lim_{n\to\infty} \hat{\mathcal{D}}_n^\epsilon\left(\boldsymbol{\theta}, \boldsymbol{g}\right) = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}(1-\theta_t)\sigma_s^2 + \frac{\theta_t\sigma_s^2}{1+g_t/\sigma_c^2}\right] \tag{5.67}$$

where $\hat{\mathcal{D}}_n^\epsilon\left(\boldsymbol{\theta}, \boldsymbol{g}\right)$ is the n-horizon average distortion under Bernoulli arrivals. Next, we upper bound the long term average distortion in this case by substituting the FFP in (5.54) setting

$$\tilde{\theta}_t(\epsilon + \tilde{g}_t) = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{5.68}$$

for all time slots t. Note that the average minimal distortion in time slot t is given by $f_\epsilon\left((1-p)^{t-1}\mu\right)$. We have the following lemma regarding f_ϵ

Lemma 5.6 *The function f_ϵ is convex and non-increasing.*

Proof: f_ϵ is non-increasing since allocating more power can only decrease the distortion. To show convexity, let $(\theta_1, \bar{g}_1)$ and $(\theta_2, \bar{g}_2)$ be the solutions achieving $f_\epsilon(x_1)$ and $f_\epsilon(x_2)$, respectively, for some $x_1, x_2 \geq 0$. Now choose $\lambda \in [0,1]$, and let $x_\lambda \triangleq \lambda x_1 + (1-\lambda)x_2$. It is direct to see that the convex combination $(\theta_\lambda, \bar{g}_\lambda) \triangleq (\lambda\theta_1 + (1-\lambda)\theta_2, \lambda\bar{g}_1 + (1-\lambda)\bar{g}_2)$ is feasible for x_λ. Therefore,

$$f_\epsilon\left(x_\lambda\right) \leq H\left(\theta_\lambda, \bar{g}_\lambda\right)$$

$$\leq \lambda H\left(\theta_1, \bar{g}_1\right) + (1-\lambda) H\left(\theta_2, \bar{g}_2\right)$$

$$= \lambda f_\epsilon\left(x_1\right) + (1-\lambda) f_\epsilon\left(x_2\right) \tag{5.69}$$

where H is as defined in (5.66), and the second inequality follows by convexity of H. ■

Next, following the same steps used in showing (5.60), by (5.67) and (5.68), we have

$$\lim_{n\to\infty} \hat{\mathcal{D}}_n^\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \leq f_\epsilon(\mu) + \frac{\sigma_s^2}{2} \tag{5.70}$$

where step (a) in (5.60) follows by Lemma 5.6. Finally, we use the above result to bound the distortion for general i.i.d. arrivals under the FFP. We basically extend the statement of Lemma 5.5 to the case with sampling costs since f_ϵ is convex and monotone. We then have

$$d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \leq \lim_{n\to\infty} \hat{\mathcal{D}}_n^\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \tag{5.71}$$

Using (5.63), (5.70), and (5.71), we have

$$f_\epsilon(\mu) \leq d_\epsilon^* \leq d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \leq f_\epsilon(\mu) + \frac{\sigma_s^2}{2} \tag{5.72}$$

It now remains to show that the FFP corresponds to (5.54). Towards that end, we solve $f_\epsilon\left(qb_t\right)$ for θ and $\bar{g}$. We first make the substitution $\bar{g} = qb_t - \theta\epsilon$ into the objective function. The problem now becomes

$$\min_{0 \le \theta \le \min\{1, qb_t/\epsilon\}} \quad \frac{\theta}{1 - \frac{\epsilon}{\sigma_c^2} + \frac{qb_t}{\theta\sigma_c^2}} - \theta \tag{5.73}$$

where the constraint $\theta \le qb_t/\epsilon$ ensures non-negativity of $\bar{g}$. One can show that the objective function above is convex in θ. Hence, we take the derivative, equate to 0, solve for θ, and then project the solution onto the feasible set to get the optimal solution of this problem [77]. This gives $\tilde{\theta}_t$ in (5.54); while $\tilde{g}_t$ in (5.54) is directly derived by substituting $g = \frac{qb_t}{\theta} - \epsilon$. ■

Note that the results in the two theorems above directly imply that the average long term distortion under the FFP proposed for both problems (5.43) and (5.39) lies within a constant additive gap from the optimal solution. We also observe that the additive gap indicated in Theorem 5.5 does not depend on the sampling cost ϵ.

5.4 Examples and Discussion

In this section we present some examples to illustrate the results of this work. We first show that the utility function $u(x) = \frac{1}{2}\log(1+x)$ considered in [54] belongs to class (A). Indeed we have $h'_\theta(x) = \frac{\theta-1}{2(1+\theta x)(1+x)}$, which is negative for all $0 < \theta < 1$, and therefore $h_\theta(x)$ is decreasing in x and does not converge to 0. We then show that the sufficient conditions of Theorem 5.2 are satisfied. We have the function

$$h(\theta) = \lim_{x\to\infty} \frac{1}{2} \log \frac{1+\theta x}{1+x} = \frac{1}{2}\log(\theta) \tag{5.74}$$

exists, and the ratio

$$r = (1-q) \lim_{t\to\infty} \frac{1 - \lim_{x\to\infty} \log(1+(1-q)^{t+1}x)/\log(1+x)}{1 - \lim_{x\to\infty} \log(1+(1-q)^{t}x)/\log(1+x)} = 1-q \tag{5.75}$$

is less than 1, and hence the gap α is finite. Furthermore, [54] showed that minimizing α over all q gives a constant additive gap, independent of q, that is equal to 0.72.

Next, we note that all bounded utility functions belong to class (B). These are functions u where there exists some constant $M < \infty$ such that $u(x) \leq M,\ \forall x$. Examples for these include: $u(x) = 1 - e^{-\beta x}$ for some $\beta > 0$, $u(x) = x/(1+x)$, and the negative distortion function $u(x) = -\frac{\sigma_s^2}{1+x/\sigma_c^2} + \sigma_s^2$. To see that these functions belong to class (B), observe that $\lim_{x\to\infty} u(x) = M$ by monotonicity of u, and hence $\lim_{x\to\infty} u(\theta x) - u(x) = 0$. We also note that class (B) is not only inclusive of bounded utility functions. For example, the unbounded function $u(x) = \sqrt{\log(1+x)}$ satisfies

$$\lim_{x\to\infty} \sqrt{\log(1+\theta x)} - \sqrt{\log(1+x)} = \frac{\log(\theta)}{\lim_{x\to\infty} \sqrt{\log(1+\theta x)} + \sqrt{\log(1+x)}} = 0 \tag{5.76}$$

and therefore belongs to class (B). For such unbounded functions in class (B), the FFP is not only within a constant additive gap of the optimal solution, but it is asymptotically optimal as well, as indicated by Theorem 5.3.

Note that one can find a (strict) lower bound on $h(\theta)$ for some utility functions if it allows more plausible computation of α, or if $h(\theta)$ itself is not direct to compute.

For instance, for any bounded utility function u, the following holds: $h(\theta) \geq (\theta - 1)M$, where M is the upper bound on u. To see this, observe that by concavity of u and the fact that $u(0) = 0$ we have

$$\inf_x u(\theta x) - u(x) \geq (\theta - 1) \sup_x u(x) \tag{5.77}$$

This gives

$$\begin{aligned}
\alpha &\geq \sum_{t=0}^{\infty} q(1-q)^t \left((1-q)^t - 1\right) M \\
&= \frac{q-1}{2-q} M \\
&\geq -\frac{1}{2} M
\end{aligned} \tag{5.78}$$

where the second inequality follows since $\frac{q-1}{2-q}$ is minimized at $q = 0$. Another example is $u(x) = \frac{1}{2}\log\left(1 + \sqrt{x}\right)$, which belongs to class (A). We observe that $h(\theta)$ in this case is lower bounded by $\frac{1}{2}\log(\theta)$. Hence, this function admits an additive gap no larger than 0.72 calculated in [54] for $u(x) = \frac{1}{2}\log(1 + x)$.

Finally, we note that the conditions of Theorem 5.2 are only sufficient for the FFP defined in (5.5) to be within an additive gap from optimal. For instance, consider $u(x) = \sqrt{x}$. This function belongs to class (A) as $h_\theta(x) = \sqrt{\theta x} - \sqrt{x}$ does not converge to 0. In fact, $h_\theta(x)$ is unbounded below and $h(\theta)$ does not exist. This means that any FFP of the form $\tilde{g}_t = \theta b_t$, for any choice of $0 < \theta < 1$, is *not* within a constant additive gap from the upper bound $\sqrt{\mu}$. However, there exists another FFP (with a different fraction than q in (5.4)) that is *optimal* in the case of Bernoulli

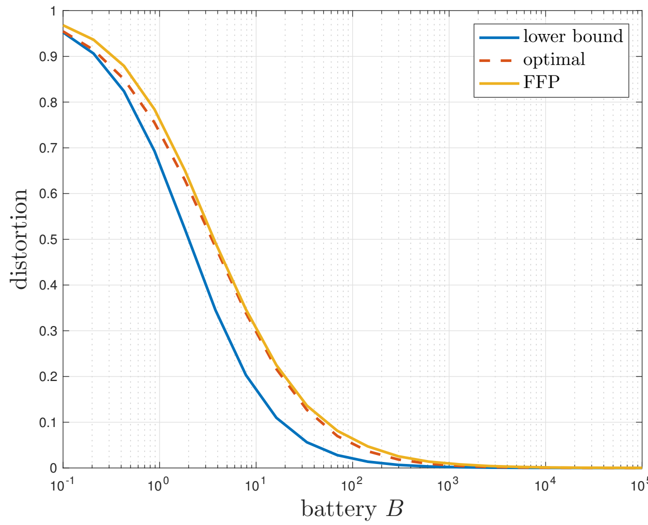


Figure 5.2: Performance of the FFP with no sampling costs.

arrivals. Since $u'(0) = \infty$, we use (5.14) to find the optimal λ, where $v(x) = 1/(4x^2)$, and substitute in (5.12) to get that the optimal transmission scheme is *fractional:* $g_t = \hat{p}\,(1-\hat{p})^{(t-1)}\,B,\ \forall t$, where the transmitted fraction $\hat{p} \triangleq 1-(1-p)^2$. This shows that one can pursue near optimality results under an FFP by further optimizing the fraction of power used in each time slot, and comparing the performance directly to the optimal solution instead of an upper bound. While in this work, we compared the lower bound achieved by the FFP to a universal upper bound that works for all i.i.d. energy arrivals.

Next, we present some examples regarding the distortion minimization setting. We set both σ_s^2 and σ_c^2 to unity, and consider a system with Bernoulli energy arrivals with probability $p = 0.5$. In Fig. 5.2, we plot the lower bound on the long term average distortion for the problem without sampling costs along with the FFP, against the battery size B. We also plot the optimal solution in this scenario. We see that the FFP performs very close to the optimal policy. We note that the

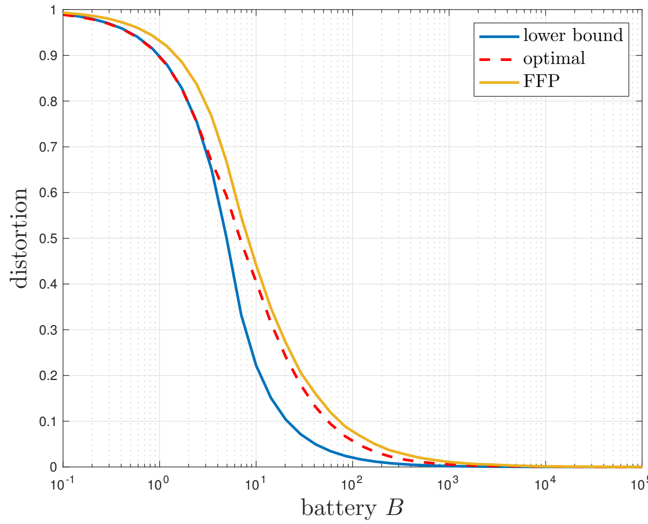


Figure 5.3: Performance of the FFP with sampling costs.

empirical gap between the optimal policy and the FFP is no larger than 0.03, while the empirical gap between the lower bound and the FFP is no larger than 0.15, which is lower than the theoretical gap of 0.5 in Theorem 5.4.

In Fig. 5.3, we plot the same curves for the problem with sampling costs. We set the sampling cost $\epsilon = 1.5$. We notice that the distortion levels are higher in general when compared to the case without sampling costs, which is mainly due to having some energy spent in sampling instead of reducing distortion. The empirical gap in this case is 0.22, which is lower than the theoretical gap of 0.5 in Theorem 5.5.

In Fig. 5.4, we show the FFP (left hand side in blue) versus the optimal policy (right hand side in red) for $B = 40$ during only one renewal period, i.e., for one energy arrival. We plot the power and the transmit duration (burstiness) during the first 10 time slots, with the height representing power and the width representing burstiness. We see that in the FFP on the left, for time slots 1 through 3, the transmission power $\tilde{g}_t$ decreases fractionally while the value of $\tilde{\theta}_t$ is constant

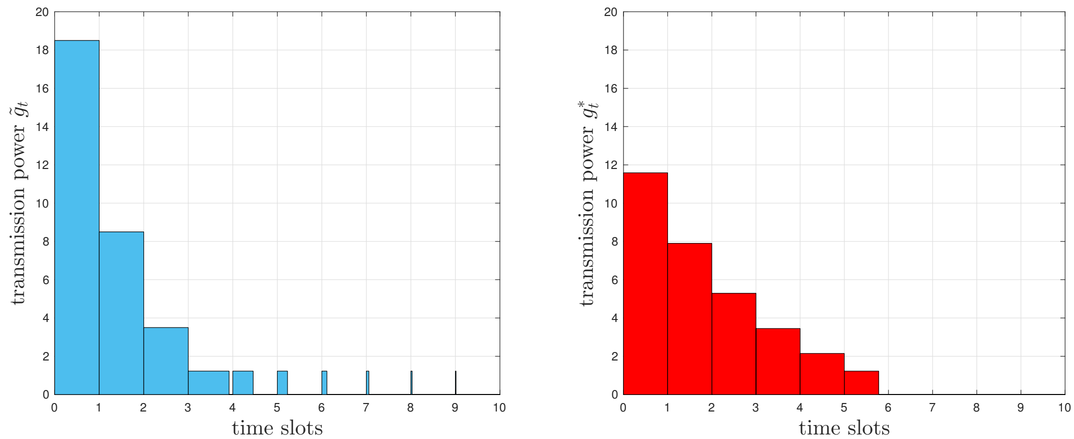


Figure 5.4: FFP (left) vs. optimal policy (right) with sampling costs and one energy arrival with $B = 40$.

at unity. Starting from time slot 4 onwards, the opposite occurs; the value of $\tilde{\theta}_t$ decreases fractionally while the transmission power $\tilde{g}_t$ is constant at 1.225. As indicated by (5.54), either the power or the transmit duration decreases fractionally while the other is constant over time. On the other hand, in the optimal policy on the right, we see that the transmission power g_t^* is decreasing all the way to the end. In this example, the last time slot of transmission is $N = 6$, and the transmission is bursty only in that time slot, as indicated by Lemma 5.4, with $\theta_6^* = 0.78$.

5.5 Conclusion

In this chapter, we considered online power scheduling policies in single-user energy harvesting channels, where the goal is to maximize the long term average utility for a general concave increasing utility function. We showed that fixed fraction policies achieve a long term average utility that lies within a constant multiplicative gap from the optimal solution for all i.i.d. energy arrivals and battery sizes. We then derived sufficient conditions on the utility function to guarantee that fixed fraction policies are within a constant additive gap from the optimal solution as well. We then considered a specific scenario where a source is aiming at sending Gaussian samples over a Gaussian channel with minimal long term average distortion. We studied this problem with and without sampling costs, and showed that fixed fraction policies are within a constant additive gap from the optimal solution.

CHAPTER 6

Mobile Energy Harvesting Nodes: Offline and Online Optimal Policies

6.1 Introduction

In this chapter, we consider a mobile energy harvesting transmitter where movement is motivated by trying to find better energy harvesting locations. Movement comes with an energy cost expenditure, and hence there exists a tradeoff between staying at the same location and moving to a new one. On one hand, the transmitter may opt not to move and use all its available energy for transmission; on the other hand, it can choose to move to a potentially better location, spending some of its available energy during the movement process, and yet harvest larger amounts of energy at the new location and achieve higher throughput. In this chapter, we characterize this tradeoff by designing throughput optimal power allocation policies subject to energy causality constraints and moving costs. In our setup, the transmitter moves along a straight line, where two energy sources are located at the opposite ends of the line. We first study the offline version of this problem where the goal is to maximize the throughput by a given deadline. Although our problem formulation is non-convex, we characterize the optimal solution in closed form in the case of a single energy arrival at each source. Then, we use this solution to characterize local optimal solutions in the case of multiple energy arrivals at each source. Next, we study the online version of this problem with i.i.d. energy arrivals at each source, and the goal is to maximize the long term average throughput. We propose an optimal *move-then-transmit* scheme where the transmitter first moves towards the source with higher mean energy arrival, stays at that source, and then starts transmission. If the transmitter has an infinite battery, it uses the optimal *best effort* transmission policy, where it transmits with the mean harvesting rate whenever feasible and stays silent otherwise. If the transmitter has a finite battery, it uses the *fixed fraction* policy, where it uses a fixed fraction of the amount of energy available in its battery for transmission in every time slot, to achieve a near optimal rate that provably lies within constant additive and multiplicative gaps from the optimal solution for all energy arrival patterns and battery sizes.

6.2 System Model and Problem Formulation

We consider a single-user AWGN channel with an energy harvesting transmitter with moving abilities. The transmitter has the ability to relocate itself to different positions in search for better energy harvesting spots. Movement is along a straight line, and energy is harvested from two energy sources located at the two opposite ends of the line, see Fig. 6.1. The transmitter's position determines how much energy is harvested from each source: the closer the transmitter is to one source, the

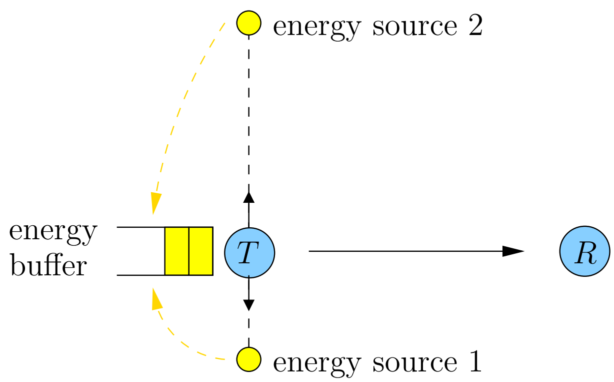


Figure 6.1: Mobile energy harvesting node moving along a straight line between two energy sources. The position of the node determines how much energy it harvests from each source.

larger the amounts of energy it harvests from that source compared to the other. In our setting, the transmitter-receiver distance is much larger than the distance between the two energy sources so as to ensure that the transmitter-receiver channel characteristics are not affected by the transmitter's movement.

We consider a time-slotted model, where the transmitter is allowed to move during a fixed portion of time at the beginning of each slot, and then starts communicating. Without loss of generality, we assume that the remaining portion of the time slot where the transmitter communicates is normalized to one time unit, so that we may use energy and power interchangeably. Throughout most of this chapter, we will consider the case where the transmitter is equipped with an infinite-sized battery to save its harvested energy. However, in some cases we will extend our analysis to the finite battery case as well. Energy arrives in packets of amounts E_{1i} and E_{2i} in slot i at the first and the second energy source, respectively. At the beginning of slot i, the transmitter relocates itself to some position x_i, and harvests energy from both sources simultaneously according to the following relationship [84–86]

$$E(i, x_i) = \frac{E_{1i}}{(x_i + \ell)^\alpha} + \frac{E_{2i}}{(L - x_i + \ell)^\alpha} \tag{6.1}$$

where α is the path loss factor, L is the distance between the two energy sources, and $\ell > 0$ is a parameter added to adjust the Friis' free space equation for short distance communication, that is, to keep the harvested energy bounded when the transmitter lies at either ends of the line. Note that $E(i, x_i)$ is the actual amount of harvested energy that enters into the battery of the transmitter.

The transmitter incurs moving costs whenever it relocates itself to a different position. We model the total moving cost up to slot k as follows

$$c_m(k) \triangleq \epsilon_m \sum_{i=1}^{k} |x_i - x_{i-1}| \tag{6.2}$$

where x_0 is the initial position of the transmitter, $\sum_{i=1}^{k} |x_i - x_{i-1}|$ represents the total distance moved by the transmitter up to slot k, and ϵ_m is the cost of movement in energy per unit distance. Since movement is not cost-free, a tradeoff arises between spending energy to move into better spots (in the sense of energy availability), and staying at the same location and spending all the available energy in communicating. We design power and movement policies that capture the optimal tradeoff of this setting.

6.2.1 Offline Problem

We first consider an offline scenario, where energy amounts are known to the transmitter prior to the start of communication. Our goal in this setting is to maximize the total number of bits delivered to the receiver by a given deadline N, subject to energy causality constraints and moving costs. The physical layer is Gaussian with unit noise power, and the transmitter uses power p_i for transmission in time slot i. We formulate the problem as follows

$$\begin{aligned} \max_{\boldsymbol{p},\boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \\ \text{s.t.} \quad & c_m(1) \leq E_0 \\ & c_m(k+1) + \sum_{i=1}^{k} p_i \leq E_0 + \sum_{i=1}^{k} E(i,x_i), \quad 1 \leq k \leq N \\ & 0 \leq x_i \leq L, \quad p_i \geq 0, \quad \forall i \end{aligned} \tag{6.3}$$

where $c_m(N+1) \triangleq c_m(N)$, and E_0 is the initial energy available at the transmitter. This initial energy enables the transmitter to relocate itself during the first slot (if needed). Note that if the transmitter needs to move in slot $k+1$, then it needs to save some energy by the end of slot k for that purpose. In other words, it should not consume all its energy in transmission by the end of slot k. That is why the energy incurred for moving up to slot $k+1$ is bounded by the residual energy remaining after slot k: $E_0 + \sum_{i=1}^{k} E(i,x_i) - p_i$. We solve problem (6.3) in Section 6.3.

6.2.2 Online Problem

We then consider an online scenario, where energy amounts are only revealed to the transmitter causally over time; the amount of energy harvested at time slot t is only known *after* moving to position x_t. We assume that energy harvesting processes at the two sources $\{E_{1i}\}$ and $\{E_{2i}\}$ follow two independent i.i.d. distributions with means μ_1 and μ_2, respectively. Only the means of the two processes are known to the transmitter prior to the start of communication. Let b_t represent the amount of energy in the battery at time slot t, and let $\mathcal{E}^t \triangleq \{E_1, E_2, \ldots, E_t\}$. A feasible online power control and movement policy $\{\boldsymbol{p}, \boldsymbol{x}\}$ is a sequence of mappings $\{x_t: \ \mathcal{E}^{t-1} \to [0, L]\}$ and $\{p_t: \ \mathcal{E}^t \to \mathbb{R}_+\}$ satisfying

$$\epsilon_m|x_1 - x_0| \leq E_0 \tag{6.4}$$

$$\epsilon_m|x_t - x_{t-1}| + p_t \leq b_t \triangleq b_{t-1} - \epsilon_m|x_{t-1} - x_{t-2}| - p_{t-1} + E(t, x_t) \tag{6.5}$$

Let us denote the above feasible set by $\mathcal{F}$. Our goal is to maximize the long term average throughput

$$r^* \triangleq \max_{\{\boldsymbol{p},\boldsymbol{x}\}\in\mathcal{F}} \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=1}^{T} \frac{1}{2}\log\left(1 + p_t\right)\right] \tag{6.6}$$

We solve problem (6.6) in Section 6.4, where we also discuss the case where the transmitter is equipped with a finite battery of size B.

6.3 Offline Setting: Problem (6.3)

In this section, we characterize the optimal solution of problem (6.3). We first note the following necessary optimality conditions.

Lemma 6.1 *In the optimal solution of (6.3), powers are non-decreasing over time.*

Proof: We show this by contradiction. Assume that at the optimal policy $\{\boldsymbol{p}^*, \boldsymbol{x}^*\}$, there exists a time slot k such that $p_k^* > p_{k+1}^*$. Keeping the movement policy $\boldsymbol{x}^*$ the same, we define another power policy $\tilde{\boldsymbol{p}}$ where only the kth and $(k+1)$st powers change to $\tilde{p}_k = \tilde{p}_{k+1} = \frac{p_k^* + p_{k+1}^*}{2}$. It is direct to see that $\{\tilde{\boldsymbol{p}}, \boldsymbol{x}^*\}$ is a feasible policy. By concavity of the log, this new policy strictly increases the objective function, and hence the original policy $\{\boldsymbol{p}^*, \boldsymbol{x}^*\}$ cannot be optimal. ■

Lemma 6.2 *In the optimal solution of (6.3), the transmitter consumes all its harvested energy by the end of communication.*

Proof: We show this by contradiction. If the statement of the lemma were not true, then we can increase the value of p_N until all energy is consumed. This strictly increases the objective function. ■

6.3.1 Single Energy Arrival

In this section we study the case where each energy source has only one energy packet arrival. That is, we have only one pair of variables (p, x) to optimize. By

Lemma 6.2, we have

$$p(x) = E_0 + \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m |x - x_0| \tag{6.7}$$

and therefore, by monotonicity of the log, problem (6.3) becomes

$$\begin{aligned} \max_x \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m |x - x_0| \\ \text{s.t.} \quad & \epsilon_m |x - x_0| \leq E_0 \\ & 0 \leq x \leq L \end{aligned} \tag{6.8}$$

Therefore, the problem now reduces to finding the optimal position x^*.

Note that there are two possible movement strategies the transmitter can make: move forward to some $x \geq x_0$, or move backward to some $x < x_0$. The transmitter chooses the movement strategy that gives the maximum objective function (and hence power/rate). To that end, we next solve the case of moving forward. The problem in this case becomes

$$\begin{aligned} \max_x \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m x \\ \text{s.t.} \quad & x_0 \leq x \leq \min\left\{\frac{E_0}{\epsilon_m} + x_0, L\right\} \triangleq x_{\max} \end{aligned} \tag{6.9}$$

Now observe that the objective function is a convex function in x that is maximized over an interval. It then follows that the optimal solution x^* has to be at the boundary of the feasible set [77], i.e.,

$$x^* \in \{x_0, x_{\max}\} \tag{6.10}$$

Hence, we pick x^* that gives the higher value after substituting in (6.7), i.e., after comparing $p\left(x_0\right)$ and $p\left(x_{\max}\right)$.

Similarly, the problem in the case of moving backward is given by

$$\begin{aligned}
\max_{x} \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} + \epsilon_m x \\
\text{s.t.} \quad & x_{\min} \triangleq \max\left\{x_0 - \frac{E_0}{\epsilon_m}, 0\right\} \leq x \leq x_0
\end{aligned} \tag{6.11}$$

which again, by convexity of the objective function, yields a solution at the boundary. That is

$$x^* \in \{x_{\min}, x_0\} \tag{6.12}$$

Hence, we pick x^* that gives the higher value after substituting in (6.7), i.e., after comparing $p\left(x_0\right)$ and $p\left(x_{\min}\right)$.

Based on the previous analysis, the optimal position in the single energy arrival scenario can only have three possible values: $x^* \in \{x_{\min}, x_0, x_{\max}\}$. This means that if the transmitter decides to move, it moves to the furthest possible distance (forward or backward) allowed by its available initial energy E_0 and the physical length of the straight line L. Therefore, the optimal power is given by

$$p^* = \max\left\{p\left(x_{\min}\right), p\left(x_0\right), p\left(x_{\max}\right)\right\} \tag{6.13}$$

and x^* is the corresponding maximizing argument.

6.3.2 Multiple Energy Arrivals

In this section we study the multiple energy arrivals setting. We note that problem (6.3) is not a convex optimization problem due to the convexity of the energy harvesting function $E(i, x_i)$ in (6.1). We therefore follow a majorization maximization argument to find a local optimal solution for this problem via successive convex optimization. Namely, we approximate $E(i, x_i)$ around some feasible point to get a convex problem, whose solution is then used to (better) approximate $E(i, x_i)$ in the next iteration. Approximate functions should be chosen carefully such that iterations converge to a local optimal solution of the original problem [47, 48]. In particular, in the $(j+1)$st iteration, we solve the following problem

$$\begin{aligned}
\max_{\boldsymbol{p},\boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \\
\text{s.t.} \quad & c_m(1) \leq E_0 \\
& c_m(k+1) + \sum_{i=1}^{k} p_i \leq E_0 + \sum_{i=1}^{k} f^{(j)}(i, x_i), \quad \forall k \\
& 0 \leq x_i \leq L, \quad p_i \geq 0, \quad \forall i
\end{aligned} \tag{6.14}$$

where $f^{(j)}(i, x_i)$ is the first order Taylor series approximation of $E(i, x_i)$ around $x_i^{(j)}$, the solution of the approximate problem in the jth iteration. That is, we have

$$f^{(j)}(i, x_i) \triangleq b_i^{(j)} + m_i^{(j)} x_i \tag{6.15}$$

where

$$b_i^{(j)} \triangleq \frac{E_{1i}}{(x_i^{(j)} + \ell)^{\alpha}} + \frac{E_{2i}}{(L - x_i^{(j)} + \ell)^{\alpha}} + \left(\frac{\alpha E_{1i}}{(x_i^{(j)} + \ell)^{\alpha+1}} - \frac{\alpha E_{2i}}{(L - x_i^{(j)} + \ell)^{\alpha+1}} \right) x_i^{(j)} \tag{6.16}$$

and

$$m_i^{(j)} \triangleq -\frac{\alpha E_{1i}}{(x_i^{(j)} + \ell)^{\alpha+1}} + \frac{\alpha E_{2i}}{(L - x_i^{(j)} + \ell)^{\alpha+1}} \tag{6.17}$$

By convexity of $E(i, x_i)$, it is direct to see that $f^{(j)}(i, x_i)$ satisfies the conditions stated in [47] that guarantee convergence of the iterative solution of problem (6.14) to a local optimal point of problem (6.3). Namely, it holds that

$$f^{(j)}(i, x_i) \leq E(i, x_i), \ \forall x_i \tag{6.18}$$

$$f^{(j)}\left(i, x_i^{(j)}\right) = E\left(i, x_i^{(j)}\right) \tag{6.19}$$

$$\frac{df^{(j)}\left(i, x_i^{(j)}\right)}{dx_i} = \frac{dE\left(i, x_i^{(j)}\right)}{dx_i} \tag{6.20}$$

We focus on problem (6.14) in the remainder of this section. In particular, we introduce some auxiliary variables $\{\delta_i\}$ to denote the amount of energy used for movement in the ith slot. That is, we have

$$\epsilon_m |x_i - x_{i-1}| = \delta_i, \quad \forall i \tag{6.21}$$

This allows us to rewrite the optimization problem as follows

$$\begin{aligned}
\max_{\boldsymbol{p},\boldsymbol{x},\boldsymbol{\delta}} \quad & \sum_{i=1}^{N} \frac{1}{2} \log(1+p_i) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_i \leq E_0 + \sum_{i=1}^{k} b_i^{(j)} + m_i^{(j)} x_i - \sum_{i=1}^{k+1} \delta_i, \quad \forall k \\
& \delta_1 \leq E_0 \\
& \epsilon_m |x_i - x_{i-1}| \leq \delta_i, \quad \forall i \\
& 0 \leq x_i \leq L, \quad p_i \geq 0, \quad \delta_i \geq 0, \quad \forall i
\end{aligned} \tag{6.22}$$

where the relaxation of the equality in (6.21) to an inequality in the above problem does not change the solution. To see this, note that if there exists some slot k such that $\delta_k^* > \epsilon_m |x_k^* - x_{k-1}^*|$, then one can simply decrease the value of δ_k^* until equality holds while keeping the values of x_k^* and x_{k-1}^* the same. This strictly increases the feasible set and thereby potentially increases the objective function. Also note that we set $\delta_{N+1} \triangleq 0$. We now have the following lemma.

Lemma 6.3 *In the optimal solution of problem (6.22), if $\delta_i^* > 0$ then the transmitter should move forward (resp. backward) during slot i if $m_i^{(j)}$ is positive (resp. negative). Conversely, if $m_i^{(j)} = 0$, then there exists an optimal policy with $\delta_i^* = 0$.*

Proof: We show this by contradiction. Assume that we have $\delta_i^* > 0$ and $m_i^{(j)} > 0$

but the transmitter moves backward during time slot i, i.e., $x_i^* < x_{i-1}^*$. Now consider the following alternative policy. Let $\delta_i = 0$, i.e., $x_i = x_{i-1}^*$, and let $\delta_{i+1} = \delta_i^* + \delta_{i+1}^*$. Since the cost to move is linear with distance, this new policy reaches the position x_{i+1}^* from x_{i-1}^* with the same cost. At the same time, since $m_i^{(j)} > 0$, this new policy harvests higher energy at slot i, and thereby achieves higher rates. Thus, the transmitter should move forward. The case where $m_i^{(j)} < 0$ implies that the transmitter should move backward can be shown using similar arguments. This proves the first part of the lemma.

To show the second part, note that since $m_i^{(j)} = 0$, moving during slot i does not make the transmitter gain any energy. Hence, by linearity of the moving cost, given any optimal policy with $\delta_i^* > 0$, setting $\delta_i = 0$ and $\delta_{i+1} = \delta_i^* + \delta_{i+1}^*$ in that case makes the transmitter harvest the same amount of energy, and reach x_{i+1}^* with the same moving cost. ■

Lemma 6.3 indicates that given the optimal amount of movement energy, the optimal movement policy is *greedy*. That is, if the transmitter moves during some time slot i, it moves towards the higher energy location in slot i without considering upcoming slots' energies. Next, we find the optimal greedy policy by decomposing problem (6.22) into inner and outer problems as follows.

6.3.2.1 Inner Problem

We first fix a feasible choice for $\{\delta_i\}$ and solve an *inner problem* for the pair $\{p_i, x_i\}$. We denote the solution of the inner problem by $R(\boldsymbol{\delta})$. By Lemma 6.3, once $\boldsymbol{\delta}$ is fixed, the position $\boldsymbol{x}$ is determined according to the sign of $\boldsymbol{m}^{(j)}$. Whence, the power $\boldsymbol{p}$ is found via directional water-filling [3]. Note that the choice of δ_i should be such that it is equal to 0 if $m_i^{(j)} = 0$, according to Lemma 6.3. In addition, we note that if we have some $\delta_i > 0$ while the greedy movement is not feasible, i.e., moving forward/backward with δ_i energy gets the transmitter outside the straight line boundaries, then surely this δ_i choice is not optimal and needs to change. How to optimally find $\{\delta_i^*\}$ is handled next.

6.3.2.2 Outer Problem

After we solve the inner problem, we find the optimal $\{\delta_i^*\}$ by solving an *outer problem* by maximizing $R(\boldsymbol{\delta})$ over the feasible choices of δ_i. We have the following lemma regarding this problem

Lemma 6.4 *$R(\boldsymbol{\delta})$ is a concave function in $\boldsymbol{\delta}$.*

Proof: Let us pick two feasible points $\boldsymbol{\delta}^{(1)}$ and $\boldsymbol{\delta}^{(2)}$ and denote the solutions of the inner problem for these two choices by $\{\boldsymbol{p}^{(1)}, \boldsymbol{x}^{(1)}\}$ and $\{\boldsymbol{p}^{(2)}, \boldsymbol{x}^{(2)}\}$, respectively. Now let $\boldsymbol{\delta}^{\theta} \triangleq \theta\boldsymbol{\delta}^{(1)} + (1-\theta)\boldsymbol{\delta}^{(2)}$ for some $0 \leq \theta \leq 1$. Next, observe that by linearity of the feasible set, the pair $\boldsymbol{p}^{(\theta)} \triangleq \theta\boldsymbol{p}^{(1)} + (1-\theta)\boldsymbol{p}^{(2)}$ and $\boldsymbol{x}^{(\theta)} \triangleq \theta\boldsymbol{x}^{(1)} + (1-\theta)\boldsymbol{x}^{(2)}$ is feasible in the inner problem for the choice of $\boldsymbol{\delta}^{(\theta)}$. Therefore, we have

$$
\begin{aligned}
R\left(\boldsymbol{\delta}^{(\theta)}\right) &\geq \sum_{i=1}^{N} \frac{1}{2} \log\left(1 + p_i^{(\theta)}\right) \\
&\geq \sum_{i=1}^{N} \frac{\theta}{2} \log\left(1 + p_i^{(1)}\right) + \frac{1-\theta}{2} \log\left(1 + p_i^{(2)}\right)
\end{aligned}
$$

$$= \theta R\left(\boldsymbol{\delta}^{(1)}\right) + (1-\theta) R\left(\boldsymbol{\delta}^{(2)}\right) \tag{6.23}$$

where the second inequality follows by concavity of the log. This concludes the proof. ■

We now solve the following outer problem

$$\begin{aligned}
\max_{\boldsymbol{\delta}} \quad & R(\boldsymbol{\delta}) \\
\text{s.t.} \quad & \delta_1 \leq E_0 \\
& \sum_{i=1}^{k+1} \delta_i \leq E_0 + \sum_{i=1}^{k} b_i^{(j)} + \left[m_i^{(j)} L\right]^+, \quad \forall k \\
& \delta_i \geq 0, \quad \forall i
\end{aligned} \tag{6.24}$$

with $\delta_{N+1} \triangleq 0$, and $[y]^+ \triangleq \max(y, 0)$. Note that the term $\left[m_i^{(j)} L\right]^+$ ensures that all the feasible range of $\{\boldsymbol{\delta}\}$ is covered in the outer problem, and that the inner problem is energy-feasible. By Lemma 6.4, the outer problem is a convex optimization problem [77]. However, not all the available energy should be used in movement, or else we achieve zero throughput. Hence, we follow an iterative water-filling algorithm to solve the outer problem similar to the one proposed in [21] that we summarize next. We add an extra $(N+1)$st slot where unused energy can be discarded. Initially, each slot is filled up by its own energy arrival and the extra $(N+1)$st slot is left empty. We allow energy/water to move to the right only if this increases the objective function. Meters are put in between slots to measure the amount of water moving forward. This allows us to pull water back to their sources if this

Algorithm 3

repeat
    Approximate $E(i, x_i)$ around the $(j-1)$st iteration's location solution $x_i^{(j-1)}$ using (6.15)-(6.17), $\forall i$.
    Fix a feasible movement energy allocation $\boldsymbol{\delta}$.
    repeat
        Solve inner problem for $R(\boldsymbol{\delta})$ as in Section 6.3.2.1.
        Solve outer problem for $\boldsymbol{\delta}^*$ as in Section 6.3.2.2.
    until Convergence of movement energy water levels.
until $\left\| \left(\boldsymbol{x}^{(j)}, \boldsymbol{p}^{(j)}\right) - \left(\boldsymbol{x}^{(j-1)}, \boldsymbol{p}^{(j-1)}\right) \right\|$ is small enough.

increases the objective function. Eventually, all the water in the $(N+1)$st slot will be discarded but can be pulled back also during the iterations if necessary. Since the objective function increases with each water flow, problem feasibility is maintained during iterations, and by convexity of the problem, iterations converge to the optimal solution. We summarize the multiple energy arrivals solution approach in Algorithm 3.

6.4 Online Setting: Problem (6.6)

In this section we discuss the solution of problem (6.6). Note that the transmitter needs to decide on both the movement and the transmission energy for each time slot during the course of communication given only causal knowledge of the harvested energy. In particular, since the energy at time slot t is revealed *after* the transmitter relocates itself to position x_t, this means that the transmitter decides on where to relocate *blindly*, i.e., before knowing what amount of energy it will harvest. We now derive an upper bound on the optimal long term average throughput under such conditions in the following lemma.

Lemma 6.5 *The optimal solution, r^*, of problem (6.6) satisfies*

$$r^* \leq \frac{1}{2}\log\left(1+\max\left\{\bar{\mu}_1, \bar{\mu}_2\right\}\right) \tag{6.25}$$

where $\bar{\mu}_1 \triangleq \frac{\mu_1}{\ell^\alpha} + \frac{\mu_2}{(L+\ell)^\alpha}$ and $\bar{\mu}_2 \triangleq \frac{\mu_1}{(L+\ell)^\alpha} + \frac{\mu_2}{\ell^\alpha}$.

Proof: First, let us take $\epsilon_m = 0$. This enlarges the feasible set $\mathcal{F}$ since now the transmitter can move without energy cost. Since $E(i, x_i)$ is convex in x_i, the movement policy in this case should be extremal; the transmitter should only be positioned at either ends of the line to harvest maximal energy. Let us assume that the transmitter chooses to be at the first source's position, i.e., at $x = 0$, for θ fraction of the time. This allows us to construct a set of time slot indices $J_1(n) \subseteq \{1, \ldots, n\}$ with $x_i = 0$ for $i \in J_1(n)$, and $\lim_{n\to\infty} |J_1(n)|/n = \theta$. Similarly, we can define $J_2(n)$ to be the time slot indices where the transmitter is located at the second source's position, $x_i = L$, with $\lim_{n\to\infty} |J_2(n)|/n = 1-\theta$. Using Jensen's inequality [77] we have

$$\begin{aligned}
r^* &\leq \lim_{T\to\infty} \frac{1}{2}\log\left(1+\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} p_t\right]\right) && (6.26)\\
&\leq \lim_{T\to\infty} \frac{1}{2}\log\left(1+\mathbb{E}\left[\frac{1}{T}\sum_{t\in J_1(T)} E(t,0) + \frac{1}{T}\sum_{t\in J_2(T)} E(t,L)\right]\right) && (6.27)\\
&= \lim_{T\to\infty} \frac{1}{2}\log\left(1+\frac{|J_1(T)|}{T}\bar{\mu}_1 + \frac{|J_2(T)|}{T}\bar{\mu}_2\right) && (6.28)\\
&= \frac{1}{2}\log\left(1+\theta\bar{\mu}_1 + (1-\theta)\bar{\mu}_2\right) && (6.29)\\
&\leq \frac{1}{2}\log\left(1+\max\left\{\bar{\mu}_1, \bar{\mu}_2\right\}\right) && (6.30)
\end{aligned}$$

where (6.27) follows by definitions of the feasible set $\mathcal{F}$. ■

Next, we propose an online feasible energy management policy and show that it achieves the upper bound in the previous lemma, and thereby proving its optimality. Let $j \triangleq \arg\max_{i\in\{1,2\}} \bar{\mu}_i$, i.e., j denotes the energy source with higher average arrival rate μ_j. Then, starting from its original position x_0, the transmitter uses all its harvested energy to move towards source j, and does not use any energy in transmission. Let us denote by n_0 the time slot at which the transmitter arrives at source j. Then, starting from time slot n_0+1 onwards, the transmitter uses all its energy in transmission, and does not use any energy in movement, i.e., it stays at source j till the end. We coin the above scheme as the *move-then-transmit* scheme. We now have the following result regarding how n_0 behaves asymptotically.

Lemma 6.6 *For all values of $\epsilon_m > 0$, $0 < L < \infty$, and $E_0 \geq 0$, it holds that $\lim_{k\to\infty} \frac{n_0}{k} = 0$ a.s.*

Proof: Let us assume without loss of generality that $\mu_2 > \mu_1$. By definition, one can write n_0 as follows

$$n_0 = \min\left\{k: \ \sum_{i=1}^{k} E(i,x_i) \geq L\epsilon_m - E_0\right\} \tag{6.31}$$

$$\leq \min\left\{k: \ \sum_{i=1}^{k} E_{1i} + E_{2i} \geq (L+\ell)^\alpha \left(L\epsilon_m - E_0\right)\right\} \tag{6.32}$$

where (6.32) follows by considering the worst case (smallest) amount of energy harvested from both sources simultaneously, i.e., assuming the transmitter is at distance L away from both sources. Now since $\{E_{1i}+E_{2i}\}$ is an i.i.d. process with mean $\mu_1 + \mu_2$, by the strong law of large numbers, we have that for fixed $\gamma, \nu > 0$, there exists a number k_0 such that $\forall k \geq k_0$ the following holds

$$\sum_{i=1}^{k} E_{1i} + E_{2i} \geq k\left(\mu_1 + \mu_2 - \nu\right) \tag{6.33}$$

with probability larger than $1-\gamma$. Whence, a further upper bound on n_0, that holds with probability larger than $1-\gamma$ is given by

$$n_0 \leq \min\left\{k \geq k_0 : \; k\left(\mu_1 + \mu_2 - \nu\right) \geq (L+\ell)^{\alpha}\left(L\epsilon_m - E_0\right)\right\} \tag{6.34}$$

$$= \max\left\{\left\lceil \frac{(L+\ell)^{\alpha}\left(L\epsilon_m - E_0\right)}{\mu_1 + \mu_2 - \nu} \right\rceil, k_0\right\} \tag{6.35}$$

Therefore, it holds that

$$\lim_{k\to\infty} \frac{n_0}{k} \leq \lim_{k\to\infty} \frac{1}{k} \max\left\{\left\lceil \frac{(L+\ell)^{\alpha}\left(L\epsilon_m - E_0\right)}{\mu_1 + \mu_2 - \nu} \right\rceil, k_0\right\} = 0 \tag{6.36}$$

with probability larger than $1-\gamma$. Since $\gamma > 0$ was arbitrary, the above is true as $\gamma \to 0$ as well. This concludes the proof. ■

Note that while staying at source j, the transmitter is harvesting i.i.d. amount of energy with an average of $\bar{\mu}_j$. Hence, the transmitter can use, e.g., the *best effort transmission* scheme introduced and analyzed in [66] to optimally manage the amounts of its harvested energy for transmission. This best effort transmission scheme achieves the capacity of an AWGN channel with an average power constraint equal to the average energy harvesting rate by basically allowing the transmitter to send with energy equal to the average harvesting rate as long as it is feasible, and stay silent otherwise [66].

Next, we state the two main results of this section. Throughout, we assume that the amounts of energy generated at the two source are bounded, i.e., there exist some $M_1 > 0$ and $M_2 > 0$ such that $E_{1i} \leq M_1$ a.s. $\forall i$ and $E_{2i} \leq M_2$ a.s. $\forall i$. It is worth noting that this boundedness assumption is satisfied naturally if the transmitter is equipped with a finite battery B, since any excess energy received above the battery capacity overflows and cannot be used. We now have the following result for the infinite battery case.

Theorem 6.1 *The move-then-transmit scheme along with best effort transmission strategy is optimal, for all values of $\epsilon_m > 0$, $0 < L < \infty$, and $E_0 \geq 0$.*

Proof: Without loss of generality let use assume that $\mu_2 > \mu_1$, and hence the transmitter initially moves towards the second source and reaches there after some n_0 time slots. We then have the following energy causality constraints for transmission

$$\frac{1}{k} \sum_{i=n_0+1}^{k} p_i \leq \frac{1}{k} \sum_{i=n_0+1}^{k} E(i, L), \quad \forall k \geq n_0 \tag{6.37}$$

Now let us examine the amounts of energy not used in transmission, i.e., during the first n_0 time slots, if the transmitter was initially located at $x_0 = L$. By Lemma 6.6 and the boundedness assumption, this amount behaves asymptotically as follows

$$\lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{n_0} E(i, L) \leq \lim_{k\to\infty} \frac{n_0}{k} \left(\frac{M_1}{(L+r)^\alpha} + \frac{M_2}{r^\alpha} \right) = 0 \quad a.s. \tag{6.38}$$

Therefore, for fixed $\gamma, \nu > 0$ we have

$$\frac{1}{k} \sum_{i=n_0+1}^{k} p_i \leq \frac{1}{k} \sum_{i=1}^{k} E(i, L) - \frac{\nu}{k}, \quad \forall k \geq n_0 \tag{6.39}$$

with probability larger than $1 - \gamma$, and k large enough. Thus, as k grows infinitely large, one can take γ and ν down to 0, which means that the energy used in movement does not have a long term average effect on the energy causality constraint set. Therefore, transmitting by the average harvesting rate at the second source $\mathbb{E}\left[E(i, L)\right] = \bar{\mu}_2$ using the best effort strategy one achieves the following rate for T large enough [66]

$$\frac{T - n_0}{T} \frac{1}{2} \log\left(1 + \bar{\mu}_2\right) - \kappa_T \tag{6.40}$$

where $\kappa_T \to 0$ as $T \to \infty$. Hence, taking the limit as $T \to \infty$, and using Lemma 6.6, one achieves a long term average throughput of $\frac{1}{2} \log\left(1 + \bar{\mu}_2\right)$, which is equal to the upper bound stated in Lemma 6.5. Therefore, the proposed scheme is optimal. ■

Next, we discuss the case where the transmitter is equipped with a finite battery of size B. Under a finite battery capacity B, reference [54] introduced a near-optimal online policy for single-user energy harvesting channels coined the *fixed fraction policy* (FFP). Under this policy, in each time slot, the transmitter uses a fixed fraction of the amount of energy available in its battery for transmission. Such fraction is given by the average harvesting rate divided by the battery capacity. It is shown in [54] that such policy achieves a long term average throughput that lies within constant multiplicative and additive gaps from the optimal solution, for all i.i.d. energy arrival patterns and battery sizes. In our setting let us define the following fraction

$$q \triangleq \frac{\max\{\bar{\mu}_1, \bar{\mu}_2\}}{B} \tag{6.41}$$

and define the transmission power at time slot t to be given by

$$p_t = \begin{cases} 0, & t \leq n_0 \\ qb_t, & t > n_0 \end{cases} \tag{6.42}$$

Since $q \leq 1$, the FFP policy above is always feasible. We now state the following result for the finite battery case.

Theorem 6.2 *The move-then-transmit scheme along with the FFP in (6.42) achieve a long term average throughput that lies within an additive gap* 0.72 *and a multiplicative gap of* 0.5 *from the optimal solution, for all values of* $\epsilon_m > 0$, $0 < L < \infty$, *and* $E_0 \geq 0$*; and for all i.i.d. energy patterns and battery sizes.*

Proof: The proof follows the same lines as in the proof of Theorem 6.1; basically, the fact that the effect of the movement strategy on the energy causality constraint set vanishes in the long term does not depend on the battery capacity, and hence it still holds. Once this is established, one can treat the problem as a single-user online problem with an energy harvesting average rate of $\bar{\mu}_j$, and use the same techniques as in [54, Theorem 2] to show that the achieved rate lies within the constant gaps

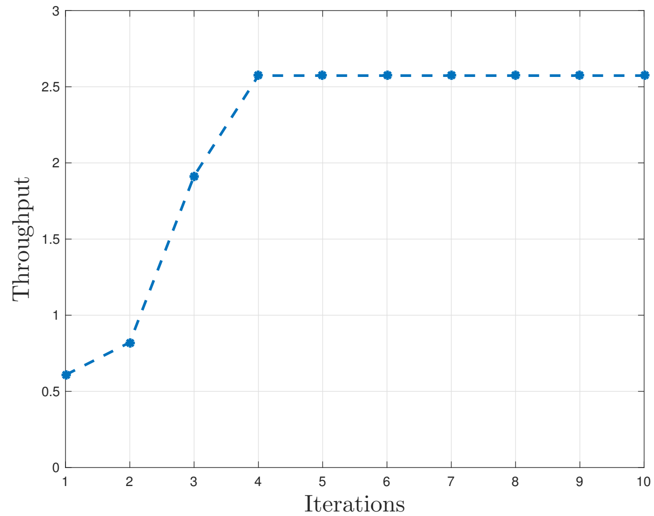


Figure 6.2: Convergence of throughput over time.

mentioned in the theorem from the upper bound stated in Lemma 6.5, and hence, the same constant gap results hold with respect to the optimal solution. ■

6.5 Numerical Results

6.5.1 Deterministic Arrivals

In this section, we present some numerical examples to further illustrate our results in the offline setting. We consider a system of four time slots. The transmitter has an initial amount of energy of $E_0 = 0.1$ energy units. The length of the straight line between the energy sources is $L = 7$ distance units, and the transmitter is initially positioned at $x_0 = 2.5$. Energies arrive at the two energy sources with amounts $\boldsymbol{E}_1 = [0, 1, 7, 5]$ and $\boldsymbol{E}_2 = [8, 5, 1, 1]$, at the first and the second energy source, respectively. The path loss factor $\alpha = 2.5$, $\ell = 0.3$, and the movement energy cost per distance $\epsilon_m = 0.5$.

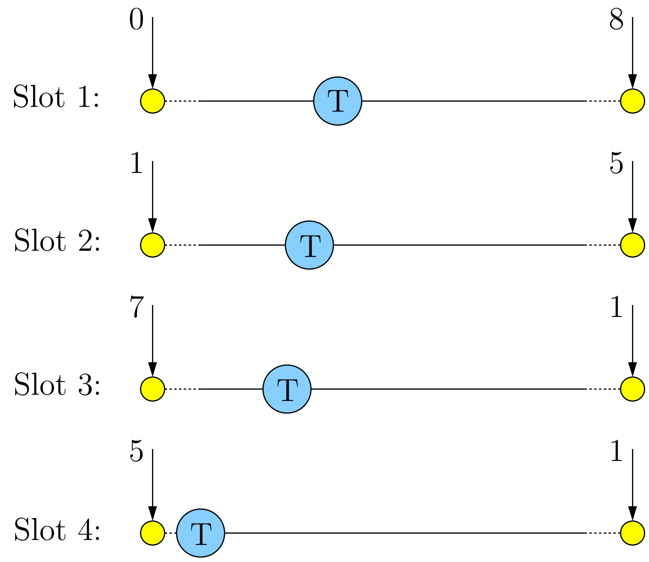


Figure 6.3: Optimal transmitter location in a four-slot system.

We solve problem (6.14) by initially approximating the energy-position function at each time slot around x_0. We then do the problem decomposition to solve for $\{\delta_i^*\}$ and $\{p_i^*, x_i^*\}$ as discussed in Sections 6.3.2.1 and 6.3.2.2. Finally, we substitute by $\{x_i^*\}$ in problem (6.14) and re-iterate until convergence. For this example, it takes 5 iterations to converge to a local optimal solution of problem (6.3). In Fig. 6.2, we show the convergence of the throughput with iterations.

In Fig. 6.3, we plot the results of this example. We show the transmitter's position at different slots in between the two energy sources. Arrows at the sources represent the amounts of energy arriving (emitted) by each source at a given time slot. From the figure, we see that the transmitter stays at its initial position in the first time slot, i.e., $x_1^* = 2.5$. This is mainly because the initial position of the transmitter is inclined towards the first source, and the fact that the energy amount at the second source is higher than that of the first source in the first time slot. One more reason for this movement behavior is that the first source receives

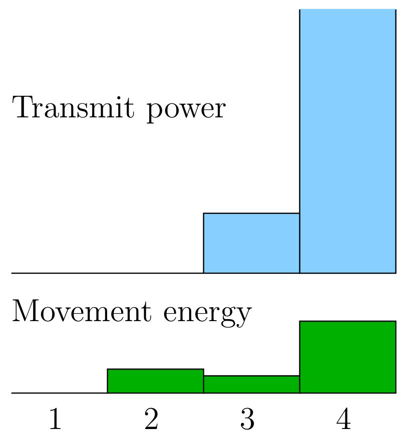


Figure 6.4: Transmit power and movement energy consumptions.

higher amounts of energy in later slots. Therefore, we see that the transmitter moves towards the first source during slots 2 and 3 until it reaches the end of the line in slot 4. The optimal position is given by $\boldsymbol{x}^* = [2.5, 1.98, 1.58, 0]$, with powers $\boldsymbol{p}^* = [0, 0, 0.68, 101.44]$, and movement energy consumption of $[0, 0.25, 0.2, 0.67]$.

We plot the optimal transmit power and movement energy consumptions over the four time slots in Fig. 6.4. The height in blue and green represents the transmit power and the movement energy costs, respectively. We see that the transmitter neither moves nor transmits during the first time slot and saves all its harvested energy for later slots' movements and transmission. It starts spending some energy in movement during the second time slot while still not transmitting, and then finally during the third and fourth time slots it both moves and transmits to the receiver, achieving a throughput of 2.57.

Next, we show the effect of the movement energy cost per unit distance, ϵ_m, on the throughput. We shift the initial position to $x_0 = 3.5$ and use the same parameter values from the previous example except that we decrease ϵ_m to 0.01. The solution

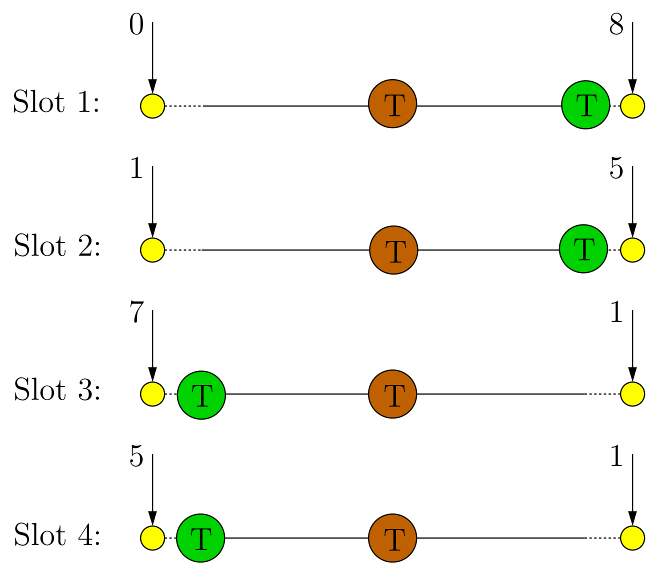


Figure 6.5: Effect of moving cost on optimal location.

in this case is $x^* = [7,7,0,0]$ with a throughput equal to 9.7. Due to the small movement energy cost, the transmitter in this case rides the energy peaks from the two sources, i.e., it harvests $E_i = \frac{1}{\ell^\alpha}\max\{E_{1i}, E_{2i}\}$, $\forall i$. The optimal location is shown by the green transmitter in Fig. 6.5. We then increase ϵ_m to 3 and re-solve. In this case, we get $x^* = [3.5, 3.5, 3.5, 3.5]$ with a throughput equal to 0.484. Due to the large movement energy cost, the transmitter does not move during the course of communication and uses all of its available energy only for transmission. The optimal location in this case is shown by the brown transmitter in Fig. 6.5.

6.5.2 Stochastic Arrivals

In this section, we present some numerical results for the online setting. We consider a system where the energy arrivals at the first source follows a uniform distribution and that at the second source follows an exponential distribution. The system parameters are the same as in the previous offline examples except that we set

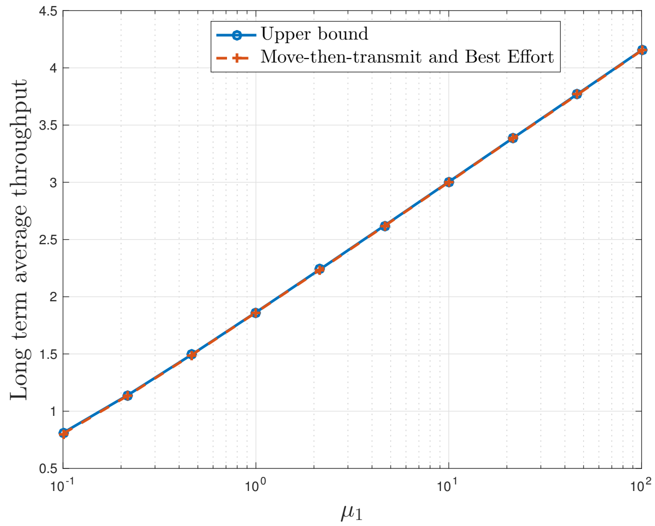


Figure 6.6: Long term average rate achieved by the proposed move-then-transmit and best effort policy, and the theoretical upper bound, versus the average harvesting rate of the first source. In this example we set $\mu_2 = 2\mu_1$.

$L = 10$ distance units and $\epsilon_m = 10$ energy units per unit distance. In Fig. 6.6 we plot the long term average rate achieved by the proposed move-then-transmit and best effort policy against μ_1. We set $\mu_2 = 2\mu_1$ in this example. We also plot the theoretical upper bound obtained in Lemma 6.5. We see that the proposed policy achieves the theoretical upper bound and that the two curves are almost identical as stated in Theorem 6.1.

Finally, we consider a transmitter with finite battery capacity B. Energy arrivals follow Bernoulli distribution with parameters 0.5 and 0.3 at the first and the second source, respectively. In Fig. 6.7, we plot the long term average throughput achieved by the proposed move-then-transmit and FFP against μ_1. We set $\mu_2 = 2 \times \frac{0.3}{0.5}\mu_2 = 1.2\mu_2$ in this example. We also scale the battery with μ_1 and set it to $B = \max\{\bar{B}_1, \bar{B}_2\}$ where $\bar{B}_1 \triangleq \frac{0.5\mu_1}{\ell^\alpha} + \frac{0.3\mu_2}{(L+\ell)^\alpha}$ and $\bar{B}_2 \triangleq \frac{0.5\mu_1}{(L+\ell)^\alpha} + \frac{0.3\mu_2}{\ell^\alpha}$. We see that the rate achieved lies within a constant gap from the upper bound as stated in

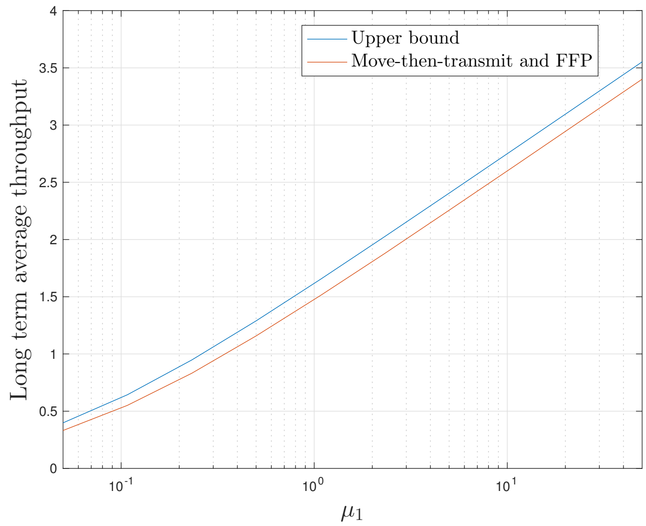


Figure 6.7: Long term average rate achieved by the proposed move-then-transmit and fixed fraction policy, and the theoretical upper bound, versus the average harvesting rate of the first source. In this example we set $\mu_2 = 1.2\mu_1$.

Theorem 6.2.

6.6 Discussion and Possible Extensions

In this section, we discuss some extensions to the problems and the model of this chapter. Regarding the movement path, we considered a one-dimensional straight line movement profile in this chapter as a first step to characterize the movement-throughput tradeoff. It would be of interest to extend the movement path to other two-dimensional or three-dimensional geometric shapes and understand the movement-throughput tradeoff in more general settings.

Regarding the energy sources, we considered the case where the sources emit energy in each time slot according to some random phenomenon. One way to extend this model is to optimize the amounts of the sources' emitted energy by introducing storage devices at the energy source side, i.e., transform the random energy source to a controlled energy sharing entity. In this case, the energy harvested at the transmitter at time slot i is given by

$$E(i, x_i) = \frac{\beta_{1i}}{(x_i + \ell)^\alpha} + \frac{\beta_{2i}}{(L - x_i + \ell)^\alpha} \tag{6.43}$$

where $\{\beta_{1i}\}$ and $\{\beta_{1i}\}$ satisfy the usual energy causality constraints

$$\sum_{i=1}^{k} \beta_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \tag{6.44}$$

$$\sum_{i=1}^{k} \beta_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \tag{6.45}$$

In other words, the two sources now generate energy with amounts $\{E_{1i}\}$ and $\{E_{2i}\}$ but only share $\{\beta_{1i}\}$ and $\{\beta_{2i}\}$ portion of them with the transmitter. We note that in this case, procrastinating policies [19] need not be optimal since the energy sharing efficiency is changing with the position of the transmitter. We also note that even with a single energy arrival at the two sources the problem now does not admit a closed form solution as shown in Section 6.3.1. This is only the case if we consider only one time slot $N = 1$. Thus, even with a single energy arrival, one has to optimize the amounts of shared energy over multiple time slots.

6.7 Conclusion

In this chapter, we considered mobility effects on energy harvesting nodes. Energy arrivals at a node depend on the node's relative position to energy emitting sources, and therefore movement is motivated by finding better energy harvesting locations. However, nodes incur a moving cost per unit distance travelled. We considered movement along a straight line, where two energy sources are located towards the opposite ends of the line. We characterized the optimal tradeoff between staying at the same spot so as to spend all available energy in transmission, and spending some energy to move to a potentially better energy location so as to achieve higher throughput. We studied this problem in both offline and online settings. In the offline setting, we designed movement and transmission policies that maximize the sum throughput by a given deadline. We first solved the case with a single energy arrival at each source, and then generalized that to the case of multiple energy arrivals. In the online setting, we proposed an optimal *move-then-transmit* scheme that maximizes the long term average throughput, where the transmitter first moves towards the energy source with higher energy harvesting mean, and then starts transmission. We analyzed the performance of this scheme under both infinite and finite battery capacities at the transmitter.

CHAPTER 7

Delay Minimal Policies in Energy Harvesting Communication Systems

7.1 Introduction

In this chapter, we characterize delay minimal power scheduling policies in energy harvesting communication systems. We consider a continuous time system where the delay experienced by each bit is given by the time spent by the bit in the queue waiting to be transmitted to its receiver. We first consider a single-user channel where the transmitter has a finite-sized battery to save its harvested energy. Data arrives during the course of communication and is saved in a finite data buffer as well. We find the optimal power policy that minimizes the average delay experienced by the bits subject to energy and data causality constraints. We characterize the optimal solution in terms of Lagrange multipliers, and calculate their values in a recursive manner. We show that, different from the existing literature, the optimum transmission power is not constant between the energy harvesting and data arrival events; the transmission power starts high, decreases linearly, and potentially reaches zero between energy harvests and data arrivals. Intuitively, untransmitted bits experience cumulative delay due to the bits to be transmitted ahead of them, and hence the reason for transmission power starting high and decreasing over time between energy harvests and data arrivals. Next, we study a multiuser version of this problem, namely a two-user broadcast channel, and characterize the optimal transmission policies that minimize the *sum* delay. For this setting, we consider the case where the transmitter has an infinite-sized battery, and that all data packets intended for the receivers are available at the beginning of the communication session. We characterize the optimal solution in terms of Lagrange multipliers, and present an iterative solution that optimally calculates their values. Our results show that in the optimal policy, both users may not be served simultaneously all the time; there may be times where only the strong user or only the weak user is served alone. We also show that the optimal policy may have *gaps* in transmission in between energy arrivals where none of the users is served, echoing the results of the single-user setting.

7.2 Single-User Channel

In this section we consider a single-user AWGN channel, see Fig. 7.1, where at arrival time t_m, $m = 0, 1, \ldots, M-1$, with $t_0 = 0$, energy is harvested at the transmitter with amount E_m and data intended for the receiver arrives with amount B_m. The transmitter saves energy and data in a battery with finite capacity $E_{\max}$ and in a data buffer with finite capacity $B_{\max}$, respectively. We denote the cumulative harvested energy and the total amounts of received data at time t by

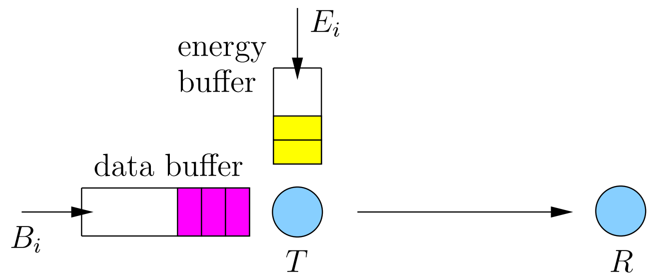


Figure 7.1: Single-user energy harvesting channel with finite-sized battery and data buffer.

$$E_a(t) = \sum_{i=0}^{m-1} E_i, \quad t_{m-1} < t \leq t_m, \quad m = 1, \dots, M \tag{7.1}$$

$$B_a(t) = \sum_{i=0}^{m-1} B_i, \quad t_{m-1} < t \leq t_m, \quad m = 1, \dots, M \tag{7.2}$$

where we define $t_M = \infty$. For a power policy $p(t)$ at time t, the cumulative consumed energy and the total departed data to the receiver at time t are given by

$$E(t) = \int_0^t p(\tau) d\tau \tag{7.3}$$

$$B(t) = \int_0^t \frac{1}{2} \log(1 + p(\tau)) d\tau \tag{7.4}$$

where log is the natural logarithm throughout this chapter. We call a policy feasible if the following is satisfied

$$E_a(t) - E_{\max} \leq E(t) \leq E_a(t), \quad \forall t \tag{7.5}$$

$$B_a(t) - B_{\max} \leq B(t) \leq B_a(t), \quad \forall t \tag{7.6}$$

The above conditions assure that the policy conforms to energy and data causality constraints, and that energy and data buffers are not overflown.

The delay experienced by each bit is the time interval from its arrival time to its actual transmission time. The total average delay for the system is given by

$$\bar{D} = \int_0^\infty t dB(t) - \int_0^\infty t dB_a(t) \tag{7.7}$$

Our objective is to characterize the optimal power policy that minimizes the total average delay in (7.7) subject to feasibility conditions in (7.5) and (7.6). For a given data arrival profile, the second term in (7.7) is constant, and therefore minimizing $\bar{D}$ is equivalent to minimizing the *gross* delay defined as

$$D = \int_0^\infty t dB(t) = \int_0^\infty \frac{t}{2} \log(1+p(t))dt \tag{7.8}$$

We note that the maximum data buffer constraint in this setting can model strict delay requirements for data packets. The optimization problem is formulated as

$$\begin{aligned}
\min_p \quad & \int_0^\infty t \log(1+p(t))dt \\
\text{s.t.} \quad & E_a(t_m) - E_{\max} \leq \int_0^{t_m} p(t)dt \leq E_a(t_m), \quad m = 1, \dots, M \\
& B_a(t_m) - B_{\max} \leq \int_0^{t_m} \log(1+p(t))dt \leq B_a(t_m), \quad m = 1, \dots, M-1 \\
& \int_0^\infty \log(1+p(t))dt = B_a(t_M) \\
& p(t) \geq 0, \quad \forall t
\end{aligned} \tag{7.9}$$

where for convenience we dropped the half term of the rate-power function[1]. We solve problem (7.9) in the remainder of this section.

7.2.1 Properties of the Optimal Solution

We note that problem (7.9) is not a convex optimization problem. However, our analysis will show that the KKT optimality conditions admit a unique, and therefore the optimal, solution. We introduce the following Lagrangian

$$\begin{aligned}\mathcal{L} = &\int_0^\infty t\log(1+p(t))dt - \int_0^\infty \eta(t)p(t)dt \\ &+ \sum_{m=1}^{M} \lambda_{1m}\left(\int_0^{t_m} p(t)dt - E_a(t_m)\right) + \sum_{m=1}^{M} \lambda_{2m}\left(E_a(t_m) - E_{\max} - \int_0^{t_m} p(t)dt\right) \\ &+ \sum_{m=1}^{M-1} \mu_{1m}\left(\int_0^{t_m} \log(1+p(t))dt - B_a(t_m)\right) \\ &+ \sum_{m=1}^{M-1} \mu_{2m}\left(B_a(t_m) - B_{\max} - \int_0^{t_m} \log(1+p(t))dt\right) \\ &- \nu\left(\int_0^\infty \log(1+p(t))dt - B_a(t_M)\right)\end{aligned} \tag{7.10}$$

where $\{\lambda_{1m}, \lambda_{2m}, \mu_{1m}, \mu_{2m}\}$, ν, and $\eta(t)$ are Lagrange multipliers. Taking the derivative with respect to $p(t)$ and equating to 0 we get the following KKT optimality conditions

$$p(t) = \left(\frac{\mu(t) - t}{\lambda(t)} - 1\right)^+ \tag{7.11}$$

[1]This is indeed without loss of optimality as the objective function and the data constraints can both be multiplied by 2.

where we have defined

$$\lambda(t) = \sum_{\{m:\ t_m \geq t\}} \lambda_{1m} - \lambda_{2m} \tag{7.12}$$

$$\mu(t) = \nu - \sum_{\{m:\ t_m \geq t\}} \mu_{1m} - \mu_{2m} \tag{7.13}$$

along with the complementary slackness conditions

$$\lambda_{1m}\left(\int_0^{t_m} p(t)dt - E_a(t_m)\right) = 0, \quad \forall m \tag{7.14}$$

$$\lambda_{2m}\left(E_a(t_m) - E_{\max} - \int_0^{t_m} p(t)dt\right) = 0, \quad \forall m \tag{7.15}$$

$$\mu_{1m}\left(\int_0^{t_m} \log(1+p(t))dt - B_a(t_m)\right) = 0, \quad m = 1, \ldots, M-1 \tag{7.16}$$

$$\mu_{2m}\left(B_a(t_m) - B_{\max} - \int_0^{t_m} \log(1+p(t))dt\right) = 0, \quad m = 1, \ldots, M-1 \tag{7.17}$$

We now state the following lemma

Lemma 7.1 *The optimal $\lambda(t)$ (resp. $\mu(t)$) is a piece wise constant function, with possible changes only if the energy (resp. data) buffer is either depleted or full.*

Proof: By the complementary slackness conditions we have

$$\lambda_{1m} = \lambda_{2m} = 0, \quad \text{if } E_a(t_m) - E_{\max} < E(t_m) < E_a(t_m) \tag{7.18}$$

$$E(t_m) = E_a(t_m), \quad \text{if } \lambda_{1m} > 0 \tag{7.19}$$

$$E(t_m) = E_a(t_m) - E_{\max}, \quad \text{if } \lambda_{2m} > 0 \tag{7.20}$$

Therefore, $\lambda(t)$ stays constant between arrival times, and can only change when $\lambda_{1m} > 0$ or $\lambda_{2m} > 0$ for some m, which occurs only if the energy buffer is either depleted or full at t_m. Similar arguments follow for $\mu(t)$. ■

By Lemma 7.1, both $\lambda(t)$ and $\mu(t)$ are sequences rather than continuous functions of time. We denote by $\{s_1, s_2, \ldots, s_L\} \subseteq \{t_0, t_1, \ldots, t_{M-1}\}$ the change times of $\lambda(t)$ and $\mu(t)$, with $s_1 = 0$. Therefore we have

$$\lambda(t) = \begin{cases} \lambda_k^c, & t \in [s_k, s_{k+1}) \\ \\ \lambda_L^c, & t \in [s_L, \infty) \end{cases}, \qquad \mu(t) = \begin{cases} \mu_k^c, & t \in [s_k, s_{k+1}) \\ \\ \mu_L^c, & t \in [s_L, \infty) \end{cases} \tag{7.21}$$

Therefore, by definition of $\{s_k\}$, at least one constraint is met with equality at s_k, $\forall k$, and no constraint is met with equality during the interval (s_{k-1}, s_k). The following lemma provides the necessary conditions for the two sequences $\{\lambda_k^c\}$ and $\{\mu_k^c\}$ to increase/decrease.

Lemma 7.2 *In the optimal policy: 1) λ_k^c is larger (resp. smaller) than λ_{k-1}^c only if the battery is full (resp. depleted) at time s_{k-1}; and 2) μ_k^c is larger (resp. smaller) than μ_{k-1}^c only if the data buffer is depleted (resp. full) at time s_{k-1}.*

Proof: By definition of $\lambda(t)$ in (7.12), the function can only increase (resp. decrease) after time s_{k-1} if $\lambda_{2m} > 0$ (resp. $\lambda_{1m} > 0$) for m such that $t_m = s_{k-1}$. By complementary slackness, the battery must be full (resp. depleted) at time s_{k-1}. The second statement of the lemma follows using similar arguments. ■

We conclude the optimality conditions by the following lemma

Lemma 7.3 *Whenever the optimal power $p(t) > 0$, it is monotonically decreasing with time.*

Proof: Let us have $p(t) > 0$ $\forall t \in (l_1, l_2)$ where (l_1, l_2) lies in between arrival times. By Lemma 7.1, we know that both $\lambda(t)$ and $\mu(t)$ are constants during that interval (say λ_l and μ_l). Hence, from (7.11), $p(t)$ is either monotonically increasing or decreasing (depending on the sign of λ_l). Now assume it is increasing during this interval, i.e., $\lambda_l < 0$, and denote $\lambda_l' = -\lambda_l$, and $\mu_l' = l_2 - \mu_l + l_1$. Now define a new power policy $p'(t) = (\mu_l' - t)/\lambda_l' - 1$, for $t \in (l_1, l_2)$. It is direct to see that both $p(t)$ and $p'(t)$ use the same energy and deliver the same data amount during (l_1, l_2), as what we did is merely flipping the curve of $p(t)$ in (l_1, l_2) around $\frac{l_1+l_2}{2}$. However, the (now decreasing) new policy $p'(t)$ does so with a strictly less delay. This is due to the multiplicative term t in the objective function; it is strictly better to use higher powers at the beginning and lower powers at the end, so that data arriving earlier in time are delivered faster. ■

By Lemma 7.3, we conclude that the optimal $\lambda(t)$ is non-negative for all t, and that it is necessary, from (7.11), to have $\mu(t) > t$ for all t before the total amount of data is delivered. Lemma 7.3 also shows that power can reach 0 in between arrivals, where the communication stops until the next energy or data arrival instant.

7.2.2 Recursive Formulas

In this section, we show how to find λ_k^c, μ_k^c, and s_k in a recursive manner. We will use these recursive formulas to construct the optimal solution in the next section.

First, assume s_k, $E(s_k)$, $B(s_k)$, and μ_k^c are known, and define the following values for all $\{m: \; t_m > s_k\}$

$$\lambda_m^{eu}: \quad E(s_k) + \int_{s_k}^{t_m} \left(\frac{\mu_k^c - t}{\lambda_m^{eu}} - 1 \right)^+ dt = E_a(t_m) \tag{7.22}$$

$$\lambda_m^{bu}: \quad B(s_k) + \int_{s_k}^{t_m} \log \left(1 + \left(\frac{\mu_k^c - t}{\lambda_m^{bu}} - 1 \right)^+ \right) dt = B_a(t_m) \tag{7.23}$$

$$\lambda_m^u = \max\{\lambda_m^{eu}, \lambda_m^{bu}\} \tag{7.24}$$

$$\lambda_m^{el}: \quad E(s_k) + \int_{s_k}^{t_m} \left(\frac{\mu_k^c - t}{\lambda_m^{el}} - 1 \right)^+ dt = E_a(t_m) - E_{\max} \tag{7.25}$$

$$\lambda_m^{bl}: \quad B(s_k) + \int_{s_k}^{t_m} \log \left(1 + \left(\frac{\mu_k^c - t}{\lambda_m^{bl}} - 1 \right)^+ \right) dt = B_a(t_m) - B_{\max} \tag{7.26}$$

$$\lambda_m^l = \min\{\lambda_m^{el}, \lambda_m^{bl}\} \tag{7.27}$$

Therefore, λ_m^u is the minimum value of λ such that either the energy or the data buffer is depleted by time t_m, i.e., an upper bound is met with equality. On the other hand, λ_m^l is the maximum value of λ such that either the energy or the data buffer is full by time t_m, i.e., a lower bound is met with equality. Let us denote $\Lambda(m) = [\lambda_m^u, \lambda_m^l]$. Hence, to maintain feasibility, we need to have $\lambda_k^c \in \Lambda(m)$ if $s_{k+1} \geq t_m$. Now define the following integers

$$m_1^{\max}(k) = \max\left\{ m: \bigcap_{i:\; t_i > s_k}^{m} \Lambda(i) \neq \emptyset \right\} \tag{7.28}$$

$$m_1^u(k) = \max\left\{ m: \lambda_m^u \in \bigcap_{i:\; t_i > s_k}^{m} \Lambda(i) \right\} \tag{7.29}$$

$$m_1^l(k) = \max\left\{ m: \lambda_m^l \in \bigcap_{i:\; t_i > s_k}^{m} \Lambda(i) \right\} \tag{7.30}$$

We now have the following lemma.

Lemma 7.4 *Assume that one has the optimal solution up to time s_k, along with μ_k^c. Then, λ_k^c and s_{k+1} are found as follows:*

$$\begin{aligned}
&\textit{If } \Lambda\left(m_1^{\max}(k)+1\right) > \bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i) \quad \Rightarrow \quad \lambda_k^c = \lambda_{m_1^l(k)}^l,\ s_{k+1} = t_{m_1^l(k)} \\
&\textit{Else, if } \Lambda\left(m_1^{\max}(k)+1\right) < \bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i) \quad \Rightarrow \quad \lambda_k^c = \lambda_{m_1^u(k)}^u,\ s_{k+1} = t_{m_1^u(k)}
\end{aligned}$$

where the comparisons of the intervals above are pointwise.

Proof: Let us assume that $\Lambda\left(m_1^{\max}(k)+1\right) > \bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i)$ and consider two different possibilities. First, if $\lambda_k^c > \lambda_{m_1^l(k)}^l$, then a lower bound will be met before $t_{m_1^l(k)}$. By Lemma 7.2, we know that $\lambda(t)$ can only increase if a lower bound is met with equality. This means that eventually the lower bound at $t_{m_1^l(k)}$ will be breached. On the other hand, if $\lambda_k^c < \lambda_{m_1^l(k)}^l$, then by definition of $m_1^l(k)$, we know that $\lambda_m^l \geq \lambda_{m_1^l(k)}^l$ for all $m:\ s_k < t_m < m_1^l(k)$. This means that only an upper bound can be met before or at $t_{m_1^l(k)}$. By Lemma 7.2, we know that $\lambda(t)$ can only decrease if an upper bound is met with equality. Therefore, $\lambda(t)$ will not increase to have a value inside $\Lambda\left(m_1^{\max}(k)+1\right)$ (which lies above $\bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i)$ by assumption) at $t_{m_1^{\max}(k)+1}$, i.e., the upper bound at $t_{m_1^{\max}(k)+1}$ will be breached. Thus, we must have $\lambda_k^c = \lambda_{m_1^l(k)}^l,\ s_{k+1} = t_{m_1^l(k)}$ in this case. Similar arguments follow for the other case when $\Lambda\left(m_1^{\max}(k)+1\right) < \bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i)$. ■

Similarly to what we did above, we can define the quantities $\left\{\mu_m^{eu}, \mu_m^{bu}, \mu_m^u\right\}$ and $\left\{\mu_m^{el}, \mu_m^{bl}, \mu_m^l\right\}$ as we did in (7.22)-(7.27) with fixed (known) λ_k^c. Further, we can also define the set $U(m) = [\mu_m^l, \mu_m^u]$, which gives rise to the following integers

$$m_2^{\max}(k) = \max\left\{ m : \bigcap_{i:\ t_i > s_k}^{m} U(i) \neq \emptyset \right\} \tag{7.31}$$

$$m_2^{u}(k) = \max\left\{ m : \mu_m^u \in \bigcap_{i:\ t_i > s_k}^{m} U(i) \right\} \tag{7.32}$$

$$m_2^{l}(k) = \max\left\{ m : \mu_m^l \in \bigcap_{i:\ t_i > s_k}^{m} U(i) \right\} \tag{7.33}$$

We now have the following lemma. The proof follows using similar arguments as in that of Lemma 7.4, and is therefore omitted for brevity.

Lemma 7.5 *Assume that one has the optimal solution up to time s_k, along with λ_k^c. Then, μ_k^c and s_{k+1} are found as follows*

$$\begin{aligned} \textit{If}\ U\left(m_2^{\max}(k)+1\right) &> \bigcap_{i:\ t_i > s_k}^{m_2^{\max}(k)} U(i) \quad \Rightarrow \quad \mu_k^c = \mu_{m_2^u(k)}^u,\ s_{k+1} = t_{m_2^u(k)} \\ \textit{Else, if}\ U\left(m_2^{\max}(k)+1\right) &< \bigcap_{i:\ t_i > s_k}^{m_2^{\max}(k)} U(i) \quad \Rightarrow \quad \mu_k^c = \mu_{m_2^l(k)}^l,\ s_{k+1} = t_{m_2^l(k)} \end{aligned}$$

where the comparisons of the intervals above are pointwise.

Lemmas 7.4 and 7.5 show how to optimally construct λ_k^c and μ_k^c, along with s_{k+1}, given μ_k^c and λ_k^c, respectively, along with the optimal solution up to s_k. In solving our problem, we neither know the optimal value of λ_1^c or μ_1^c in order to apply those lemmas, and hence, we need to assume some initialization values for either of them in order to start computing the remaining ones recursively. It then remains to find out if such initializations were erroneous, and how to adjust them if this were the case. In addition to that issue, we also note that Lemmas 7.4 and 7.5 only give the value of s_{k+1}. One needs either λ_{k+1}^c or μ_{k+1}^c along the way in order to reapply the results of the lemmas and move forward to find s_{k+2}. We address these issues formally through the next series of lemmas. Throughout the lemmas, we first assume a value for μ_k^c and find the corresponding values of λ_k^c and s_{k+1} by Lemma 7.4. We then assess the optimality of the assumed μ_k^c according to the constraints met at s_{k+1}. The next lemma will help in that assessment.

Lemma 7.6 *Given a time interval $[t_a, t_b]$, and a power policy $p_0(t)$, if we define another power policy $p_1(t)$ that consumes the same amount of energy during $[t_a, t_b]$, and has a slower decline, then the policy $p_1(t)$ departs more data during that interval. Similarly, if we define another power policy $p_2(t)$ that departs the same amount of data during $[t_a, t_b]$, and has a slower decline, then the policy $p_2(t)$ consumes less energy during that interval.*

Proof: Assume without loss of generality that $E_i(t_a) = B_i(t_a) = 0$, for $i = 0, 1, 2$. Since we have $E_1(t_b) = E_0(t_b)$, and that $p_1(t)$ declines slower than $p_0(t)$, therefore it must hold that $E_0(t) = \int_{t_a}^{t} p_0(\tau)d\tau \geq \int_{t_a}^{t} p_1(\tau)d\tau = E_1(t)$ $\forall t \in [t_a, t_b]$, i.e., $p_0(t)$ majorizes $p_1(t)$ in the interval $[t_a, t_b]$. By concavity of the log, it then follows that $B_1(t_b) = \int_{t_a}^{t_b} \frac{1}{2}\log(1 + p_1(t))dt > \int_{t_a}^{t_b} \frac{1}{2}\log(1 + p_0(t))dt = B_0(t_b)$ by the theory of continuous majorization [87]. This proves the first part of the lemma.

We prove the second part by contradiction. Assume $E_2(t_b) \geq E_0(t_b)$. Since $p_2(t)$ declines slower than $p_0(t)$, therefore there must exist some point $t' \in (t_a, t_b]$ at which $E_2(t') = E_0(t')$ with $E_0(t) \geq E_2(t)$ $\forall t \in [t_a, t']$. Using the first assertion of the lemma, we have

$$B_2(t') > B_0(t') \tag{7.34}$$

Since $E_2(t_b) \geq E_0(t_b)$, then we must have

$$B_2(t_b) - B_2(t') > B_0(t_b) - B_0(t') \tag{7.35}$$

From (7.34) and (7.35), we get $B_2(t_b) > B_0(t_b)$, which contradicts the assumption that both policies depart the same amount of data. Therefore we must have $E_2(t_b) < E_0(t_b)$. ■

Next, we use the results in Lemma 7.6 to prove the statements in the following lemmas.

Lemma 7.7 *If an energy constraint is binding at s_{k+1}, while data constraints are not, and if $\mu_k^c > s_{k+1}$, then we have $\mu_{k+1}^c = \mu_k^c$. Otherwise, μ_k^c is not optimal, and needs to increase. Similarly, if a data constraint is binding at s_{k+1}, while energy constraints are not, and if $s_{k+1} < t_M = \infty$, then we have $\lambda_{k+1}^c = \lambda_k^c$. Otherwise, μ_k^c is not optimal, and needs to decrease.*

Proof: By complementary slackness, we know that we must have $\mu_{k+1}^c = \mu_k^c$ since the data constraints are not binding at s_{k+1}. However, if $\mu_k^c < s_{k+1}$, then by (7.11), $p(t) = 0 \ \forall t \geq s_{k+1}$, and the transmitter will not be able to deliver the required amount of data to the receiver. Hence, μ_k^c needs to increase in order to maintain feasibility of the problem. This proves the first part of the lemma. To show the second part, we also note that by complementary slackness, we must have $\lambda_{k+1}^c = \lambda_k^c$ since the energy constraints are not binding at s_{k+1}. However, if $s_{k+1} = \infty$, i.e., we reached the end of the communication session, then we can use some extra amounts of energy to decrease the delay as follows: decrease the value of μ_k^c and decrease that of λ_k^c such that the amounts of departed bits in (s_k, ∞) stays the same. This makes the power in the interval (s_k, ∞) be of a faster decline, i.e., finish transmission faster, and in turn by Lemma 7.6 will consume more energy, which is feasible since the energy constraints are not binding. ■

Lemma 7.8 *If the battery is empty at* s_{k+1}*, and the data buffer is overflown, then* μ_k^c *is not optimal and needs to increase. Similarly, if the data buffer is empty at* s_{k+1}*, and the battery is overflown, then* μ_k^c *is not optimal and needs to decrease.*

Proof: To show the first part, let us increase the value of μ_k^c and increase that of λ_k^c such that the consumed energy in the interval $(s_k, s_{k+1}]$ stays the same. This means that the power in the interval $(s_k, s_{k+1}]$ will have a slower decline. By Lemma 7.6, this new policy departs more bits, and prevents the overflow of the data buffer. Similarly, for the second part, let us decrease the value of μ_k^c and decrease that of λ_k^c such that the data delivered in the interval $(s_k, s_{k+1}]$ stays the same. This means that the power in the interval $(s_k, s_{k+1}]$ will have a faster decline, i.e., finish transmission faster, and in turn by Lemma 7.6 will consume more energy and prevent the overflow of the battery. ■

The next two lemmas deal with the cases where both data and energy constraints are binding at s_{k+1}. In such cases, we re-solve a shifted problem starting at s_{k+1} recursively using the above analysis, with initial conditions as indicated by the binding constraints at s_{k+1}, e.g., a full/empty data/energy buffer, and denote the optimal Lagrange multipliers of this shifted problem by $\{\bar{\lambda}_i, \bar{\mu}_i\}_{i=k+1}^{M}$. We then compare the values of those Lagrange multipliers obtained from the shifted problem to λ_k^c and μ_k^c and examine their optimality as follows.

Lemma 7.9 *If the battery is empty (resp. full) and the data buffer is full (resp. empty) at s_{k+1}, and the solution of the shifted problem satisfies: $\bar{\lambda}_{k+1} \leq \lambda_k^c$ and $\bar{\mu}_{k+1} \leq \mu_k^c$ (resp. $\bar{\lambda}_{k+1} \geq \lambda_k^c$ and $\bar{\mu}_{k+1} \geq \mu_k^c$), then the solution of the shifted problem, as well as the pair (λ_k^c, μ_k^c), is optimal. Otherwise, μ_k^c is not optimal and needs to increase (resp. decrease).*

Proof: We first note that the conditions of optimality stated in the lemma are those stated in Lemma 7.2. If these are not satisfied, and the battery is empty while the data buffer is full at s_{k+1}, then we can increase the value of μ_k^c and increase that of λ_k^c such that the consumed energy in $(s_k, s_{k+1}]$ stays the same. This means that the power in the interval $(s_k, s_{k+1}]$ will have a slower decline. By Lemma 7.6, this new policy departs more bits, which is feasible since the data buffer is full at s_{k+1}, and eventually achieves less delay. The proof of the other scenario stated in the lemma where the battery is full and the data buffer is empty at s_{k+1} follows using similar arguments as in the proof of the second part of Lemma 7.8. ■

Lemma 7.10 *If both the battery and the data buffer are empty (resp. full) at s_{k+1}, and the solution of the shifted problem satisfies: $\bar{\lambda}_{k+1} \leq \lambda_k^c$ and $\bar{\mu}_{k+1} \geq \mu_k^c$ (resp. $\bar{\lambda}_{k+1} \geq \lambda_k^c$ and $\bar{\mu}_{k+1} \leq \mu_k^c$), then the solution of the shifted problem, as well as the* *pair* (λ_k^c, μ_k^c)*, is optimal. Otherwise, if* $\bar{\lambda}_{k+1} > \lambda_k^c$ *(resp.* $\bar{\mu}_{k+1} > \mu_k^c$*), then* μ_k^c *is not optimal and needs to increase. On the other hand, if* $\bar{\mu}_{k+1} < \mu_k^c$ *(resp.* $\bar{\lambda}_{k+1} < \lambda_k^c$*), then* μ_k^c *is not optimal and needs to decrease.*

Proof: We first note that the conditions of optimality stated in the lemma are those stated in Lemma 7.2. If both the battery and the data buffer are empty at s_{k+1}, and $\bar{\lambda}_{k+1} > \lambda_k^c$, then we can increase the value of μ_k^c and increase that of λ_k^c such that the amount of data delivered in $(s_k, s_{k+1}]$ stays the same. This means that the power in the interval $(s_k, s_{k+1}]$ will have a slower decline. By Lemma 7.6, this new policy consumes a smaller amount of energy, i.e., energy constraints will not be binding at s_{k+1}, and therefore we will have $\lambda_{k+1}^c = \lambda_k^c$. On the other hand if $\bar{\mu}_{k+1} < \mu_k^c$, then we decrease the value of μ_k^c and decrease that of λ_k^c such that the amount of energy consumed in $(s_k, s_{k+1}]$ stays the same. This means that the power in the interval $(s_k, s_{k+1}]$ will have a faster decline. By Lemma 7.6, this new policy delivers a smaller amount of data, i.e., data constraints will not be binding at s_{k+1}, and therefore we will have $\mu_{k+1}^c = \mu_k^c$. The proof of the other scenario stated in the lemma where both the battery and the data buffer are full follows using similar arguments. ■

It is clear from the above recursive formulas that the optimal Lagrange multipliers can only have one unique set of values. This shows that the KKT conditions have a unique solution for this problem, as mentioned in the beginning of the analysis in Section 7.2.1.

7.2.3 Constructing the Optimal Solution

In this section, we summarize the solution of the single-user problem. We first initialize by setting $s_1 = 0$, and choosing a value for μ_1^c. We then find the value of λ_1^c and s_1 by Lemma 7.4. Next, we check the constraints at s_1 and use Lemmas 7.7, 7.8, 7.9, and 7.10 to assess the optimality of the initialized μ_1^c. This results into one of the following cases: 1) the value of μ_2^c or λ_2^c is given because μ_1^c is optimal; 2) μ_1^c is not optimal and needs to increase or decrease; 3) the optimal solution of the problem is obtained according to Lemmas 7.9 and 7.10. In case 3, we need to solve a shifted problem starting at s_2; we do so by initializing a value of μ_2^c and continue as discussed above. In case 2, one can find the optimal μ_1^c by using, e.g., a bisection search. In case 1, we either use Lemma 7.4 to find λ_2^c and s_3 if μ_2^c was given, or use Lemma 7.5 to find μ_2^c and s_3 if λ_2^c was given; we then repeat the above constraints' checks at s_3, and so on. We stop when all data is transmitted under the above conditions.

7.3 Broadcast Channel

In this section, we consider an energy harvesting two-user broadcast channel, see Fig. 7.2, where at time t_m, $m = 0, 1, \ldots, M-1$, with $t_0 = 0$, energy is harvested with amount E_m. Unlike the single-user problem, the data packets in this broadcast setting are available before the communication starts, in amounts B_1 and B_2, for the first and the second user, respectively.

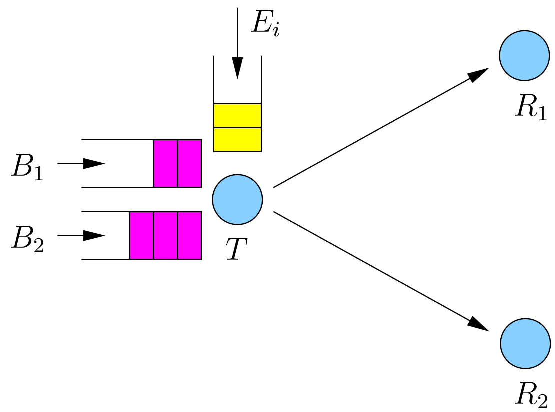


Figure 7.2: Two-user energy harvesting broadcast channel.

The physical layer is a degraded broadcast channel,

$$Y_j = X + Z_j, \quad j = 1, 2 \tag{7.36}$$

where X is the transmitted signal, Y_j is the received signal of user j, and Z_j is the Gaussian noise at receiver j with variance σ_j^2. We assume $\sigma_1^2 = 1 < \sigma_2^2 \triangleq \sigma^2$, i.e., the first user is stronger. The capacity region for this channel is [49]

$$r_1 \leq \frac{1}{2} \log \left(1 + \alpha P\right), \quad r_2 \leq \frac{1}{2} \log \left(1 + \frac{(1-\alpha) P}{\alpha P + \sigma^2}\right) \tag{7.37}$$

where α is the fraction of the total power assigned to the first (stronger) user, and log is the natural logarithm. Working on the boundary of the capacity region we have,

$$P = e^{2(r_1+r_2)} + \left(\sigma^2 - 1\right) e^{2r_2} - \sigma^2 \triangleq g\left(r_1, r_2\right) \tag{7.38}$$

which is the minimum power needed to achieve rates r_1 and r_2, at the first and the second user, respectively. Note that $g(r_1, r_2)$ is strictly convex in (r_1, r_2) [77]. We call a policy feasible if the following are satisfied:

$$\int_0^t g\left(r_1(\tau), r_2(\tau)\right) d\tau \leq E_a(t), \quad \forall t \tag{7.39}$$

$$\int_0^\infty r_1(t)dt = B_1 \tag{7.40}$$

$$\int_0^\infty r_2(t)dt = B_2 \tag{7.41}$$

where the first constraint is the energy causality constraint with $E_a(t)$ as defined in (7.1), and the remaining two are to ensure data delivery to both users.

As discussed in the single-user scenario, the average gross delay experienced by each user is given by

$$D_1 = \int_0^\infty r_1\left(t\right) t dt \tag{7.42}$$

$$D_2 = \int_0^\infty r_2\left(t\right) t dt \tag{7.43}$$

Note that, unlike the single-user scenario, in this two-user setting, there is a tradeoff between the delays experienced by the two users. This tradeoff can be characterized by developing the *delay region*, similar to *departure region* in [7], where all achievable (D_1, D_2) can be plotted. It can be shown that this region is strictly convex, and in order to achieve pareto-optimum delay points, one needs to solve *weighted sum delay minimization* problems in the form of $\min \mu_1 D_1 + \mu_2 D_2$ subject to energy causality constraints. We focus on the *sum* delay minimization problem by taking $\mu_1 = \mu_2 = 1$. Therefore, in this section, we consider the following optimization problem:

$$
\begin{aligned}
\min_{r_1, r_2} \quad & \int_0^\infty r_1(\tau)\,\tau d\tau + \int_0^\infty r_2(\tau)\,\tau d\tau \\
\text{s.t.} \quad & \int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau \leq E_a(t_m), \quad m = 1, \ldots, M \\
& \int_0^\infty r_1(\tau) d\tau = B_1 \\
& \int_0^\infty r_2(\tau) d\tau = B_2 \\
& r_1(t) \geq 0, \quad r_2(t) \geq 0, \quad \forall t
\end{aligned}
\tag{7.44}
$$

7.3.1 Minimum Sum Delay Policy

We note that (7.44) is a convex optimization problem [77]. We solve using a Lagrangian approach:

$$
\begin{aligned}
\mathcal{L} = & \int_0^\infty r_1(\tau)\,\tau d\tau + \int_0^\infty r_2(\tau)\,\tau d\tau + \sum_{m=1}^{M} \lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau - E_a(t_m) \right) \\
& - \nu_1 \left(\int_0^\infty r_1(\tau) d\tau - B_1 \right) - \nu_2 \left(\int_0^\infty r_2(\tau) d\tau - B_2 \right) \\
& - \int_0^\infty \gamma_1(\tau) r_1(\tau) d\tau - \int_0^\infty \gamma_2(\tau) r_2(\tau) d\tau
\end{aligned}
\tag{7.45}
$$

where $\{\lambda_m\}$, ν_1, ν_2, $\gamma_1(t)$, and $\gamma_2(t)$ are Lagrange multipliers. KKT optimality conditions are:

$$
t + \lambda(t) \frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} - \nu_1 - \gamma_1(t) = 0 \tag{7.46}
$$

$$
t + \lambda(t) \frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} - \nu_2 - \gamma_2(t) = 0 \tag{7.47}
$$

where we have:

$$\lambda(t) = \sum_{\{m:t_m \geq t\}} \lambda_m \tag{7.48}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} = 2e^{2(r_1(t)+r_2(t))} \tag{7.49}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} = 2e^{2(r_1(t)+r_2(t))} + 2\left(\sigma^2 - 1\right) e^{2r_2(t)} \tag{7.50}$$

along with the complementary slackness conditions:

$$\lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau - E_a(t_m) \right) = 0, \quad \forall m \tag{7.51}$$

$$\nu_1 \left(\int_0^{\infty} r_1(\tau) d\tau - B_1 \right) = 0, \quad \gamma_1(t) r_1(t) = 0 \quad \forall t \tag{7.52}$$

$$\nu_2 \left(\int_0^{\infty} r_2(\tau) d\tau - B_2 \right) = 0, \quad \gamma_2(t) r_2(t) = 0 \quad \forall t \tag{7.53}$$

From the above KKT conditions, we can write the rates and total power expressions in terms of the Lagrange multipliers. First, we write the rate expressions as:

$$r_1(t) = \frac{1}{2} \log \left(\frac{\left(\sigma^2 - 1\right)\left(\gamma_1(t) + \nu_1 - t\right)}{\gamma_2(t) - \gamma_1(t) + \nu_2 - \nu_1} \right) \tag{7.54}$$

$$r_2(t) = \frac{1}{2} \log \left(\frac{\gamma_2(t) - \gamma_1(t) + \nu_2 - \nu_1}{\lambda(t)\left(\sigma^2 - 1\right)} \right) \tag{7.55}$$

We now state the following result.

Lemma 7.11 *The optimal Lagrange multipliers* (ν_1^*, ν_2^*) *satisfy:* $\nu_1^* < \nu_2^* < \sigma^2 \nu_1^*$.

Proof: We show this by contradiction. Assume $\nu_2^* \leq \nu_1^*$. Then, by (7.55), the value of $r_2(t)$ is well-defined only if $\gamma_2(t) > 0\ \forall t$, which means by complementary slackness that $r_2(t) = 0\ \forall t$. Therefore, assuming $B_2 > 0$, the weak user will never get to receive any of its data. This proves the first inequality.

To show the second inequality, assume $\sigma^2 \nu_1^* \leq \nu_2^*$. Thus,

$$\frac{(\sigma^2 - 1)(\nu_1 - t)}{\gamma_2(t) + \nu_2 - \nu_1} \leq 1, \quad \forall t, \gamma_2(t) \geq 0 \tag{7.56}$$

Therefore, the right hand side of (7.54) can only be positive if $\gamma_1(t) > 0$, but this means, by complementary slackness, that $r_1(t) = 0$, which is a contradiction. Hence, $r_1(t) = 0\ \forall t$, and, assuming $B_1 > 0$, the strong user will never get to receive any of its data. ■

Next, we characterize the optimal total transmit power $g(r_1(t), r_2(t))$ by the following lemma.

Lemma 7.12 *In the optimal policy, the total transmit power $g(r_1(t), r_2(t))$ is given by*

$$g(r_1(t), r_2(t)) = \max\left\{\frac{\nu_2 - t}{\lambda(t)} - \sigma^2, \frac{\nu_1 - t}{\lambda(t)} - 1\right\}^+ \tag{7.57}$$

Proof: From (7.47) and (7.50), we have

$$g(r_1(t), r_2(t)) = \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \tag{7.58}$$

Since from (7.49) and (7.50) we always have

$$\frac{\partial g(r_1(t), r_2(t))}{\partial r_2(t)} - \sigma^2 \geq \frac{\partial g(r_1(t), r_2(t))}{\partial r_1(t)} - 1 \tag{7.59}$$

with equality iff $r_2(t) = 0$, from (7.46) and (7.47), we have

$$\frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \geq \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 \tag{7.60}$$

Thus, if $r_2(t) > 0$, by complementary slackness $\gamma_2(t) = 0$, and the total power is given by

$$\begin{aligned} g(r_1(t), r_2(t)) &= \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 && (7.61)\\ &> \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 && (7.62)\\ &\geq \frac{\nu_1 - t}{\lambda(t)} - 1 && (7.63) \end{aligned}$$

On the other hand, if $r_2(t) = 0$ and $r_1(t) > 0$, we have

$$\begin{aligned} g(r_1(t), r_2(t)) &= \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 && (7.64)\\ &= \frac{\nu_1 - t}{\lambda(t)} - 1 && (7.65)\\ &\geq \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 && (7.66) \end{aligned}$$

Finally, if both rates are zero, then the total power is zero. Combining this with the above gives (7.57). ■

The above lemma shows that the optimal power decreases with time between energy harvests, and can reach zero before increasing again with the next energy harvest. The following lemmas characterize the structure of the optimal policy.

Lemma 7.13 *In the optimal policy, the transmission starts by sending data to the strong user, and finishes by sending data to the weak user.*

Proof: We show this by contradiction. Assume that the transmission starts by sending data to the weak user only, i.e., $r_2(0) > r_1(0) = 0$.[2] By complementary slackness, we have $\gamma_2(0) = 0$. By Lemma 7.11, since $\sigma^2\nu_1 > \nu_2$, we have

$$\frac{(\sigma^2 - 1)(\gamma_1(0) + \nu_1)}{\nu_2 - \nu_1 - \gamma_1(0)} > 1, \quad \forall \gamma_1(0) \geq 0 \tag{7.67}$$

which implies, by (7.54), that $r_1(0) > 0$, which is a contradiction. For the second part of the lemma, assume that the transmission ends at some time t_f with $r_1(t_f) > r_2(t_f) = 0$. By Lemma 7.12, we know that this can only occur if $\lambda(t_f) > \frac{\nu_2 - \nu_1}{\sigma^2 - 1} \triangleq \lambda_{th}$. Since $\lambda(t)$ is non-increasing, we have $\lambda(t) \geq \lambda(t_f)$, $\forall t \leq t_f$. This means that $\lambda(t)$ does not fall below λ_{th} throughout the transmission, which is equivalent to saying, again by Lemma 7.12, that the weak user does not receive any of its data, which is a contradiction. ■

Lemma 7.14 *For $t < t_{th} \triangleq \frac{\sigma^2\nu_1 - \nu_2}{\sigma^2 - 1}$, if the transmitter is sending data, then it is sending to the strong user.*

[2] Extension of the contradiction arguments in this lemma to an ϵ-length interval, $\epsilon > 0$, follows directly.

Proof: We show this by contradiction. Assume that for some $t < t_{th}$ data is sent only to the weak user, i.e., we have $r_1(t) = 0$ and $r_2(t) > 0$. By complementary slackness, we have $\gamma_2(t) = 0$. Since $t < t_{th}$, it follows by simple manipulations that the numerator of the term inside the log in (7.54) is strictly larger than its denominator $\forall \gamma_1(t) \geq 0$, i.e., $r_1(t) > 0$, which is a contradiction. The only case where $r_1(t) = 0$ for some $t < t_{th}$ is when $\gamma_2(t) > 0$, which means by complementary slackness that $r_2(t) = 0$. ■

7.3.1.1 Modes of Operation

There can be four different modes of operation at a given time, depending on which user is receiving data. The first mode is when only the strong user is receiving data, i.e., $r_1(t) > 0$ and $r_2(t) = 0$. By Lemma 7.12, this can be the case only if $\lambda(t) \geq \lambda_{th} = \frac{\nu_2 - \nu_1}{\sigma^2 - 1}$. In this mode, we have the total power and the strong user's rate given by

$$g(r_1(t), 0) = \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{7.68}$$

$$r_1(t) = \frac{1}{2} \log \left(\frac{\nu_1 - t}{\lambda(t)} \right) \tag{7.69}$$

The second mode of operation is when both users are receiving data, i.e., $r_1(t) > 0$ and $r_2(t) > 0$. Again by Lemma 7.12, this can be the case only if $\lambda(t) < \lambda_{th}$. Moreover, by (7.54), we also need $t < t_{th} = \frac{\sigma^2 \nu_1 - \nu_2}{\sigma^2 - 1}$. In this mode, the total power and the users' rates are given by

$$g(r_1(t), r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{7.70}$$

$$r_1(t) = \frac{1}{2}\log\left(\frac{(\sigma^2 - 1)(\nu_1 - t)}{\nu_2 - \nu_1}\right) \tag{7.71}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2 - \nu_1}{\lambda(t)(\sigma^2 - 1)}\right) \tag{7.72}$$

The third mode of operation is when only the weak user is receiving data, i.e., $r_1(t) = 0$ and $r_2(t) > 0$. For this to occur we need both $\lambda(t) < \lambda_{th}$ and $t \geq t_{th}$. The total power and the weak user's rate are then given by

$$g(0, r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{7.73}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2 - t}{\lambda(t)\sigma^2}\right) \tag{7.74}$$

The fourth mode is when both rates (and the power) are zero. We denote this mode as a communication *gap*. These gaps may occur, for instance, if there is a small amount of energy in the battery that is insufficient to deliver all the data, and a large amount of energy arrives later. The transmitter may then finish up this small amount of energy to send some bits out and wait for additional energy to send the remaining bits.

7.3.1.2 Finding the value of $\lambda(t)$

We next characterize the rates and powers. The following lemma shows that $\lambda(t)$ is a piecewise constant function.

Lemma 7.15 *In the optimal policy, the Lagrange multiplier function $\lambda(t)$ is piecewise constant, with possible changes only when energy is depleted.*

Proof: By the complementary slackness conditions on $\lambda(t)$,

$$\lambda_m^* = 0, \quad \text{if } E^*(t_m) < E_a(t_m) \tag{7.75}$$

$$E^*(t_m) = E_a(t_m), \quad \text{if } \lambda_m^* > 0 \tag{7.76}$$

Therefore, $\lambda(t)$ remains constant between energy harvests, and can only decrease when $\lambda_m > 0$ for some m, which happens only when energy is depleted. ■

By Lemma 7.15, $\lambda(t)$ is a sequence rather than a continuous function of time. We denote the times of change of $\lambda(t)$ by $\{s_1, s_2, \ldots, s_L\}$ with $s_1 = 0$, and the values of $\lambda(t)$ between such times by

$$\lambda(t) = \begin{cases} \lambda_k^c, & t \in [s_k, s_{k+1}) \\ \lambda_L^c, & t \in [s_L, \infty) \end{cases} \tag{7.77}$$

Next, we characterize the optimal $\{\lambda_k^c\}$ sequentially. Determining the value of λ_k^c requires the knowledge of ν_1^* and ν_2^*, and also which mode of operation is active during the interval $[s_k, s_{k+1})$. Let us define $B_j(t)$ as the total amount of bits transmitted to user j by time t. The next lemma shows how to compute λ_k^c given the mode of operation. The proof uses similar steps as in the proof of Lemma 7.4 in the single-user setting and is omitted for brevity.

Lemma 7.16 *Given a mode of operation, with the optimal ν_1^*, ν_2^*, λ_l^c, s_l, $\forall l < k$, define the following quantities $\forall m$: $t_m > s_k$*

$$\bar{\lambda}_m : \quad E^*(s_k) + \int_{s_k}^{t_m} g(r_1(\tau), r_2(\tau))^+ d\tau = E_a(t_m) \tag{7.78}$$

$$\tilde{\lambda}_1 : \quad B_1^*(s_k) + \int_{s_k}^{\infty} r_1(\tau)^+ d\tau = B_1 \tag{7.79}$$

$$\tilde{\lambda}_2 : \quad B_2^*(s_k) + \int_{s_k}^{\infty} r_2(\tau)^+ d\tau = B_2 \tag{7.80}$$

where r_1, r_2, and $g(r_1, r_2)$ are defined by the mode of operation in Section 7.3.1.1, with the convention that $\tilde{\lambda}_j = 0$ whenever a mode of operation has $r_j = 0$, $j = 1, 2$. Then, the optimal λ_k^c for this mode of operation is given by

$$\lambda_k^c = \max\{\bar{\lambda}_m, \tilde{\lambda}_1, \tilde{\lambda}_2\}, \quad \forall m : t_m > s_k \tag{7.81}$$

The results in Lemma 7.16 imply that one has to know the mode of operation before computing the optimal values of the Lagrange multipliers. Note that communication gaps occur naturally due to the $(\cdot)^+$ operation in these expressions. In the next section, we develop an iterative solution that computes $\{\lambda_k^c\}$ based on an initial assignment of the mode of operation and the values of ν_1, ν_2. The solution is based on the necessary conditions stated in the previous lemmas. By Lemma 7.11, we know that the optimal values of ν_1, ν_2 lie in a cone in $\mathbb{R}^2_{++}$. We also know, by Lemmas 7.12 and 7.13, that the communication stops if $t > \nu_2$. Therefore, we find an upper bound on the value of ν_2^* as follows. First, we move all of the energy to t_{M-1}, the arrival time of the last energy packet, and start the communication from there. Second, we solve this single energy arrival problem and find its optimal ν_2^* which we denote by ν_2^{single}. Therefore, an upper bound on ν_2^* of the multiple energy arrival problem is

$$\nu_2^* \leq \nu_2^{\text{single}} + t_{M-1} \triangleq \nu^{\text{ub}} \tag{7.82}$$

Once this upper bound is found, one can perform a two-dimensional grid search over the feasible region of ν_1, ν_2:

$$\mathcal{R}_{\nu_1\nu_2} = \left\{\nu_1, \nu_2 : 0 < \nu_1 < \nu_2 < \sigma^2\nu_1, \nu_2 \leq \nu^{\text{ub}}\right\} \tag{7.83}$$

Next, we analyze the single energy arrival case to characterize the upper bound on ν_2^*.

7.3.1.3 Single Energy Arrival

For the single energy arrival case, we first note that there can be no communication gaps, as this can only increase the delay. We also note that since there is only one value of λ, corresponding to only one energy arrival constraint, the optimal power is given by the first term in (7.57). If not, then the weak user will never receive its data. Hence, the first mode of operation where only the strong user is receiving data never occurs. Thus, the optimal total power is given by

$$p_s(t) = \frac{\nu_2 - t}{\lambda} - \sigma^2, \quad \forall t \leq t_f \triangleq \nu_2 - \lambda\sigma^2 \tag{7.84}$$

where the subscript s denotes single arrival, and t_f is such that $p_s(t)$ is non-negative. From the above, we also note that λ cannot be 0, or else the power is infinitely large. Since $\lambda > 0$, by complementary slackness, the transmitter has to consume all of its energy by the end of transmission. This simplifies the single energy arrival problem, as in this case, we have all the three constraints, both users' data and transmitter's energy, met with equality. Therefore, we can solve for the optimal values of the Lagrange multipliers satisfying the following:

$$\int_0^{t_{th}} \frac{1}{2} \log\left(\frac{(\sigma^2 - 1)(\nu_1 - t)}{\nu_2 - \nu_1}\right) dt = B_1 \tag{7.85}$$

$$\frac{t_{th}}{2} \log\left(\frac{\nu_2 - \nu_1}{\lambda(\sigma^2 - 1)}\right) + \int_{t_{th}}^{t_f} \frac{1}{2} \log\left(\frac{\nu_2 - t}{\lambda\sigma^2}\right) dt = B_2 \tag{7.86}$$

$$\int_0^{t_f} p_s(t) dt = E \tag{7.87}$$

The above three equations are direct consequences of the modes of operation analysis in Section 7.3.1.1. These can be further simplified into:

$$\frac{\nu_1}{2} \log\left(\frac{(\sigma^2 - 1)\nu_1}{\nu_2 - \nu_1}\right) = B_1 \tag{7.88}$$

$$\frac{\nu_2}{2} \log\left(\frac{\nu_2 - \nu_1}{\lambda(\sigma^2 - 1)}\right) = B_2 \tag{7.89}$$

$$\frac{(\nu_2 - \lambda\sigma^2)^2}{2\lambda} = E \tag{7.90}$$

Note that (7.88)-(7.90) have three equations in three unknowns, and can be solved numerically for the values of λ^*, ν_1^*, and ν_2^*. Note from the above analysis that, since we always start with the second mode of operation, where both users receive data, in this setting, we have $\lambda < \lambda_{th}$. This implies that $t_f > t_{th}$, and enables the following stronger version of Lemma 7.13.

Lemma 7.17 *In the optimal policy solving (7.44), transmission always ends by sending data only to the weak user.*

Proof: In the single energy arrival case, since $t_f > t_{th}$, we always end transmission by sending data only to the weak user. In the multiple arrival case, the last energy arrival can be viewed as a single energy arrival problem with the remaining data in the data buffers as modified constraints. Then the single energy arrival result applies, yielding the stated result. ■

We have now characterized how to get the upper bound ν^{ub} in (7.82). In the next section we present an iterative method to find the optimal Lagrange multipliers solving problem (7.44).

7.3.2 Iterative Solution

The analysis presented in Lemma 7.16 describes an optimal method of finding $\{\lambda_k^c\}$ given ν_1^* and ν_2^*. To find the latter two, we perform a grid search over the region $\mathcal{R}_{\nu_1\nu_2}$, which is fully characterized by the single arrival analysis. We perform the search as follows. We fix $(\nu_1, \nu_2) \in \mathcal{R}_{\nu_1\nu_2}$, and solve for $\{\lambda_k^c\}$ to acquire a transmission policy accordingly. We denote by Mode 1, Mode 2, and Mode 3, the mode of operation where data is sent only to the strong user, both users, and only to the weak user, respectively. Since Mode 1 can only occur at the beginning, we assume that the transmission starts according to that mode, and compute the corresponding λs by Lemma 7.16. If these λs are all less than λ_{th}, then they are correct. We move to Mode 2 once we get a value of λ larger than λ_{th}. We stay at Mode 2 until the time passes t_{th}, then move to Mode 3 till the end of communication. By Lemma 7.17, we know that Mode 3 always exists. The transmission then ends whenever the weak user's data or the transmission energy is finished.

After we find the transmission policy, we check whether the data buffers of both users are empty. If this is the case, then by the convexity of the problem, this policy is optimal as we have thus found a feasible policy satisfying the KKT conditions [77]. Note that we might end up with a policy that either does not finish up all the users' data, or even transmits more than the available. If either is the case, we re-solve using another (ν_1, ν_2) point. We summarize how to find the optimal (ν_1, ν_2) iteratively as follows. We initialize by setting $\nu_1 = \epsilon$ and $\nu_2 = \nu_1 + \epsilon$ for some $\epsilon > 0$ small enough. We then solve for $\{\lambda_k^c\}$ as described above. If we do not reach a feasible KKT point, we increase ν_2 by another ϵ and repeat. We keep doing this until we reach a feasible KKT point, or ν_2 becomes larger than $\min\{\sigma^2\nu_1, \nu^{\text{ub}}\}$. In the latter case, we increase ν_1 by ϵ and repeat the whole procedure again. Since the region $\mathcal{R}_{\nu_1\nu_2}$ is bounded, iterations are guaranteed to find the optimal solution.

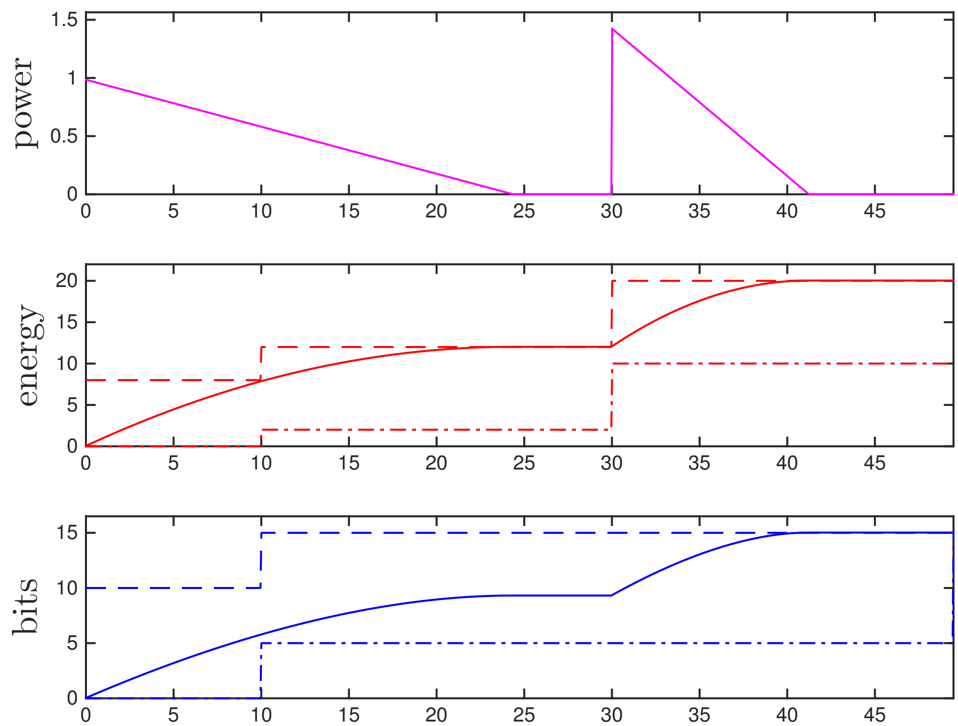


Figure 7.3: Optimal solution for a single-user system with 3 energy arrivals and 2 data arrivals.

7.4 Numerical Results

In this section, we present some numerical examples to further illustrate the results in this chapter. We begin by considering a single use channel with $E_{\max} = B_{\max} = 10$ units. Energy arrives with amounts of $[8, 12, 20]$ at times $t = [0, 10, 30]$, while data arrives with amounts of $[10, 15]$ at times $t = [0, 10]$. In Fig. 7.3 we show the delay minimal solution in this setting. We see that the power is monotonically decreasing between arrival times, and actually drops to 0 before the last energy arrival. The optimal energy and data profiles are also shown in the figure. The upper and lower dotted lines represent the upper and lower constraints, respectively, as dictated by

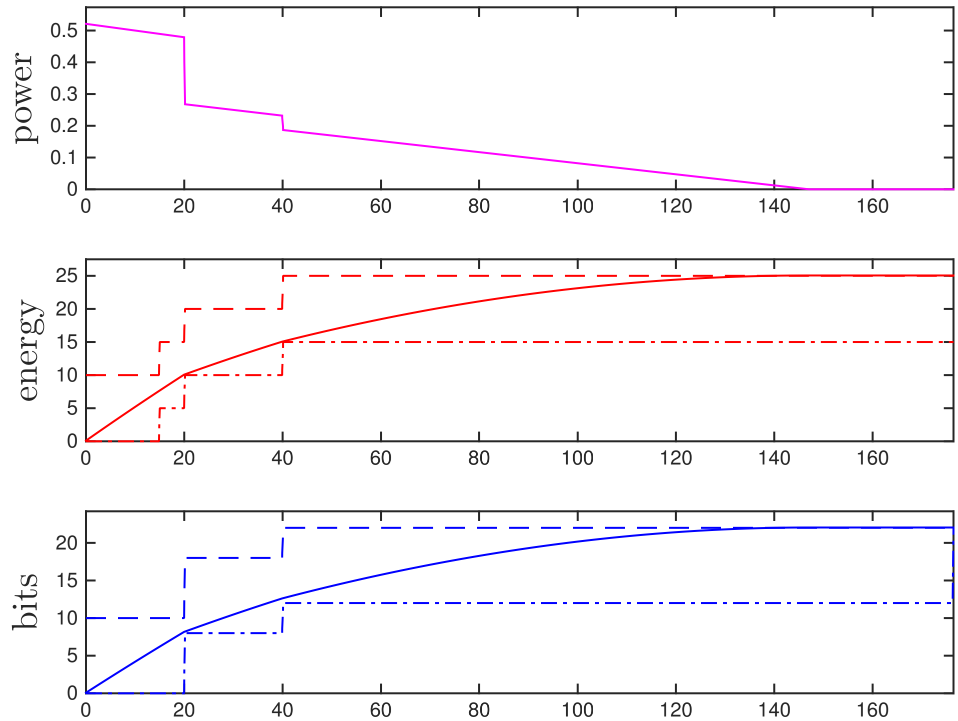


Figure 7.4: Effects of having a finite-sized battery and data buffer in a single-user system.

the arrival profile and the value of the finite-sized battery or data buffer.

In Fig. 7.3, we see that the size of the buffers is not a bottleneck to the system. We therefore consider another example where energy arrives with amounts of $[10, 15, 20, 25]$ at times $t = [0, 15, 20, 40]$, while data arrives with amounts of $[10, 18, 22]$ at times $t = [0, 20, 40]$, and plot the optimal solution in Fig. 7.4. We see that the power in this case does not drop down to 0 until at the end of communication, and that its slope changes when the optimal energy or data profiles hit the lower bounds indicated by the size of the buffers.

Next, we present a numerical example to illustrate the results of the broadcast setting. We consider a system where energy arrives with values $[6, 10, 4, 5]$ at times

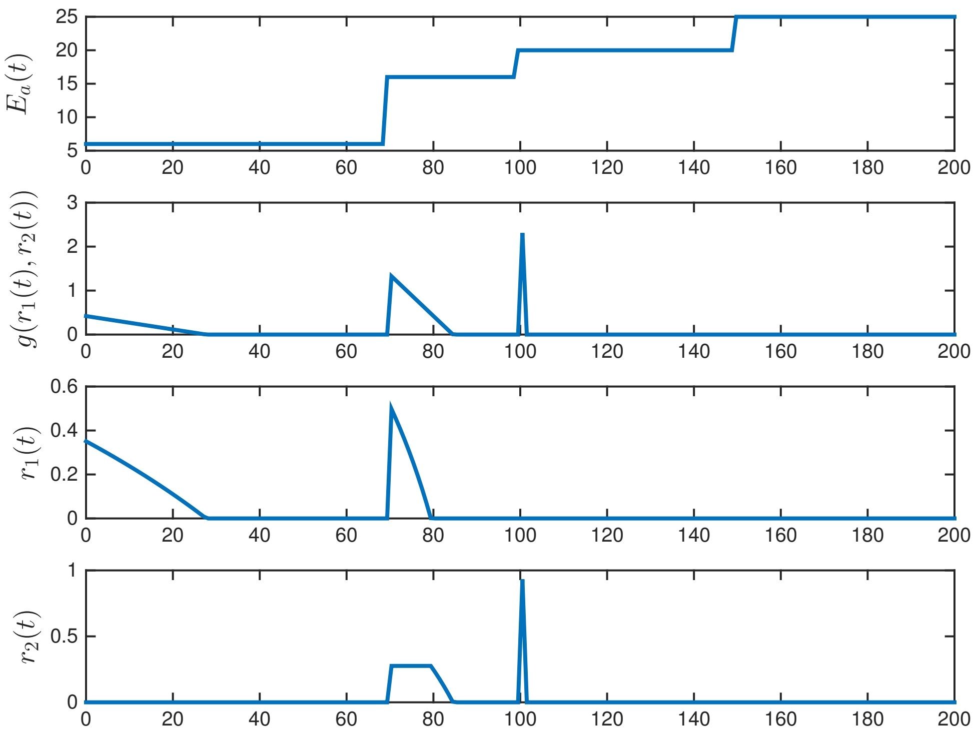


Figure 7.5: Optimal power and rates for a system with four energy arrivals.

$t = [0, 70, 100, 150]$, with amounts of data $B_1 = 8$ and $B_2 = 4.25$ intended for the strong and the weak user, respectively. We first find the upper bound on ν_2^* by solving the single energy arrival case by setting $E = 25$ in (7.90) and finding the value of ν_2^{single}. Adding $t_{M-1} = 150$, we get $\nu^{\text{ub}} \simeq 170$. We then apply the iterative solution described in Section 7.3.2 to find the optimal total power allocation for the multiple arrival case and the corresponding users' rates. These are shown in Fig. 7.5 as a function of time. We see that all four modes of operation are present in this example: the transmitter begins by sending data only to the strong user (Mode 1) until it consumes the initial energy arrival, and stays silent until the next energy arrival, then it sends data to both users simultaneously (Mode 2) until all strong

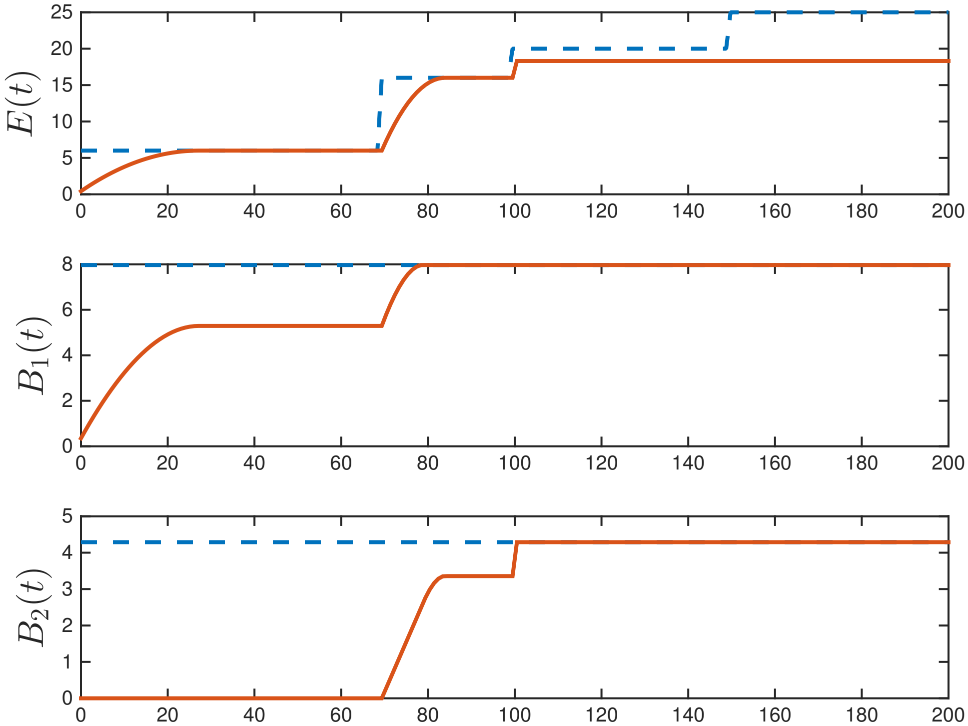


Figure 7.6: Optimal energy and data consumption.

user's data is finished, which occurs at $t_{th} \simeq 79.4$. Then, it starts sending data only to the weak user (Mode 3), before keeping silent until the third energy arrival, and then finishes up the weak user's data. Note that the fourth energy arrival is not used in this example. In Fig. 7.6, we show the corresponding optimal total energy and data consumption for this policy as a function of time.

Finally, we compare this to the transmission completion time minimization problem in [7] with the same data values and energy arrival profile. The optimal transmission completion time is equal to $T^* = 90$. Calculating the delay achieved by such policy gives $D \simeq 717.2$. On the other hand, our delay minimizing policy achieves a smaller delay of $D^* \simeq 593.3$, however, it takes a larger amount of time to finish $T \simeq 101.5$. This shows that there exists a tradeoff between delay minimization and transmission completion time minimization, and that the two problems are different, even when all data is available before the start of communication. That is, finishing data delivery by a minimum time, and having data experience minimum overall delay yield different optimum policies.

7.5 Conclusion

In this chapter, we considered delay minimization in energy harvesting communication channels. First, we studied the single-user channel where the transmitter has a finite-sized battery and data buffer, and energy and data packets become available at the transmitter during the course of communication. We determined the optimum power control policy in terms of the Lagrange multiplier functions. We identified the properties of these functions and gave a method that evaluates them recursively. We proposed a solution which iteratively updates the initial value of a Lagrange multiplier, and obtains the optimum power allocation policy. The optimal power values start high, decrease linearly, potentially reaching zero between energy harvests and data arrivals. This policy is different from the piecewise constant power policies of the existing literature which focus on minimizing a deadline by which all packets are transmitted or maximizing the throughput before a fixed deadline. Initial high powers in our case make sure that the delay does not accumulate by transmitting data at faster rates first, then decreasing the rate gradually.

Next, we considered a two-user energy harvesting broadcast channel and characterized the minimal sum delay policy subject to energy harvesting constraints, when the transmitter has an infinite-sized battery, and all data intended for both users is available before transmission. We showed that the optimal power is decreasing between energy harvests, and that there can be times when data is sent only to the strong user, both users, or only to the weak user. We also showed that there can be communication gaps where the transmitter is silent between energy arrivals. We presented a method to find the optimal policy iteratively.

CHAPTER 8

Age-Minimal Transmission in Energy Harvesting Two-hop Networks

8.1 Introduction

In this chapter, we consider an energy harvesting two-hop network where a source is communicating to a destination through a relay. During a given communication session time, the source collects measurement updates from a physical phenomenon and sends them to the relay, which then forwards them to the destination, see Fig. 8.1. The objective is to send these updates to the destination as *timely* as possible; namely, such that the total *age of information* is minimized by the end of the communication session, subject to energy causality constraints at the source and the relay, and data causality constraints at the relay. Both the source and the relay use fixed, yet possibly different, transmission rates. Hence, each update packet incurs fixed non-zero transmission delays. We first solve the single-hop version of this problem, and then show that the two-hop problem is solved by treating the source and relay nodes as one combined node, with some parameter transformations, and solving a single-hop problem between that combined node and the destination.

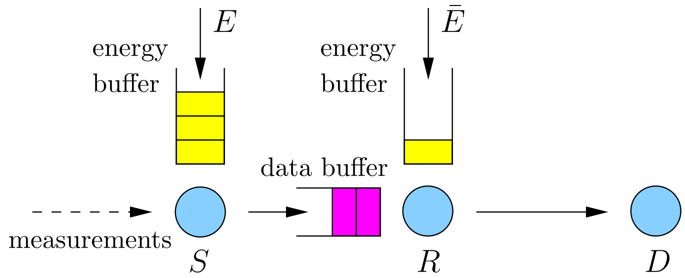


Figure 8.1: Energy harvesting two-hop network. The source collects measurements and sends them to the destination through the relay.

8.2 System Model and Problem Formulation

A source node acquires measurement updates from some physical phenomenon and sends them to a destination, through the help of a half-duplex relay, during a communication session of duration T time units. Updates need to be sent as *timely* as possible; namely, such that the total *age of information* is minimized by time T. The age of information metric is defined as

$$a(t) \triangleq t - U(t), \quad \forall t \tag{8.1}$$

where $U(t)$ is the time stamp of the latest received update packet at the destination, i.e., the time at which it was acquired at the source. Without loss of generality, we assume $a(0) = 0$. The objective is to minimize the following quantity

$$A_T \triangleq \int_0^T a(t)dt \tag{8.2}$$

Both the source and the relay depend on energy harvested from nature to transmit their data, and are equipped with infinite-sized batteries to save their incoming energy. Energy arrives in packets of amounts E and $\bar{E}$ at the source and the relay, respectively. Update packets are of equal length, and are transmitted using *fixed* rates at the source and the relay. We assume that one update transmission consumes one energy packet at a given node, and hence the number of updates is equal to the minimum of the number of energy arrivals at the source and the relay. Under a fixed rate policy, each update takes d and $\bar{d}$ amount of time to get through the source-relay channel and the relay-destination channel, respectively[1].

Source energy packets arrive at times $\{s_1, s_2, \ldots, s_N\} \triangleq \boldsymbol{s}$, and relay energy packets arrive at times $\{\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_N\} \triangleq \bar{\boldsymbol{s}}$, where without loss of generality we assume that both the source and the relay receive N energy packets, since each update consumes one energy packet in transmission from either node, and hence any extra energy arrivals at either the source or the relay cannot be used. Let t_i and $\bar{t}_i$ denote the transmission time of the ith update at the source and the relay, respectively. We first impose the following constraints

$$t_i \geq s_i, \; \bar{t}_i \geq \bar{s}_i, \qquad 1 \leq i \leq N \tag{8.3}$$

representing the *energy causality constraints* [1] at the source and the relay, which mean that no energy packet can be used before being harvested. Next, we must

[1] d can be considered, for instance, equal to B/r where B is the update packet length in bits and $r = g(E)$ is the transmission rate in bits/time units, where g is some increasing function representing the rate-energy relationship.

have

$$t_i + d \leq \bar{t}_i, \quad 1 \leq i \leq N \tag{8.4}$$

to ensure that the relay does not forward an update before receiving it from the source, which represents the *data causality constraints* [1]. We also have the *service time constraints*

$$t_i + d \leq t_{i+1}, \quad \bar{t}_i + \bar{d} \leq \bar{t}_{i+1}, \qquad 1 \leq i \leq N-1 \tag{8.5}$$

which ensure that there can only be one transmission at a time at the source and the relay. Hence, d and $\bar{d}$ represent the service (busy) time of the source and relay servers, respectively.

Transmission times at the source and the relay should also be related according to the half-duplex nature of the relay operation. For that, we must have the *half-duplex constraints*

$$(t_i, t_i + d) \cap (\bar{t}_j, \bar{t}_j + \bar{d}) = \emptyset, \quad \forall i, j \tag{8.6}$$

where $\emptyset$ denotes the empty set, since the relay cannot receive and transmit simultaneously. These constraints enforce that either the source transmits a new update after the relay finishes forwarding the prior one, i.e., $t_{i+1} \geq \bar{t}_i + \bar{d}$ for some i; or that the source delivers a new update before the relay starts transmitting the prior one, i.e., $t_{i+k} + d \leq \bar{t}_i$ for some i and k. The latter case means that there are $k+1$

update packets waiting in the relay's data buffer just before time $\bar{t}_i$. We prove that this case is not age-optimal. To see this, consider the example of having $k+1=2$ updates packets in the relay's data buffer waiting for service. The relay in this case has two choices at its upcoming transmission time: 1) forward the first update followed by the second one sometime later, or 2) forward the second update only and ignore the first one. These two choices yield different age evolution curves. We observe, geometrically, that A_T under choice 2 is strictly less than that under choice 1. Since the source under choice 2 consumes an extra energy packet to send the first update unnecessarily, it should instead save this energy packet to send a new update after the first one is forwarded by the relay. Therefore, it is optimal to replace the half-duplex constraints in (8.6) by the following reduced ones

$$\bar{t}_i + \bar{d} \leq t_{i+1}, \quad 1 \leq i \leq N-1 \tag{8.7}$$

Next, observe that (8.5) can be removed from the constraints since it is implied by (8.4) and (8.7). In conclusion, the constraints are now those in (8.3), (8.4), and (8.7).

Finally, we add the following constraint to ensure reception of all updates by time T

$$\bar{t}_N + \bar{d} \leq T \tag{8.8}$$

In Fig. 8.2, we present an example of the age of information in a system with

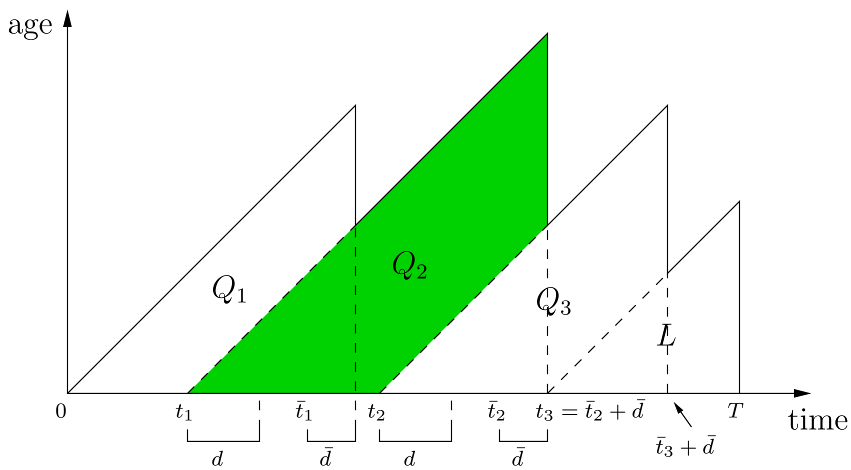


Figure 8.2: Age evolution in a two-hop network with three updates.

3 updates. The area under curve representing A_T is given by the sum of the areas of the trapezoids Q_1, Q_2, and Q_3, in addition to the area of the triangle L. The area of Q_2 for instance is given by $\frac{1}{2}\left(\bar{t}_2+\bar{d}-t_1\right)^2-\frac{1}{2}\left(\bar{t}_2+\bar{d}-t_2\right)^2$. The objective is to choose feasible transmission times for the source and the relay such that A_T is minimized. Computing the area under the age curve for general N arrivals, we formulate the problem as follows

$$\begin{aligned}
\min_{\boldsymbol{t},\bar{\boldsymbol{t}}} \quad & \sum_{i=1}^{N} \left(\bar{t}_i+\bar{d}-t_{i-1}\right)^2-\left(\bar{t}_i+\bar{d}-t_i\right)^2+\left(T-t_N\right)^2 \\
\text{s.t.} \quad & t_i \geq s_i,\ \bar{t}_i \geq \bar{s}_i, \quad 1 \leq i \leq N \\
& t_i + d \leq \bar{t}_i, \quad 1 \leq i \leq N \\
& \bar{t}_i + \bar{d} \leq t_{i+1}, \quad 1 \leq i \leq N
\end{aligned} \tag{8.9}$$

with $t_0 \triangleq 0$ and $t_{N+1} \triangleq T$.

We note that the energy arrival times $\boldsymbol{s}$ and $\bar{\boldsymbol{s}}$, the transmission delays d and $\bar{d}$, the session time T, and the number of energy arrivals N, are such that problem

(8.9) has a feasible solution. This is true only if

$$T \geq \bar{s}_i + (N - i + 1)\,\bar{d}, \quad \forall i \tag{8.10}$$

$$T \geq s_i + (N - i + 1)\left(d + \bar{d}\right), \quad \forall i \tag{8.11}$$

where (8.10) (resp. (8.11)) ensures that the ith energy arrival time at the relay (resp. source) is small enough to allow the reception of the upcoming $N - i$ updates within time T.

8.3 Solution Building Block: The Single-User Channel

In this section, we solve the single-user version of problem (8.9); namely, when the source is communicating directly with the destination. We use the solution to the single-user problem in this section as a building block to solve problem (8.9) in the next section. In Fig. 8.3, we show an example of the age evolution in a single-user setting. The area of Q_2 is now given by $\frac{1}{2}\left(t_2 + d - t_1\right)^2 - \frac{1}{2}d^2$. We compute the area under the age curve for general N arrivals and formulate the single-user problem as follows

$$\begin{aligned} \min_{\boldsymbol{t}} \quad & \sum_{i=1}^{N} \left(t_i + d - t_{i-1}\right)^2 + \left(T - t_N\right)^2 \\ \text{s.t.} \quad & t_i \geq s_i, \quad 1 \leq i \leq N \\ & t_i + d \leq t_{i+1}, \quad 1 \leq i \leq N \end{aligned} \tag{8.12}$$

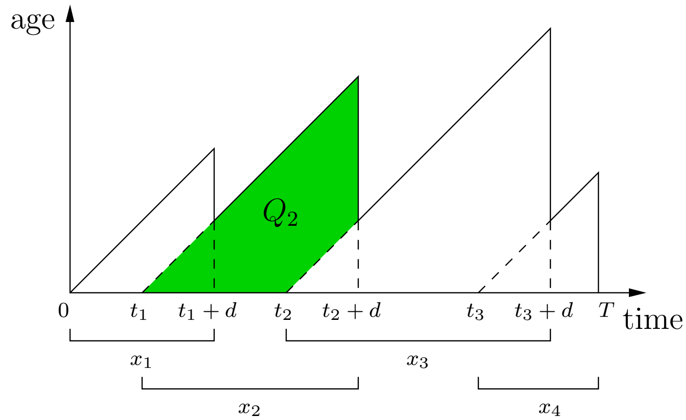


Figure 8.3: Age evolution using in a single-user channel with three updates.

where the second constraints are the service time constraints.

We note that reference [76] considered problem (8.12) when the transmission delay $d = 0$. We extend their results for a positive delay (and hence a finite transmission rate) in this section. We first introduce the following change of variables: $x_1 \triangleq t_1 + d$; $x_i \triangleq t_i - t_{i-1} + d$, $2 \leq i \leq N$; and $x_{N+1} \triangleq T - t_N$. These variables must satisfy $\sum_{i=1}^{N+1} x_i = T + Nd$, which reflects the dependent relationship between the new variables $\{x_i\}$. This can also be seen from Fig. 8.3. Substituting by $\{x_i\}$ in problem (8.12), we get the following equivalent problem

$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & \sum_{i=1}^{N+1} x_i^2 \\
\text{s.t.} \quad & \sum_{i=1}^{k} x_i \geq s_k + kd, \quad 1 \leq k \leq N \\
& x_i \geq 2d, \quad 2 \leq i \leq N \\
& x_{N+1} \geq d \\
& \sum_{i=1}^{N+1} x_i = T + Nd
\end{aligned}
\tag{8.13}
$$

Observe that problem (8.13) is a convex problem that can be solved by standard techniques [77]. For instance, we introduce the following Lagrangian

$$
\begin{aligned}
\mathcal{L} = \sum_{i=1}^{N+1} x_i^2 - \sum_{k=1}^{N} \lambda_k \left(\sum_{i=1}^{k} x_i - s_k - kd \right) - \sum_{i=2}^{N} \eta_i \left(x_i - 2d \right) \\
- \eta_{N+1} \left(x_{N+1} - d \right) + \nu \left(\sum_{i=1}^{N+1} x_i - T - Nd \right) \qquad (8.14)
\end{aligned}
$$

where $\{\lambda_1, \ldots, \lambda_N, \eta_2 \ldots, \eta_{N+1}, \nu\}$ are Lagrange multipliers, with $\lambda_i, \eta_i \geq 0$ and $\nu \in \mathbb{R}$. Differentiating with respect to x_i and equating to 0 we get the following KKT conditions

$$x_1 = \sum_{k=1}^{N} \lambda_k - \nu \qquad (8.15)$$

$$x_i = \sum_{k=i}^{N} \lambda_k + \eta_i - \nu, \quad 2 \leq i \leq N \qquad (8.16)$$

$$x_{N+1} = \eta_{N+1} - \nu \qquad (8.17)$$

along with complementary slackness conditions

$$\lambda_k \left(\sum_{i=1}^{k} x_i - s_k - kd \right) = 0, \quad 1 \leq k \leq N \qquad (8.18)$$

$$\eta_i \left(x_i - 2d \right) = 0, \quad 1 \leq i \leq N \qquad (8.19)$$

$$\eta_{N+1}(x_{N+1} - d) = 0 \qquad (8.20)$$

We now have the following lemmas characterizing the optimal solution of problem (8.13): $\{x_i^*\}$. Lemmas 8.1 and 8.3 show that the sequence $\{x_i^*\}_{i=2}^{N+1}$ is non-increasing, and derive necessary conditions for it to strictly decrease. On the other hand, Lemma 8.2 shows that x_1^* can be smaller or larger than x_2^*, and derives necessary conditions for the two cases.

Lemma 8.1 *For* $2 \leq i \leq N-1$, $x_i^* \geq x_{i+1}^*$. *Furthermore,* $x_i^* > x_{i+1}^*$ *only if* $\sum_{j=1}^{i} x_j^* = s_i + id$.

Proof: We show this by contradiction. Assume that for some $i \in \{2, \ldots, N-1\}$ we have $x_i^* < x_{i+1}^*$. By (8.16), this is equivalent to having $\lambda_i + \eta_i < \eta_{i+1}$, i.e., $\eta_{i+1} > 0$, which implies by complementary slackness in (8.19) that $x_{i+1}^* = 2d$. This means that $x_i^* < 2d$, i.e., infeasible. Therefore $x_i^* \geq x_{i+1}^*$ holds. This proves the first part of the lemma.

To show the second part, observe that since $x_i^* > x_{i+1}^*$ if and only if $\lambda_i + \eta_i > \eta_{i+1}$, then either $\lambda_i > 0$ or $\eta_i > 0$. If $\eta_i > 0$, then by (8.19) we must have $x_i^* = 2d$, which renders $x_{i+1}^* < 2d$, i.e., infeasible. Therefore, η_i cannot be positive and we must have $\lambda_i > 0$. By complementary slackness in (8.18), this implies that $\sum_{j=1}^{i} x_j^* = s_i + id$. ■

Lemma 8.2 $x_1^* > x_2^*$ *only if* $x_1^* = s_1 + d$*; while* $x_1^* < x_2^*$ *only if* $x_i^* = 2d$*, for* $2 \leq i \leq N$.

Proof: The necessary condition for x_1^* to be larger than x_2^* can be shown using the same arguments as in the proof of the second part of Lemma 8.1, and is omitted for brevity. Let us now assume that x_1^* is smaller than x_2^*. By (8.15) and (8.16), this occurs if and only if $\eta_2 > \lambda_1$, which implies that $x_2^* = 2d$ by complementary slackness in (8.19). Finally, by Lemma 8.1, we know that $\{x_i^*\}_{i=2}^N$ is non-increasing; since they are all bounded below by $2d$, and $x_2^* = 2d$, then they must all be equal to $2d$. ■

Lemma 8.3 *$x_N^* \geq x_{N+1}^*$. Furthermore, $x_N^* > x_{N+1}^*$ only if at least: 1) $\sum_{i=1}^N x_i^* = s_N + Nd$, or 2) $x_N^* = 2d$ occurs.*

The proof of Lemma 8.3 is along the same lines of the proofs of the previous two lemmas and is omitted for brevity.

We will use the results of Lemmas 8.1, 8.2, and 8.3 to derive the optimal solution of problem (8.13). To do so, one has to consider the relationship between the parameters of the problem: T, d, and N. For instance, one expects that if the session time T is much larger than the minimum inter-update time d, then the energy causality constraints will be binding while the constraints enforcing one update at a time will not be, and vice versa. We formalize this idea by considering two different cases as follows.

8.3.1 $Nd \leq T < (N+1)d$

We first note that Nd is the least value that T can have for problem (8.13) to admit a feasible solution. In this case, the following theorem shows that the optimal solution is achieved by sending all updates back to back with the minimal inter-update time possible to allow the reception of all of them by the end of the relatively small session time T.

Theorem 8.1 *Let $Nd \leq T < (N+1)d$. Then, the optimal solution of problem (8.13) is given by*

$$x_1^* = \max\left\{\frac{T-(N-2)d}{2}, s_1 + d\right\} \tag{8.21}$$

$$x_i^* = 2d, \quad 2 \leq i \leq N \tag{8.22}$$

$$x_{N+1}^* = T - (N-2)d - x_1^* \tag{8.23}$$

Proof: We first argue that if $x_1^* \geq x_2^*$, then $\sum_{i=1}^{N+1} x_i^* \geq (2N+1)d$. The last constraint in problem (8.13) then implies that $T \geq (N+1)d$, which is infeasible in this case. Therefore, we must have $x_1^* < x_2^*$. By Lemma 8.2, this occurs only if $x_i^* = 2d$ for $2 \leq i \leq N$. Hence, we set $x_{N+1} = T - (N-2)d - x_1$, and observe that problem (8.13) in this case reduces to a problem in only one variable x_1 as follows

$$\begin{aligned} \min_{x_1} \quad & x_1^2 + (T-(N-2)d - x_1)^2 \\ \text{s.t.} \quad & s_1 + d \leq x_1 \leq T-(N-1)d \end{aligned} \tag{8.24}$$

whose solution is given by projecting the critical point of the objective function onto the feasible interval since the problem is convex [77]. This directly gives (8.21). ■

8.3.2 $T \geq (N+1)d$

In this case, we propose an algorithmic solution that is based on the necessary optimality conditions in Lemmas 8.1, 8.2, and 8.3. We first solves problem (8.13) without considering the service time constraints, i.e., assuming that the set of constraints $\{x_i \geq 2d,\ 2 \leq i \leq N;\ x_{N+1} \geq d\}$ is not active. We then check if any of these abandoned constraints is not satisfied, and optimally alter the solution to make it feasible.

Let us denote by (P^e) problem (8.13) without the set of constraints $\{x_i \geq 2d,\ 2 \leq i \leq N;\ x_{N+1} \geq d\}$, i.e., considering only the energy causality constraints. We then introduce the following algorithm to solve problem (P^e)

Definition 8.1 (Inter-Update Balancing Algorithm) *Start by computing*

$$i_1 \triangleq \arg\max\left\{s_1, \frac{s_2}{2}, \ldots, \frac{s_N}{N}, \frac{T-d}{N+1}\right\} \tag{8.25}$$

where the set is indexed as $\{1, \ldots, N+1\}$*, and then set*

$$x_1^* = \cdots = x_{i_1}^* = \max\left\{s_1, \frac{s_2}{2}, \ldots, \frac{s_N}{N}, \frac{T-d}{N+1}\right\} + d \tag{8.26}$$

If $i_1 = N+1$ *stop, else compute*

$$i_2 \triangleq \arg\max\left\{s_{i_1+1} - s_{i_1}, \frac{s_{i_1+2} - s_{i_1}}{2}, \ldots, \frac{s_N - s_{i_1}}{N - i_1}, \frac{T - d - s_{i_1}}{N + 1 - i_1}\right\} \tag{8.27}$$

where the set is indexed as $\{i_1+1, \ldots, N+1\}$*, and then set*

$$x_{i_1+1}^* = \cdots = x_{i_2}^* = \max\left\{s_{i_1+1} - s_{i_1}, \frac{s_{i_1+2} - s_{i_1}}{2}, \ldots, \frac{s_N - s_{i_1}}{N - i_1}, \frac{T - d - s_{i_1}}{N + 1 - i_1}\right\} + d \tag{8.28}$$

If $i_2 = N+1$ stop, else continue with computing i_3 as above. The algorithm is guaranteed to stop since it will at most compute i_{N+1} which is equal to $N+1$ by construction.

Note that while computing i_k, if the $\arg\max$ is not unique, we pick the largest maximizer. Observe that the algorithm equalizes the x_i's as much as allowed by the energy causality constraints. Let $\{\bar{x}_i\}_{i=1}^N$ be the output of the Inter-Update Balancing algorithm and let $\{x_i^e\}_{i=1}^N$ denote the optimal solution of problem (P^e). We now have the following results

Lemma 8.4 *$\{\bar{x}_i\}_{i=1}^N$ is a non-increasing sequence, and $\bar{x}_j > \bar{x}_{j+1}$ only if $\sum_{i=1}^j \bar{x}_i = s_j + jd$.*

Proof: We show this by induction. Clearly, we have $\bar{x}_1 = \bar{x}_2 = \cdots = \bar{x}_{i_1} = \frac{s_{i_1}}{i_1} + d$ by construction. Now assume that $\{\bar{x}_i\}_{i=1}^{i_k}$ is non-increasing, and consider $\{\bar{x}_i\}_{i=1}^{i_{k+1}}$. We know that $\bar{x}_{i_k+1} = \bar{x}_{i_k+2} = \cdots = \bar{x}_{i_{k+1}} = \frac{s_{i_{k+1}} - s_{i_k}}{i_{k+1} - i_k} + d$ by construction. We now proceed by contradiction; assume that $\bar{x}_{i_{k+1}} > \bar{x}_{i_k}$. This means that the following holds

$$\frac{s_{i_{k+1}} - s_{i_k}}{i_{k+1} - i_k} > \frac{s_{i_k} - s_{i_{k-1}}}{i_k - i_{k-1}} \tag{8.29}$$

or equivalently

$$\frac{i_{k+1} - i_k}{i_{k+1} - i_{k-1}} s_{i_{k-1}} + \frac{i_k - i_{k-1}}{i_{k+1} - i_{k-1}} s_{i_{k+1}} > s_{i_k} \tag{8.30}$$

Next, observe that the following holds by construction when choosing x_{i_k}

$$\frac{s_{i_k} - s_{i_{k-1}}}{i_k - i_{k-1}} \geq \frac{s_{i_{k+1}} - s_{i_{k-1}}}{i_{k+1} - i_{k-1}} \tag{8.31}$$

which is equivalent to

$$\frac{i_{k+1} - i_k}{i_{k+1} - i_{k-1}} s_{i_{k-1}} + \frac{i_k - i_{k-1}}{i_{k+1} - i_{k-1}} s_{i_{k+1}} \leq s_{i_k} \tag{8.32}$$

This contradicts (8.30), and proves the first part of the lemma.

Now let us show the second part. Assume that $\bar{x}_{j+1} < \bar{x}_j$. Then necessarily we must have $j = i_k$ for some i_k, or else they should be equal. Therefore, by construction, we have

$$\begin{aligned}\sum_{i=1}^{i_k} \bar{x}_i =& (i_k - i_{k-1}) \left(\frac{s_{i_k} - s_{i_{k-1}}}{i_k - i_{k-1}} + d \right) + (i_{k-1} - i_{k-2}) \left(\frac{s_{i_{k-1}} - s_{i_{k-2}}}{i_{k-1} - i_{k-2}} + d \right) \\ &+ \cdots + i_1 \left(\frac{s_{i_1}}{i_1} + d \right) \\ =& s_{i_k} + i_k d \end{aligned} \tag{8.33}$$

This concludes the proof. ■

Lemma 8.5 $x_i^e = \bar{x}_i$, $1 \leq i \leq N$.

Proof: Let $\{\bar{x}_i\}_{i=1}^N$ be the output of the Inter-Update Balancing algorithm and let $\{x_i^e\}_{i=1}^N$ denote the optimal solution of problem (P^e). We first show that $\{\bar{x}_i\}_{i=1}^N$ is feasible. Let j be such that $i_k < j \le i_{k+1}$. Then

$$\begin{aligned}
\sum_{i=1}^{j} \bar{x}_i &= \sum_{i=1}^{i_k} \bar{x}_i + \sum_{i=i_k+1}^{j} \bar{x}_i \\
&= s_{i_k} + i_k d + (j - i_k)\left(\frac{s_{i_{k+1}} - s_{i_k}}{i_{k+1} - i_k} + d\right) && (8.34) \\
&\ge s_{i_k} + jd + (j - i_k)\left(\frac{s_j - s_{i_k}}{j - i_k}\right) && (8.35) \\
&= s_j + jd && (8.36)
\end{aligned}$$

where (8.34) follows by (8.33), and (8.35) follows since, by construction, we have

$$\frac{s_{i_{k+1}} - s_{i_k}}{i_{k+1} - i_k} \ge \frac{s_j - s_{i_k}}{j - i_k}, \quad \forall i_k < j \le i_{k+1} \tag{8.37}$$

Finally, note that the stopping criterion of the algorithm is when $i_L = N + 1$ for some i_L. Whence, we have

$$\begin{aligned}
\sum_{i=1}^{N+1} \bar{x}_i =& i_1\left(\frac{s_{i_1}}{i_1} + d\right) + (i_2 - i_1)\left(\frac{s_{i_2} - s_{i_1}}{i_2 - i_1} + d\right) \\
&+ \cdots + (N + 1 - i_{L-1})\left(\frac{T - d - s_{i_{L-1}}}{N + 1 - i_{L-1}} + d\right) \\
=& (N+1)d + (T - d) = T + Nd && (8.38)
\end{aligned}$$

This shows that that $\{\bar{x}_i\}_{i=1}^{N}$ is feasible.

Next, we show that $x_i^e = \bar{x}_i, \ \forall i$. We show this by contradiction. Let $x_i^e = \bar{x}_i$ for $1 \le i \le m-1$ and let $x_m^e \ne \bar{x}_m$, i.e., m is the first time index at which the two sequences are different. We now consider two cases as follows.

First, assume $x_m^e < \bar{x}_m$. Note that it must be the case that $i_{k_1} < m \leq i_k$ for some i_k. Therefore, $\bar{x}_i = \bar{x}_m$, $\forall m \leq i \leq i_k$, by construction. By (8.16), we have that $\{x_i^e\}$ is non-increasing since $\eta_i = 0$ and $\lambda_i \geq 0$. Therefore, $x_i^e < \bar{x}_i$, $\forall m \leq i \leq i_k$, and hence $\sum_{i=1}^{i_k} x_i^e < \sum_{i=1}^{i_k} \bar{x}_i = s_{i_k} + i_k d$, i.e., the allegedly-optimal policy is not feasible. Therefore $x_m^e \geq \bar{x}_m$

Second, assume $x_m^e > \bar{x}_m$. Since $\sum_{i=1}^{N+1} x_i^e = \sum_{i=1}^{N+1} \bar{x}_i$, therefore there must exist some time index $l > m$ such that $x_l^e < \bar{x}_l$. Now let $\epsilon \triangleq \min\{x_m^e - \bar{x}_m, \bar{x}_l - x_l^e\}$, and consider a new policy $\{\tilde{x}_i\}$ which is equal to $\{x_i^e\}$ except at time indices m and l, with $\tilde{x}_m = x_m^e - \epsilon$ and $\tilde{x}_l = x_l^e + \epsilon$. Since $\tilde{x}_m \geq \bar{x}_m$, the new policy is feasible. In addition, by convexity of the square function, the following holds [77]

$$(\tilde{x}_m)^2 + (\tilde{x}_l)^2 < (x_m^*)^2 + (x_l^*)^2 \tag{8.39}$$

which means that the new policy achieves a lower age, rendering $\{x_i^e\}$ suboptimal.

The above arguments show that we must have $x_i^e = \bar{x}_i$, $\forall i$. This completes the proof. ■

We note that Lemma 8.5 is similar to [76, Theorem 1]. In fact, the Inter-Update Balancing algorithm reduces to the optimal offline algorithm proposed in [76] when $d = 0$. When $d > 0$, some change of parameters can still show the equivalence. The next corollary now follows.

Corollary 8.1 *Consider problem (P^e) with the additional constraint that* $\sum_{i=1}^{j} x_i = s_j + jd$ *holds for some* $j \leq N$. *Then, the optimal solution of the problem, under this condition, for time indices not larger than* j *is given by* $\{x_i^e\}_{i=1}^{j}$.

Proof: This direct by setting $T' \triangleq s_j + d$ and $N' \triangleq j - 1$, and applying the Inter-Update Balancing algorithm on the problem with a reduced number of variables $\{x_1, \ldots, x_{N'+1}\}$. ■

The following theorem shows that the optimal solution of problem (8.13), $\{x_i^*\}$, is found by equalizing the inter-update times as much as allowed by the energy causality constraints. If such equalization does not satisfy the minimal inter-update time constraints, we force it to be exactly equal to such minimum and adjust the last variable x_{N+1} accordingly.

Theorem 8.2 *Let $T \geq (N+1)d$. If $x_i^e \geq 2d$, $2 \leq i \leq N$ and $x_{N+1}^e \geq d$, then $x_i^* = x_i^e$, $\forall i$. Else, let n_0 be the first time index at which $\{x_i^e\}$ is not feasible in problem (8.13). Then, we have $n_0 \leq N$. If $n_0 > 2$, we have*

$$x_i^* = x_i^e, \quad 1 \leq i \leq n_0 - 1 \tag{8.40}$$

$$x_i^* = 2d, \quad n_0 \leq i \leq N \tag{8.41}$$

$$x_{N+1}^* = T + Nd - \sum_{i=1}^{N} x_i^* \tag{8.42}$$

Otherwise, for $n_0 = 2$, $\{x_i^*\}$ is given by the above if $x_1^e = s_1 + d$, else $\{x_i^*\}$ is given by (8.21)-(8.23).

Proof: The first part of the theorem follows directly since the solution of the less constrained problem (P^e) is optimal if feasible in problem (8.13). Next, we prove the second part.

We first show that $n_0 \leq N$ by contradiction. Assume that $n_0 = N + 1$, i.e.,

$x_{N+1}^e < d$ and $x_N^e \geq 2d > x_{N+1}^e$. By Lemma 8.4, this means that $\sum_{i=1}^{N} x_i^e = s_N + Nd$. Hence, $x_{N+1}^e = T + Nd - s_N - Nd = T - s_N$, which cannot be less than d by the feasibility assumption in (8.10). Thus, $n_0 \leq N$.

Now let $n_0 > 2$ and observe that $x_{n_0}^e < 2d \leq x_{n_0-1}$. Thus, by Lemma 8.4, we must have $\sum_{i=1}^{n_0-1} x_i^e = s_{n_0-1} + (n_0-1)d$. Now let us show that the proposed policy is feasible; we only need to check whether $x_{N+1}^* \geq d$. Towards that, we have

$$\begin{aligned} x_{N+1}^* &= T + Nd - \sum_{i=1}^{n_0-1} x_i^* - (N - n_0 + 1)\, 2d \\ &= T - s_{n_0-1} - (N - n_0 + 1)d \geq d \end{aligned} \tag{8.43}$$

where the last inequality follows by the feasibility assumption in (8.10). Therefore, the proposed policy is feasible.

We now show that it is optimal as follows. Assume that there exists another policy $\{\tilde{x}_i\}$ that achieves a lower age than $\{x_i^*\}$. We now have two cases. First, assume that $\sum_{i=1}^{n_0-1} \tilde{x}_i = s_{n_0-1} + (n_0-1)d$. then by Corollary 8.1 we must have $\tilde{x}_i = x_i^*$ for $1 \leq i \leq n_0 - 1$. Now for $n_0 \leq i \leq N$, if $\tilde{x}_i > x_i^* = 2d$, this means that $\tilde{x}_{N+1} < x_{N+1}^*$ to satisfy the last constraint in (8.13). Since $\sum_{i=n_0}^{N+1} \tilde{x}_i = \sum_{i=n_0}^{N+1} x_i^*$, then by convexity of the square function, $\sum_{i=n_0}^{N+1} (\tilde{x}_i)^2 > \sum_{i=n_0}^{N+1} (x_i^*)^2$ [77], and hence $\{\tilde{x}_i\}$ cannot be optimal. Second, assume that $\sum_{i=1}^{n_0-1} \tilde{x}_i > s_{n_0-1} + (n_0-1)d = \sum_{i=1}^{n_0-1} x_i^*$. Since $\tilde{x}_i \geq x_i^* = 2d$ for $n_0 \leq i \leq N$, and $\sum_{i=1}^{N+1} \tilde{x}_i = \sum_{i=1}^{N+1} x_i^*$, then we must have $\tilde{x}_{N+1} < x_{N+1}^*$. Thus, $\sum_{i=1}^{N+1} (\tilde{x}_i)^2 > \sum_{i=1}^{N+1} (x_i^*)^2$ by convexity of the square function [77], and $\{\tilde{x}_i\}$ cannot be optimal.

Finally, let $n_0 = 2$. If $x_1^e = s_1 + d$, then the proof follows by the arguments for the $n_0 > 2$ case. Else if $x_1^e > s_1 + d$, then $x_1^e = x_2^e \geq x_{N+1}^e$ by Lemma 8.4. Since $\{x_i^e\}_{i=2}^N$ have to increase to at least $2d$, then $x_1^e + x_{N+1}^e$ has to decrease to satisfy the last constraint in (8.13). However, one cannot increase x_1^e to $2d$ or more and compensate that by decreasing x_{N+1}^e, by convexity of the square function. Thus, $x_1^* < x_2^*$, and Lemma 8.2 shows that the results of Theorem 8.1 follow to give (8.21)-(8.23). ■

8.4 Two-Hop Network: Solution of Problem (8.9)

We now discuss how to use the results of the single-user problem to solve problem (8.9). We have the following theorem.

Theorem 8.3 *The optimal solution of problem (8.9) is given by the optimal solution of problem (8.12) after replacing s_i by $\max\{\bar{s}_i, s_i + d\}$, $\forall i$; d by $d + \bar{d}$; and T by $T + d$.*

Proof: Let f denote the objective function of problem (8.9). Differentiating f with respect to t_i, $i \leq N - 1$, we get $\frac{\partial f}{\partial t_i} = 2\left(\bar{t}_i + \bar{d} - t_i\right) - 2\left(\bar{t}_{i+1} + \bar{d} - t_i\right)$, which is negative since $\bar{t}_{i+1} > \bar{t}_i$. We also have $\frac{\partial f}{\partial t_N} = 2\left(\bar{t}_N + \bar{d} - t_N\right) - 2\left(T - t_N\right)$, which is non-positive since $\bar{t}_N + \bar{d} \leq T$. Thus, f is decreasing in $\{t_i\}_{i=1}^{N-1}$ and non-increasing in t_N. Therefore, the optimal $\{t_i^*\}$ satisfies the data causality constraints in (8.4) with equality for all updates so as to be the largest possible and achieve the smallest

A_T. Setting $t_i = \bar{t}_i - d,\ \forall i$ in problem (8.9) we get

$$f = \sum_{i=1}^{N} \left(\bar{t}_i + \bar{d} + d - \bar{t}_{i-1}\right)^2 - N\left(\bar{d} + d\right)^2 + \left(T + d - \bar{t}_N\right)^2 \tag{8.44}$$

with the constraints now being

$$\bar{t}_i \geq s_i + d,\ \bar{t}_i \geq \bar{s}_i, \quad \forall i \tag{8.45}$$

$$\bar{t}_i + \bar{d} + d \leq \bar{t}_{i+1}, \quad 1 \leq i \leq N-1 \tag{8.46}$$

$$\bar{t}_N + \bar{d} \leq T \tag{8.47}$$

We now see that minimizing f subject to the above constraints is exactly the same as solving problem (8.12) after applying the change of parameters mentioned in the theorem. ■

Theorem 8.3 shows that the source should send its updates *just in time* as the relay is ready to forward, and no update should wait for service in the relay's data buffer. Thus, the source and the relay act as one combined node that can send updates whenever it receives combined energy packets at times $\{\max\{\bar{s}_i, s_i + d\}\}$. This fundamental observation can be generalized to multi-hop networks as well. Given $M > 1$ relays, each node should send updates just in time as the following node is ready to forward, until reaching destination.

8.5 Numerical Results

We now present some numerical examples to further illustrate our results. A two-hop network has energy arriving at times $\boldsymbol{s} = [2, 6, 7, 11, 13]$ at the source, and $\bar{\boldsymbol{s}} = [1, 4, 9, 10, 15]$ at the relay. A source transmission takes $d = 1$ time unit to reach the relay; a relay transmission takes $\bar{d} = 2$ time units to reach the destination. Session time is $T = 19$. We apply the change of parameters in Theorem 8.3 to get new energy arrival times $\boldsymbol{s} = [3, 7, 9, 12, 15]$, new transmission delay $d = 3$, and new session time $T = 20$. Then, we solve problem (8.13) to get the optimal inter-update times, using the new parameters. Note that $T \geq (N+1)d = 18$, whence the optimal solution is given by Theorem 8.2. We apply the Inter-Update Balancing algorithm to get $\boldsymbol{x}^e = [6.5, 6.5, 5.67, 5.67, 5.67, 5]$. Hence, the first infeasible inter-update time occurs at $n_0 = 3$ ($x_3^e < 2d = 6$). Thus, we set: $x_1^* = x_1^e$ and $x_2^* = x_2^e$; $x_3^* = x_4^* = x_5^* = 2d$; and $x_6^* = T + Nd - \sum_{i=1}^{5} x_i^*$. We see that $\boldsymbol{x}^* = [6.5, 6.5, 6, 6, 6, 4]$ satisfies the conditions stated in Lemmas 8.1, 8.2, and 8.3.

We consider another example where energy arrives at times $\boldsymbol{s} = [0, 4, 4, 9, 13]$ and $\bar{\boldsymbol{s}} = [1, 3, 6, 10, 12]$, with $T = 16$. Applying the change of parameters in Theorem 8.3 we get $T = 17 < (N+1)d = 18$, and hence we use the results of Theorem 8.1 to get $\boldsymbol{x}^* = [5, 6, 6, 6, 6, 3]$. We then increase T to 18. This is effectively 19 according to Theorem 8.3, and therefore we apply Theorem 8.2 results. The Inter-Update Balancing algorithm gives $\boldsymbol{x}^e = [5.8, 5.8, 5.8, 5.8, 5.8, 5]$, and hence $n_0 = 2$. Since $x_1^e > s_1 + d = 4$, then the optimal solution is given by (8.21)-(8.23) as $\boldsymbol{x}^* = [5, 6, 6, 6, 6, 5]$.

8.6 Conclusion

In this chapter, we proposed age-minimal policies in energy harvesting two-hop networks with fixed transmission delays. The optimal policy is such that the relay's data buffer should not contain any packets waiting for service; the source should send an update to the relay just in time as the relay is ready to forward. This let us treat the source and relay nodes as one combined node communicating with the destination node, and reduce the two-hop problem to a single hop one. We solved the single hop problem by balancing inter-update times to the extent allowed by energy arrival times and transmission delays.

CHAPTER 9

Conclusions

In this dissertation, we characterized optimal energy management policies in energy harvesting communication networks while taking into account various system costs.

In Chapter 2, we considered receiver decoding costs, where energy harvesting receivers spend an amount of energy to decode their intended messages. We modeled the decoding energy as a convex increasing function of the incoming data rate. This introduced a further coupling between transmitters and receivers of the network. We characterized throughput-optimal policies in single-user and multi-user settings by treating decoding costs as generalized data arrivals and moving all constraints to the transmitter side.

In Chapter 3, we studied the impact of decoding costs on energy harvesting cooperative multiple access channels, where users cooperate in the physical layer to achieve higher rates. We showed that, depending on the relative values of the decoding costs, data cooperation between the users might achieve lower rates than directly sending to the receiver. In the case when cooperation is beneficial, we determined the optimal distribution of harvested energy to decoding and cooperative forwarding.

In Chapter 4, we investigated the addition of processing costs on top of decoding costs in two-way energy harvesting channels, where users incur a processing energy cost whenever they are operating. Due to processing costs, transmission can be bursty; the users communicate only during a portion of the time. We designed throughput-optimal schemes under both decoding and processing costs in a single setting.

In Chapter 5, we focused on online settings. We characterized online power control policies that maximize the long term average utility of single-user energy harvesting channels with finite batteries, for some concave increasing utility function. We showed that fixed fraction policies perform within constant multiplicative and additive gaps from the optimal solution for all energy arrivals and battery sizes. We also considered a specific scenario of distortion minimization with and without sampling costs.

In Chapter 6, we considered another aspect of system costs in energy harvesting single-user channels, that is, the cost of movement in search of better energy harvesting locations. We characterized the optimal throughput-movement tradeoff, in offline and online settings, for a transmitter moving along a straight line and communicating with a receiver.

We then considered different performance metrics, other than the throughput metric considered in previous chapters. In particular, in Chapter 7, we studied the issue of transmission delay. We defined the delay as the time elapsed from arrival to departure of data units, and characterized delay minimal transmission policies in single-user and broadcast energy harvesting channels. Different from conventional throughput-optimal policies, delay minimal policies give higher priorities to earlier arriving data units compared to later arriving ones, and may have communication gaps in between energy or data arrivals.

Finally, in Chapter 8, we considered the metric of age of information in energy harvesting two-hop networks, where a transmitter is sending status updates of a physical phenomenon to a receiver through the help of a relay. With the age of information defined as the time elapsed since the freshest update has reached the destination, we showed that age minimal policies are such that the transmitter should send updates to the relay just in time as the relay is ready to forward them to the destination.

The contents of Chapter 2 are published in [79,88], Chapter 3 in [89], Chapter 4 in [81,82,90], Chapter 5 in [91,92], Chapter 6 in [93], Chapter 7 in [94], and Chapter 8 in [95].

Bibliography

[1] J. Yang and S. Ulukus. Optimal packet scheduling in an energy harvesting communication system. *IEEE Trans. Comm.*, 60(1):220–230, January 2012.

[2] K. Tutuncuoglu and A. Yener. Optimum transmission policies for battery limited energy harvesting nodes. *IEEE Trans. Wireless Comm.*, 11(3):1180–1189, March 2012.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. Transmission with energy harvesting nodes in fading wireless channels: Optimal policies. *IEEE JSAC*, 29(8):1732–1743, September 2011.

[4] C. K. Ho and R. Zhang. Optimal energy allocation for wireless communications with energy harvesting constraints. *IEEE Trans. Signal Proc.*, 60(9):4808–4818, September 2012.

[5] J. Yang and S. Ulukus. Optimal packet scheduling in a multiple access channel with energy harvesting transmitters. *Journal of Comm. and Networks*, 14(2):140–150, April 2012.

[6] Z. Wang, V. Aggarwal, and X. Wang. Iterative dynamic water-filling for fading multiple-access channels with energy harvesting. *IEEE JSAC*, 33(3):382–395, March 2015.

[7] J. Yang, O. Ozel, and S. Ulukus. Broadcasting with an energy harvesting rechargeable transmitter. *IEEE Trans. Wireless Comm.*, 11(2):571–583, February 2012.

[8] O. Ozel, J. Yang, and S. Ulukus. Optimal broadcast scheduling for an energy harvesting rechargebale transmitter with a finite capacity battery. *IEEE Trans. Wireless Comm.*, 11(6):2193–2203, June 2012.

[9] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal. Optimal packet scheduling on an energy harvesting broadcast link. *IEEE JSAC*, 29(8):1721–1731, September 2011.

[10] H. Erkal, F. M. Ozcelik, and E. Uysal-Biyikoglu. Optimal offline broadcast scheduling with an energy harvesting transmitter. *EURASIP Journal Wireless Comm. Networking*, 2013(1):1–20, July 2013.

[11] K. Tutuncuoglu and A. Yener. Sum-rate optimal power policies for energy harvesting transmitters in an interference channel. *Journal Comm. Networks*, 14(2):151–161, April 2012.

[12] D. Gunduz and B. Devillers. Two-hop communication with energy harvesting. In *IEEE CAMSAP*, December 2011.

[13] C. Huang, R. Zhang, and S. Cui. Throughput maximization for the Gaussian relay channel with energy harvesting constraints. *IEEE JSAC*, 31(8):1469–1479, August 2013.

[14] O. Orhan and E. Erkip. Optimal transmission policies for energy harvesting two-hop networks. In *CISS*, March 2012.

[15] I. Ahmed, A. Ikhlef, R. Schober, and R. K. Mallik. Power allocation in energy harvesting relay systems. In *IEEE VTC*, May 2012.

[16] Y. Luo, J. Zhang, and K. Ben Letaief. Optimal scheduling and power allocation for two-hop energy harvesting communication systems. *IEEE Trans. Wireless Comm.*, 12(9):4729–4741, September 2013.

[17] B. Varan and A. Yener. Delay constrained energy harvesting networks with limited energy and data storage. *IEEE JSAC*, 34(5):1550–1564, May 2016.

[18] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus. Energy cooperation in energy harvesting communications. *IEEE Trans. Comm.*, 61(12):4884–4898, December 2013.

[19] K. Tutuncuoglu and A. Yener. Energy harvesting networks with energy cooperation: Procrastinating policies. *IEEE Trans. Comm.*, 63(11):4525–4538, November 2015.

[20] B. Gurakan and S. Ulukus. Energy harvesting diamond channel with energy cooperation. In *IEEE ISIT*, July 2014.

[21] B. Gurakan and S. Ulukus. Cooperative diamond channel with energy harvesting nodes. *IEEE JSAC*, 34(5):1604–1617, May 2016.

[22] K. Tutuncuoglu, A. Yener, and S. Ulukus. Optimum policies for an energy harvesting transmitter under energy storage losses. *IEEE JSAC*, 33(3):476–481, March 2015.

[23] D. Gunduz and B. Devillers. A general ramework for the optimization of energy harvesting communication systems with battery imperfections. *Journal Comm. Networks*, 14(2):130–139, April 2012.

[24] O. Orhan, D. Gunduz, and E. Erkip. Throughput maximization for an energy harvesting communication system with processing cost. In *IEEE ITW*, September 2012.

[25] J. Xu and R. Zhang. Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power. *IEEE JSAC*, 32(2):322–332, February 2014.

[26] O. Orhan, D. Gunduz, and E. Erkip. Energy harvesting broadband communication systems with processing energy cost. *IEEE Trans. Wireless Comm.*, 13(11):6095–6107, November 2014.

[27] O. Ozel, K. Shahzad, and S. Ulukus. Optimal energy allocation for energy harvesting transmitters with hybrid energy storage and processing cost. *IEEE Trans. Signal Proc.*, 62(12):3232–3245, June 2014.

[28] M. Gregori and M. Payaró. Throughput maximization for a wireless energy harvesting node considering power consumption. In *IEEE VTC*, September 2012.

[29] Q. Bai, J. Li, and J. A Nossek. Throughput maximizing transmission strategy of energy harvesting nodes. In *IEEE IWCLD*, November 2011.

[30] M. Gregori and M. Payaró. Optimal power allocation for a wireless mutli-antenna energy harvesting node with arbitrary input distribution. In *IEEE ICC*, June 2012.

[31] Y. Luo, J. Zhang, and K. Ben Letaief. Training optimization for energy harvesting communication systems. In *IEEE Globecom*, December 2012.

[32] A. Nayyar, T. Basar, D. Teneketzis, and V. V. Veeravalli. Optimal strategies for communication and remote estimation with an energy harvesting sensor. *IEEE Trans. Auto. Control*, 58(9):2246–2260, September 2013.

[33] C. Huang, R. Zhang, and S. Cui. Optimal power allocation for outage probability minimization in fading channels with energy harvesting constraints. *IEEE Trans. Wireless Comm.*, 13(2):1074–1087, February 2014.

[34] O. Orhan, D. Gunduz, and E. Erkip. Delay-constrained distortion minimization for energy harvesting transmission over a fading channel. In *IEEE ISIT*, July 2013.

[35] O. Orhan, D. Gunduz, and E. Erkip. Source channel coding under energy, delay, and buffer constraints. *IEEE Trans. Wireless Comm.*, 14(7):3836 – 3849, July 2015.

[36] B. Varan, K. Tutuncuoglu, and A. Yener. Energy harvesting communications with continuous energy arrivals. In *UCSD ITA*, Feb 2014.

[37] K. Tutuncuoglu and A. Yener. Communicating with energy harvesting transmitters and receivers. In *UCSD ITA*, February 2012.

[38] H. Mahdavi-Doost and R. D. Yates. Energy harvesting receivers: Finite battery capacity. In *IEEE ISIT*, July 2013.

[39] R. D. Yates and H. Mahdavi-Doost. Energy harvesting receivers: Optimal sampling and decoding policies. In *IEEE GlobalSIP*, December 2013.

[40] H. Mahdavi-Doost and R. D. Yates. Fading channels in energy-harvesting receivers. In *CISS*, March 2014.

[41] R. Nagda, S. Satpathi, and R. Vaze. Optimal offline and competitive online strategies for transmitter-receiver energy harvesting. In *IEEE ICC*, June 2015. Longer version available: arXiv:1412.2651v2.

[42] P. Grover, K. Woyach, and A. Sahai. Towards a communication-theoretic understanding of system-level power consumption. *IEEE JSAC*, 29(8):1744–1755, September 2011.

[43] J. Rubio, A. Pascual-Iserte, and M. Payaró. Energy-efficient resource allocation techniques for battery management with energy harvesting nodes: a practical approach. In *Euro. Wireless Conf.*, April 2013.

[44] P. Rost and G. Fettweis. On the transmission-computation-energy tradeoff in wireless and fixed networks. In *IEEE Globecom Workshop Green Comm.*, December 2010.

[45] S. Cui, A. J. Goldsmith, and A. Bahai. Power estimation for Viterbi decoders. Technical report, Wireless System Lab, Stanford Univ., CA, 2003. Available Online: http://wsl.stanford.edu/publications.html.

[46] A. El Gamal and Y. Kim. *Network Information Theory*. Cambridge University Press, 2011.

[47] B. R. Marks and G. P. Wright. A general inner approximation algorithm for nonconvex mathematical programs. *Operations research*, 26(4):681–683, July-August 1978.

[48] M. Chiang, C. W. Tan, D. P. Palomar, D. O'Neill, and D. Julian. Power control by geometric programming. *IEEE Trans. Wireless Comm.*, 6(7):2640–2651, July 2007.

[49] T. Cover and J. A. Thomas. *Elements of Information Theory*. 2006.

[50] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity – part I: System description. *IEEE Trans. Comm.*, 51(11):1927–1938, November 2003.

[51] N. Su, O. Kaya, S. Ulukus, and M. Koca. Cooperative multiple access under energy harvesting constraints. In *IEEE Globecom*, December 2015.

[52] B. Gurakan, B. Sisman, O. Kaya, and S. Ulukus. Energy and data cooperation in energy harvesting multiple access channel. In *IEEE WCNC*, April 2016.

[53] P. Youssef-Massaad, L. Zheng, and M. Medard. Bursty transmission and glue pouring: on wireless channels with overhead costs. *IEEE Trans. Wireless Comm.*, 7(12):5188–6194, December 2008.

[54] D. Shaviv and A. Ozgur. Universally near optimal online power control for energy harvesting nodes. *IEEE JSAC*, 34(12):3620–3631, December 2016.

[55] A. Baknina and S. Ulukus. Optimal and near-optimal online strategies for energy harvesting broadcast channels. *IEEE JSAC*, 34(12):3696–3708, December 2016.

[56] H. A. Inan and A. Ozgur. Online power control for the energy harvesting multiple access channel. In *IEEE WiOpt*, May 2016.

[57] A. Baknina and S. Ulukus. Online policies for multiple access channel with common energy harvesting source. In *IEEE ISIT*, July 2016.

[58] A. Baknina and S. Ulukus. Online scheduling for energy harvesting channels with processing costs. *IEEE Trans. Green Comm. and Networking*, 1(3):281–293, September 2017.

[59] A. Baknina and S. Ulukus. Online scheduling for an energy harvesting link with processing costs. In *IEEE Globecom*, December 2016.

[60] A. Baknina and S. Ulukus. Online scheduling for energy harvesting two-way channels with processing costs. In *IEEE Globecom*, December 2016.

[61] M. B. Khuzani and P. Mitran. On online energy harvesting in multiple access communication systems. *IEEE Trans. Info. Theory*, 60(3):1883–1898, March 2014.

[62] F. Amirnavaei and M. Dong. Online power control optimization for wireless transmission with energy harvesting and storage. *IEEE Trans. Wireless Comm.*, 15(7):4888–4901, July 2016.

[63] B. T. Bacinoglu and E. Uysal-Biyikoglu. Finite horizon online lazy scheduling with energy harvesting transmitters over fading channels. In *IEEE ISIT*, June 2014. Longer version available: arXiv:1702.06390v1.

[64] Y. Shi, L. Xie, Y. T. Hou, and H. D. Sheali. On renewable sensor networks with wireless energy transfer. In *IEEE Infocom*, April 2011.

[65] A. H. Coarasa, P. Nintaanavongsa ans S. Sanyal, and K. R. Chowdhury. Impact of mobile transmitter sources on radio frequency wireless energy harvesting. In *IEEE ICNC*, January 2013.

[66] O. Ozel and S. Ulukus. Achieving AWGN capacity under stochastic energy harvesting. *IEEE Trans. Info. Theory*, 58(10):6471–6483, October 2012.

[67] J. Yang and S. Ulukus. Delay-minimal transmission for energy constrained wireless communications. In *IEEE ICC*, 2008.

[68] S. Kaul, R. Yates, and M. Gruteser. Real-time status: How often should one update? In *IEEE Infocom*, March 2012.

[69] R. Yates and S. Kaul. Real-time status updating: Multiple sources. In *IEEE ISIT*, July 2012.

[70] C. Kam, S. Kompella, and A. Ephremides. Age of information under random updates. In *IEEE ISIT*, July 2013.

[71] M. Costa, M. Codreanu, and A. Ephremides. Age of information with packet management. In *IEEE ISIT*, June 2014.

[72] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis. Age and value of information: Non-linear age case. Available Online: arXiv:1701.06927v1.

[73] Y. Sun, E. Uysal-Biyikoglu, R. Yates, C. E. Koksal, and N. B. Shroff. Update or wait: How to keep your data fresh. In *IEEE Infocom*, April 2016.

[74] A. M. Bedewy, Y. Sun, and N. B. Shroff. Age-optimal information updates in multihop networks. Available Online: arXiv:1701.05711v3.

[75] R. D. Yates. Lazy is timely: Status updates by an energy harvesting source. In *IEEE ISIT*, June 2015.

[76] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu. Age of information under energy replenishment constraints. In *UCSD ITA*, February 2015.

[77] S. P. Boyd and L. Vandenberghe. *Convex Optimization*. 2004.

[78] A. A. D'Amico, L. Sanguinetti, and D. P. Palomar. Convex seperable problems with linear constraints in signal processing and communications. *IEEE Trans. Signal Proc.*, 62(22):6045–6058, November 2014.

[79] A. Arafa and S. Ulukus. Single-user and multiple-access channels with energy harvesting transmitters and receivers. In *IEEE GlobalSIP*, December 2014.

[80] O. Kaya and S. Ulukus. Power control for fading cooperative multiple access channels. *IEEE Trans. Wireless Comm.*, 6(8):2915–2923, August 2007.

[81] A. Arafa, A. Baknina, and S. Ulukus. Energy harvesting two-way channel with decoding costs. In *IEEE ICC*, May 2016.

[82] A. Arafa, A. Baknina, and S. Ulukus. Optimal policies in energy harvesting two-way channels with processing costs. In *IEEE WiOpt*, May 2016.

[83] S. M. Ross. *Stochastic Processes*. 1996.

[84] S. He, J. Chen, F. Jiang, D. Yau, G. Xing, and Y. Sun. Energy provisioning in wireless rechargeable sensor networks. *IEEE Trans. Mobile Computing*, 12(10):1931–1942, October 2013.

[85] L. Fu, P. Cheng, Y. Gu, J. Chen, and T. He. Minimizing charging delay in wireless rechargeable sensor networks. In *IEEE Infocom*, April 2013.

[86] K. Huang and V. Lau. Enabling wireless power transfer in cellular networks: Architecture, modeling and deployment. *IEEE Trans. Wireless Comm.*, 13(2):902–912, February 2014.

[87] I. Olkin and A. W. Marshall. *Inequalities: Theory of Majorization and its Applications*. 2014.

[88] A. Arafa and S. Ulukus. Optimal policies for wireless networks with energy harvesting transmitters and receivers: Effects of decoding costs. *IEEE JSAC*, 33(12):2611–2625, December 2015.

[89] A. Arafa, O. Kaya, and S. Ulukus. Energy harvesting cooperative multiple access channel with decoding costs. In *EUSIPCO*, August 2016.

[90] A. Arafa, A. Baknina, and S. Ulukus. Energy harvesting two-way channels with decoding and processing costs. *IEEE Trans. Green Comm. and Networking*, 1(1):3–16, March 2017.

[91] A. Arafa, A. Baknina, and S. Ulukus. Energy harvesting networks with general utility functions: Near optimal online policies. In *IEEE ISIT*, June 2017.

[92] A. Arafa and S. Ulukus. Near optimal online distortion minimization for energy harvesting nodes. In *IEEE ISIT*, June 2017.

[93] A. Arafa and S. Ulukus. Mobile energy harvesting policies. In *IEEE ICC*, May 2017.

[94] M. Fu, A. Arafa, S. Ulukus, and W. Chen. Delay minimal policies in energy harvesting broadcast channels. In *IEEE ICC*, May 2016.

[95] A. Arafa and S. Ulukus. Age-minimal transmission in energy harvesting two-hop networks. In *IEEE Globecom*, December 2017. Available Online: arXiv:1704.08679v1.