Delay Minimal Policies in Energy Harvesting Broadcast Channels

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***Abstract*— We consider a two-user energy harvesting broadcast channel, and characterize the delay minimal transmission policies that minimize the *total* delay experienced by the data packets in the system. We consider a continuous time system where the delay experienced by each bit is given by the time spent by the bit in the queue waiting to be transmitted to its receiver. We consider the case where all data packets are available at the transmitter at the beginning of the communication session. We characterize the optimal solution in terms of the Lagrange multipliers, and present an iterative algorithm that optimally calculates their values. Our results show that in the optimal policy, both users may not be served simultaneously all the time; there may be times where only the strong user or only the weak user is served alone. We also show that the optimal policy may have *gaps* in transmission where none of the users is served until the next energy arrival.**

I. Introduction

We consider an energy harvesting broadcast channel where the transmitter relies solely on energy harvested from nature to deliver data packets to two users on the downlink. We assume that the data packets are available at the transmitter at the beginning, and the energy arrives (is harvested) throughout the communication session. The transmitter needs to adapt its transmission power to its energy harvesting profile. Optimal resource allocation and scheduling policies have been considered for various energy harvesting communication models. Earlier works focus on throughput maximization and transmission completion time minimization policies for single user settings [1]–[4], multiple access channels [5], broadcast channels [6], relay and two-hop networks [7], [8]. Later works introduce energy sharing concepts to improve system throughput [9], and incorporate system aspects of energy harvesting into problem formulations, such as energy losses [10], transmission processing costs [11], and receiver decoding costs [12].

Reference [13] revisits the single-user energy harvesting communication system and considers the problem of *delay minimization*, as opposed to transmission completion time (e.g., deadline) minimization or throughput maximization. The delay experienced by each bit is given by the amount of time the bit spends in the queue waiting to be transmitted. Reference [13] shows that unlike most results in the literature on energy harvesting, the optimal transmit power in this case is not piece-wise constant; it decreases between energy harvests and may drop to zero before the next energy harvest. The intuition for this is that the later sent bits in the queue experience cumulative delay including the delays of the earlier sent bits; therefore, earlier sent bits need to be sent out faster which necessitates using higher powers earlier.

In this paper, we consider a multi-user version of the problem studied in [13]. We consider a two user energy harvesting broadcast channel where the data packets intended for both users are available before the transmission starts. In this system, there is a trade-off between the delays experienced by both users; as more resources (power) is allocated to a user, its delay decreases while the delay of the other user increases. We consider the minimization of the *sum* delay in the system. We formulate the problem using a Lagrangian framework, and express the optimal solution in terms of Lagrange multipliers. We develop an iterative algorithm that solves the optimum Lagrange multipliers by enforcing the KKT optimality conditions. We show that the optimal transmission power decreases between energy harvests, and may possibly hit zero before the next energy harvest, yielding communication *gaps*, where no data is transmitted. During active communication data may be sent to both users, or only to the stronger, or only to the weaker user, depending on the energy harvesting profile.

Finally, we contrast our work with [6] which developed an algorithm that minimized the transmission completion time, i.e., a time by which all data is delivered to users. To that end, [6] studies the throughput maximization problem, and shows that, for general priorities, there exists a *cut-off power* level such that only the total power above this level is used to serve the weaker user. In particular, for sum throughput maximization, this cut-off is infinity, and all power is allocated to packets sent to the stronger user. In contrast, in our *sum delay minimization* problem, the weaker user always gets a share of the transmitted power, as otherwise, its delay becomes unbounded. In our work, we show that there exists a *cut-off time*, beyond which data is sent only to the weaker user.

II. System Model

We consider an energy harvesting two-user broadcast channel, where energy is harvested at times $\{t_0, t_1, \ldots, t_{M-1}\}$ in amounts $\{E_0, E_1, \ldots, E_{M-1}\}$, respectively, with $t_0 = 0$. We denote the cumulative harvested energy as:

$$E_a(t) = \sum_{i=0}^{m-1} E_i, \quad t_{m-1} < t \leq t_m, \quad m = 1, \ldots, M \qquad (1)$$

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where we define $t_M = \infty$. The data packets are available before the communication starts, in amounts B_1 and B_2, for the first and the second user, respectively.

The physical layer is a degraded broadcast channel,

$$Y_j = X + Z_j, \quad j = 1, 2 \tag{2}$$

where X is the transmitted signal, Y_j is the received signal of user j, and Z_j is the Gaussian noise at receiver j with variance σ_j^2. We assume $\sigma_1^2 = 1 < \sigma_2^2 \triangleq \sigma^2$, i.e., the first user is stronger. The capacity region for this channel is [14]

$$r_1 \leq \frac{1}{2}\log\left(1 + \alpha P\right), \quad r_2 \leq \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma^2}\right) \tag{3}$$

where α is the fraction of the total power assigned to the first (stronger) user, and $\log$ is the natural logarithm. Working on the boundary of the capacity region we have,

$$P = e^{2(r_1+r_2)} + \left(\sigma^2 - 1\right)e^{2r_2} - \sigma^2 \triangleq g\left(r_1, r_2\right) \tag{4}$$

which is the minimum power needed to achieve rates r_1 and r_2, at the first and the second user, respectively. Note that $g(r_1, r_2)$ is strictly convex in (r_1, r_2) [15]. We call a policy feasible if the following are satisfied:

$$\int_0^t g\left(r_1(\tau), r_2(\tau)\right)d\tau \leq E_a(t), \quad \forall t \tag{5}$$

$$\int_0^\infty r_1(t)dt = B_1 \tag{6}$$

$$\int_0^\infty r_2(t)dt = B_2 \tag{7}$$

where the first constraint is the energy causality constraint, and the remaining two are to ensure data delivery to both users.

The average delay experienced by each user is given by [13]

$$D_1 = \int_0^\infty r_1\left(t\right)tdt \tag{8}$$

$$D_2 = \int_0^\infty r_2\left(t\right)tdt \tag{9}$$

Note that, each delay expression is the integral of rate multiplied by time as in [13]. Unlike [13], in this two-user setting, there is a trade-off between the delays experienced by the two users. This trade-off can be characterized by developing the *delay region*, similar to *departure region* in [6], where all achievable (D_1, D_2) can be plotted. In can be shown that this region is strictly convex, and in order to achieve, pareto-optimum delay points, one needs to solve *weighted sum delay minimization* problems in the form of $\min \mu_1 D_1 + \mu_2 D_2$ subject to energy causality constraints. We skip such a general treatment due to space limitations here, and instead, focus on the *sum* delay minimization problem by taking $\mu_1 = \mu_2 = 1$. Therefore, in this paper, we consider the following optimization problem:

$$\begin{aligned}
\min_{r_1, r_2} \quad & \int_0^\infty r_1\left(\tau\right)\tau d\tau + \int_0^\infty r_2\left(\tau\right)\tau d\tau \\
\text{s.t.} \quad & \int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right)d\tau \leq E_a(t_m), \quad m = 1, \dots, M \\
& \int_0^\infty r_1(\tau)d\tau = B_1 \\
& \int_0^\infty r_2(\tau)d\tau = B_2 \\
& r_1(t) \geq 0, \quad r_2(t) \geq 0, \quad \forall t
\end{aligned} \tag{10}$$

III. Minimum Sum Delay Policy

We note that (10) is a convex optimization problem [15]. We solve using a Lagrangian approach:

$$\begin{aligned}
\mathcal{L} = & \int_0^\infty r_1\left(\tau\right)\tau d\tau + \int_0^\infty r_2\left(\tau\right)\tau d\tau \\
& + \sum_{m=1}^{M} \lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right)d\tau - E_a(t_m)\right) \\
& - \nu_1 \left(\int_0^\infty r_1(\tau)d\tau - B_1\right) - \nu_2 \left(\int_0^\infty r_2(\tau)d\tau - B_2\right) \\
& - \int_0^\infty \gamma_1(\tau)r_1(\tau)d\tau - \int_0^\infty \gamma_2(\tau)r_2(\tau)d\tau
\end{aligned} \tag{11}$$

where $\{\lambda_m\}$, ν_1, ν_2, $\gamma_1(t)$, and $\gamma_2(t)$ are the Lagrange multipliers. KKT optimality conditions are:

$$t + \lambda(t)\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} - \nu_1 - \gamma_1(t) = 0 \tag{12}$$

$$t + \lambda(t)\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} - \nu_2 - \gamma_2(t) = 0 \tag{13}$$

where we have:

$$\lambda(t) = \sum_{\{m: t_m \geq t\}} \lambda_m \tag{14}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} = 2e^{2(r_1(t)+r_2(t))} \tag{15}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} = 2e^{2(r_1(t)+r_2(t))} + 2\left(\sigma^2 - 1\right)e^{2r_2(t)} \tag{16}$$

along with the complementary slackness conditions:

$$\lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right)d\tau - E_a(t_m)\right) = 0, \quad \forall m \tag{17}$$

$$\nu_1 \left(\int_0^\infty r_1(\tau)d\tau - B_1\right) = 0, \quad \gamma_1(t)r_1(t) = 0 \quad \forall t \tag{18}$$

$$\nu_2 \left(\int_0^\infty r_2(\tau)d\tau - B_2\right) = 0, \quad \gamma_2(t)r_2(t) = 0 \quad \forall t \tag{19}$$

From the above KKT conditions, we can write the rates and total power expressions in terms of the Lagrange multipliers. First, we write the rates expressions as:

$$r_1(t) = \frac{1}{2}\log\left(\frac{\left(\sigma^2 - 1\right)\left(\gamma_1\left(t\right) + \nu_1 - t\right)}{\gamma_2\left(t\right) - \gamma_1\left(t\right) + \nu_2 - \nu_1}\right) \tag{20}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\gamma_2\left(t\right) - \gamma_1\left(t\right) + \nu_2 - \nu_1}{\lambda\left(t\right)\left(\sigma^2 - 1\right)}\right) \tag{21}$$

We now state the following result.

Lemma 1 *The optimal Lagrange multipliers (ν_1^*, ν_2^*) satisfy: $\nu_1^* < \nu_2^* < \sigma^2\nu_1^*$.*

Proof: We show this by contradiction. Assume $\nu_2^* \leq \nu_1^*$. Then, by (21), the value of $r_2(t)$ is well-defined only if $\gamma_2(t) > 0$ $\forall t$, which means by complementary slackness that $r_2(t) = 0$ $\forall t$. Therefore, assuming $B_2 > 0$, the weak user will never get to receive any of its data. This proves the first inequality.

To show the second inequality, assume $\sigma^2 \nu_1^* \leq \nu_2^*$. Thus,

$$\frac{\left(\sigma^2 - 1\right)\left(\nu_1 - t\right)}{\gamma_2\left(t\right) + \nu_2 - \nu_1} \leq 1, \quad \forall t, \gamma_2(t) \geq 0 \tag{22}$$

Therefore, the right hand side of (20) can only be positive if $\gamma_1(t) > 0$, but this means, by complementary slackness, that $r_1(t) = 0$, which is a contradiction. Hence, $r_1(t) = 0$ $\forall t$, and, assuming $B_1 > 0$, the strong user will never get to receive any of its data. ■

Next, we characterize the optimal total transmit power $g\left(r_1(t), r_2(t)\right)$ by the following lemma.

Lemma 2 *In the optimal policy, the total transmit power* $g\left(r_1(t), r_2(t)\right)$ *is given by*

$$g(r_1(t), r_2(t)) = \max\left\{\frac{\nu_2 - t}{\lambda(t)} - \sigma^2, \frac{\nu_1 - t}{\lambda(t)} - 1\right\}^+ \tag{23}$$

Proof: From (13) and (16), we have

$$g(r_1(t), r_2(t)) = \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \tag{24}$$

Since from (15) and (16) we always have

$$\frac{\partial g(r_1(t), r_2(t))}{\partial r_2(t)} - \sigma^2 \geq \frac{\partial g(r_1(t), r_2(t))}{\partial r_1(t)} - 1 \tag{25}$$

with equality iff $r_2(t) = 0$, from (12) and (13), we have

$$\frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \geq \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 \tag{26}$$

Thus, if $r_2(t) > 0$, by complementary slackness $\gamma_2(t) = 0$, and the total power is given by

$$g(r_1(t), r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{27}$$

$$> \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 \tag{28}$$

$$\geq \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{29}$$

On the other hand, if $r_2(t) = 0$ and $r_1(t) > 0$, we have

$$g(r_1(t), r_2(t)) = \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \tag{30}$$

$$= \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{31}$$

$$\geq \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{32}$$

Finally, if both rates are zero, then the total power is zero. Combining this with the above gives (23). ■

The above lemma shows that the optimal power decreases with time between energy harvests, and can reach zero before increasing again with the next energy harvest. The following lemmas characterize the structure of the optimal policy.

Lemma 3 *In the optimal policy, the transmission starts by sending data to the strong user, and finishes by sending data to the weak user.*

Proof: We show this by contradiction. Assume that the transmission starts by sending data to the weak user only, i.e., $r_2(0) > r_1(0) = 0$.[1] By complementary slackness, we have $\gamma_2(0) = 0$. By Lemma 1, since $\sigma^2 \nu_1 > \nu_2$, we have

$$\frac{\left(\sigma^2 - 1\right)\left(\gamma_1(0) + \nu_1\right)}{\nu_2 - \nu_1 - \gamma_1(0)} > 1, \quad \forall \gamma_1(0) \geq 0 \tag{33}$$

which implies, by (20), that $r_1(0) > 0$, which is a contradiction. For the second part of the lemma, assume that the transmission ends at some time t_f with $r_1(t_f) > r_2(t_f) = 0$. By Lemma 2, we know that this can only occur if $\lambda(t_f) > \frac{\nu_2 - \nu_1}{\sigma^2 - 1} \triangleq \lambda_{th}$. Since $\lambda(t)$ is non-increasing, we have $\lambda(t) \geq \lambda(t_f)$, $\forall t \leq t_f$. This means that $\lambda(t)$ does not fall below λ_{th} throughout the transmission, which is equivalent to saying, again by Lemma 2, that the weak user does not receive any of its data, which is a contradiction. ■

Lemma 4 *For* $t < t_{th} \triangleq \frac{\sigma^2 \nu_1 - \nu_2}{\sigma^2 - 1}$, *if the transmitter is sending data, then it is sending to the strong user.*

Proof: We show this by contradiction. Assume that for some $t < t_{th}$ data is sent only to the weak user, i.e., we have $r_1(t) = 0$ and $r_2(t) > 0$. By complementary slackness, we have $\gamma_2(t) = 0$. Since $t < t_{th}$, it follows by simple manipulations that the numerator of the term inside the log in (20) is strictly larger than its denominator $\forall \gamma_1(t) \geq 0$, i.e., $r_1(t) > 0$, which is a contradiction. The only case where $r_1(t) = 0$ for some $t < t_{th}$ is when $\gamma_2(t) > 0$, which means by complementary slackness that $r_2(t) = 0$. ■

A. Modes of Operation

There can be four different modes of operation at a given time, depending on which user is receiving data. The first mode is when only the strong user is receiving data, i.e., $r_1(t) > 0$ and $r_2(t) = 0$. By Lemma 2, this can be the case only if $\lambda(t) \geq \lambda_{th} = \frac{\nu_2 - \nu_1}{\sigma^2 - 1}$. In this mode, we have the total power and the strong user's rate given by

$$g(r_1(t), 0) = \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{34}$$

$$r_1(t) = \frac{1}{2}\log\left(\frac{\nu_1 - t}{\lambda(t)}\right) \tag{35}$$

The second mode of operation is when both users are receiving data, i.e., $r_1(t) > 0$ and $r_2(t) > 0$. Again by Lemma 2, this can be the case only if $\lambda(t) < \lambda_{th}$. Moreover, by (20),

[1] Extension of the contradiction arguments in this lemma to an ϵ-length interval, $\epsilon > 0$, follows directly.

we also need $t < t_{th} = \frac{\sigma^2\nu_1-\nu_2}{\sigma^2-1}$. In this mode, the total power and the users' rates are given by

$$g(r_1(t), r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{36}$$

$$r_1(t) = \frac{1}{2}\log\left(\frac{(\sigma^2-1)(\nu_1 - t)}{\nu_2-\nu_1}\right) \tag{37}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2-\nu_1}{\lambda(t)(\sigma^2-1)}\right) \tag{38}$$

The third mode of operation is when only the weak user is receiving data, i.e., $r_1(t) = 0$ and $r_2(t) > 0$. For this to occur we need both $\lambda(t) < \lambda_{th}$ and $t \geq t_{th}$. The total power and the weak user's rate are then given by

$$g(0, r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{39}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2 - t}{\lambda(t)\sigma^2}\right) \tag{40}$$

The fourth mode is when both rates (and the power) are zero. We denote this mode as a communication *gap*. These gaps may occur, for instance, if there is a small amount of energy in the battery that is insufficient to deliver all the data, and a large amount of energy arrives later. The transmitter may then finish up this small amount of energy to send some bits out and wait for additional energy to send the remaining bits.

B. Finding the value of $\lambda(t)$

We next characterize the rates and powers. The following lemma shows that $\lambda(t)$ is a piecewise constant function.

Lemma 5 *In the optimal policy, the Lagrange multiplier function $\lambda(t)$ is piecewise constant, with possible changes only when energy is depleted.*

Proof: By the complementary slackness conditions on $\lambda(t)$,

$$\lambda_m^* = 0, \quad \text{if } E^*(t_m) < E_a(t_m) \tag{41}$$

$$E^*(t_m) = E_a(t_m), \quad \text{if } \lambda_m^* > 0 \tag{42}$$

Therefore, $\lambda(t)$ remains constant between energy harvests, and can only decrease when $\lambda_m > 0$ for some m, which happens only when energy is depleted. ■

By Lemma 5, $\lambda(t)$ is a sequence rather than a continuous function of time. We denote the times of change of $\lambda(t)$ by $\{s_1, s_2, \ldots, s_L\}$ with $s_1 = 0$, and the values of $\lambda(t)$ between such times by

$$\lambda(t) = \begin{cases} \lambda_k^c, & t \in [s_k, s_{k+1}) \\ \lambda_L^c, & t \in [s_L, \infty) \end{cases} \tag{43}$$

Next, we characterize the optimal $\{\lambda_k^c\}$ sequentially. Determining the value of λ_k^c requires the knowledge of ν_1^* and ν_2^*, and also which mode of operation is active during the interval $[s_k, s_{k+1})$. Let us define $B_j(t)$ as the total amount of bits transmitted to user j by time t. The next theorem shows how to compute λ_k^c given the mode of operation. The proof uses similar steps as in [13, Lemmas 2 and 3] and is omitted here.

Theorem 1 *Given a mode of operation, with the optimal ν_1^*, ν_2^*, λ_l^c, s_l, $\forall l < k$, define the following quantities $\forall m$: $t_m > s_k$*

$$\bar{\lambda}_m: \quad E^*(s_k) + \int_{s_k}^{t_m} g(r_1(\tau), r_2(\tau))^+ d\tau = E_a(t_m) \tag{44}$$

$$\tilde{\lambda}_1: \quad B_1^*(s_k) + \int_{s_k}^{\infty} r_1(\tau)^+ d\tau = B_1 \tag{45}$$

$$\tilde{\lambda}_2: \quad B_2^*(s_k) + \int_{s_k}^{\infty} r_2(\tau)^+ d\tau = B_2 \tag{46}$$

where r_1, r_2, and $g(r_1, r_2)$ are defined by the mode of operation in Section III-A, with the convention that $\tilde{\lambda}_j = 0$ whenever a mode of operation has $r_j = 0$, $j = 1, 2$. Then, the optimal λ_k^c for this mode of operation is given by

$$\lambda_k^c = \max\{\bar{\lambda}_m, \tilde{\lambda}_1, \tilde{\lambda}_2\}, \quad \forall m : t_m > s_k \tag{47}$$

The results in Theorem 1 imply that one has to know the mode of operation before computing the optimal values of the Lagrange multipliers. Note that communication gaps occur naturally due to the $(\cdot)^+$ operation in these expressions. In the next section, we develop an iterative algorithm that computes $\{\lambda_k^c\}$ based on an initial assignment of the mode of operation and the values of ν_1, ν_2. The algorithm is based on the necessary conditions stated in the previous lemmas. By Lemma 1, we know that the optimal values of ν_1, ν_2 lie in a cone in $\mathbb{R}_{++}^2$. We also know, by Lemmas 2 and 3, that the communication stops if $t > \nu_2$. Therefore, we find an upper bound on the value of ν_2^* as follows. First, we move all of the energy to t_{M-1}, the arrival time of the last energy packet, and start the communication from there. Second, we solve this single energy arrival problem and find its optimal ν_2^* which we denote by ν_2^{single}. Therefore, an upper bound on ν_2^* of the multiple energy arrival problem is

$$\nu_2^* \leq \nu_2^{\text{single}} + t_{M-1} \triangleq \nu^{\text{ub}} \tag{48}$$

Once this upper bound is found, one can perform a two-dimensional grid search over the feasible region of ν_1, ν_2:

$$\mathcal{R}_{\nu_1\nu_2} = \left\{\nu_1, \nu_2 : 0 < \nu_1 < \nu_2 < \sigma^2\nu_1, \nu_2 \leq \nu^{\text{ub}}\right\} \tag{49}$$

Next, we analyze the single energy arrival case to characterize the upper bound on ν_2^*.

C. Single Energy Arrival

For the single energy arrival case, we first note that there can be no communication gaps, as this can only increase the delay. We also note that since there is only one value of λ, corresponding to only one energy arrival constraint, the optimal power is given by the first term in (23). If not, then the weak user will never receive its data. Hence, the first mode of operation where only the strong user is receiving data never occurs. Thus, the optimal total power is given by

$$p_s(t) = \frac{\nu_2 - t}{\lambda} - \sigma^2, \quad \forall t \leq t_f \triangleq \nu_2 - \lambda\sigma^2 \tag{50}$$

where the subscript s denotes single arrival, and t_f is such that $p_s(t)$ is non-negative. From the above, we also note that

Algorithm 1 Solving for $\{\lambda_k^c\}$ given (ν_1, ν_2)

Assume transmission starts with Mode 1
repeat
    Find the next λ_k^c using (47)
until $\lambda_k^c < \lambda_{th}$
Set the mode of operation as Mode 2
repeat
    Find the next λ_k^c using (47)
until $t > t_{th}$
Set the mode of operation as Mode 3
repeat
    Find the next λ_k^c using (47)
until Weak user's data or transmission energy is finished

Algorithm 2 Finding the optimal (ν_1^*, ν_2^*)

Fix $\nu_1 = \epsilon > 0$ small enough
while $\nu_1 \leq \nu^{\text{ub}}$ **do**
    Fix $\nu_2 = \nu_1 + \epsilon$
    while $\nu_2 \leq \min\{\sigma^2\nu_1, \nu^{\text{ub}}\}$ **do**
        Solve for $\{\lambda_k^c\}$ via Algorithm 1
        if $B_1(\infty) = B_1$ and $B_2(\infty) = B_2$ **then**
            Declare current policy as optimal
        else
            $\nu_2 \leftarrow \nu_2 + \epsilon$
        end if
    end while
    $\nu_1 \leftarrow \nu_1 + \epsilon$
end while

λ cannot be 0, or else the power is infinitely large. Since $\lambda > 0$, by complementary slackness, the transmitter has to consume all of its energy by the end of transmission. This simplifies the single energy arrival problem, as in this case, we have all the three constraints, both users' data and transmitter's energy, met with equality. Therefore, we can solve for the optimal values of the Lagrange multipliers satisfying the following:

$$\int_0^{t_{th}} \frac{1}{2}\log\left(\frac{(\sigma^2-1)(\nu_1-t)}{\nu_2-\nu_1}\right)dt = B_1 \quad (51)$$

$$\frac{t_{th}}{2}\log\left(\frac{\nu_2-\nu_1}{\lambda(\sigma^2-1)}\right) + \int_{t_{th}}^{t_f} \frac{1}{2}\log\left(\frac{\nu_2-t}{\lambda\sigma^2}\right)dt = B_2 \quad (52)$$

$$\int_0^{t_f} p_s(t)dt = E \quad (53)$$

The above three equations are direct consequences of the modes of operation analysis in Section III-A. These can be further simplified into:

$$\frac{\nu_1}{2}\log\left(\frac{(\sigma^2-1)\nu_1}{\nu_2-\nu_1}\right) = B_1 \quad (54)$$

$$\frac{\nu_2}{2}\log\left(\frac{\nu_2-\nu_1}{\lambda(\sigma^2-1)}\right) = B_2 \quad (55)$$

$$\frac{\left(\nu_2-\lambda\sigma^2\right)^2}{2\lambda} = E \quad (56)$$

Note that (54)-(56) have three equations in three unknowns, and can be solved numerically for the values of λ^*, ν_1^*, and ν_2^*. Note from the above analysis that, since we always start with the second mode of operation, where both users receive data, in this setting, we have $\lambda < \lambda_{th}$. This implies that $t_f > t_{th}$, and enables the following stronger version of Lemma 3.

Lemma 6 *In the optimal policy solving (10), transmission always ends by sending data only to the weak user.*

Proof: In the single energy arrival case, since $t_f > t_{th}$, we always end transmission by sending data only to the weak user. In the multiple arrival case, the last energy arrival can be viewed as a single energy arrival problem with the remaining data in the data buffers as modified constraints. Then the single energy arrival result applies, yielding the stated result. ■

We have now characterized how to get the upper bound ν^{ub} in (48). In the next section we present an iterative algorithm to find the optimal Lagrange multipliers solving problem (10).

IV. Iterative Solution

The analysis presented in Theorem 1 describes an optimal method of finding $\{\lambda_k^c\}$ given ν_1^* and ν_2^*. To find the latter two, we perform a grid search over the region $\mathcal{R}_{\nu_1\nu_2}$, which is fully characterized by the single arrival analysis. We perform the search as follows. We fix $(\nu_1, \nu_2) \in \mathcal{R}_{\nu_1\nu_2}$, and solve for $\{\lambda_k^c\}$ to acquire a transmission policy accordingly. We denote by Mode 1, Mode 2, and Mode 3, the mode of operation where data is sent only to the strong user, both users, and only to the weak user, respectively. Since Mode 1 can only occur at the beginning, we assume that the transmission starts according to that mode, and compute the corresponding λs by Theorem 1. If these λs are all less than λ_{th}, then they are correct. We move to Mode 2 once we get a value of λ larger than λ_{th}. We stay at Mode 2 until the time passes t_{th}, then move to Mode 3 till the end of communication. By Lemma 6, we know that Mode 3 always exists. The transmission then ends whenever the weak user's data or the transmission energy is finished. We summarize these in Algorithm 1 above.

After we find the transmission policy, we check whether the data buffers of both users are empty. If this is the case, then by the convexity of the problem, this policy is optimal as we have thus found a feasible policy satisfying the KKT conditions [15]. Note that we might end up with a policy that either does not finish up all the users' data, or even transmits more than the available. If either is the case, we re-solve using another (ν_1, ν_2) point. Since the region $\mathcal{R}_{\nu_1\nu_2}$ is bounded, iterations are guaranteed to find the optimal solution according to the above stopping criterion. We summarize how to find the optimal (ν_1, ν_2) in Algorithm 2 above.

V. Numerical Results

We present a numerical example to illustrate the results in this paper. We consider a system where energy arrives with values $[6, 10, 4, 5]$ at times $t = [0, 70, 100, 150]$, with amounts of data $B_1 = 8$ and $B_2 = 4.25$ intended for the strong and the

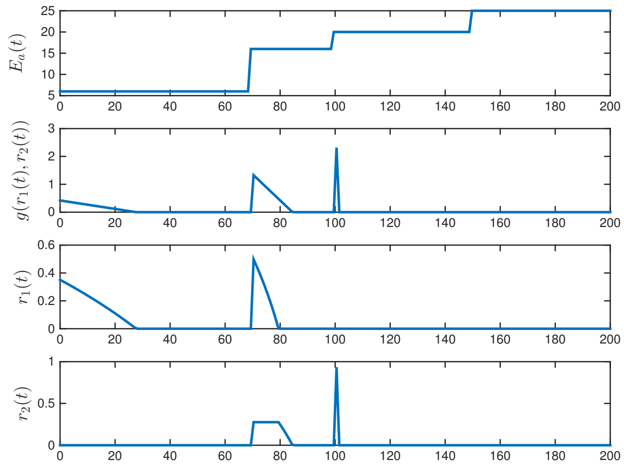

Fig. 1. Optimal power and rates for a system with four energy arrivals.

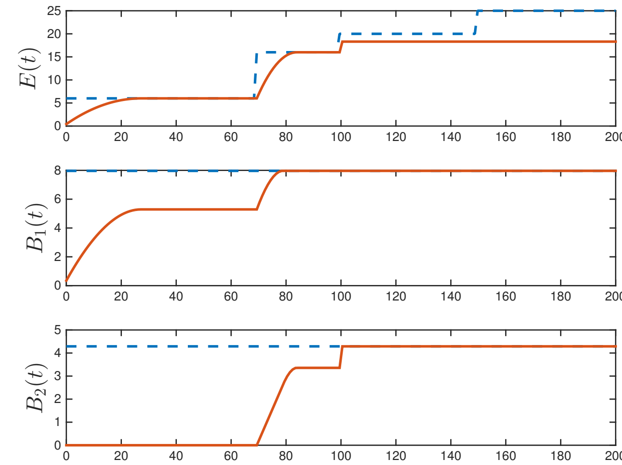

Fig. 2. Optimal energy and data consumption.

weak user, respectively. We first find the upper bound on ν_2^* by solving the single energy arrival case by setting $E = 25$ in (56) and finding the value of ν_2^{single}. Adding $t_{M-1} = 150$, we get $\nu^{\text{ub}} \simeq 170$. We then apply Algorithm 2 to find the optimal total power allocation for the multiple arrival case and the corresponding users' rates. These are shown in Fig. 1 as a function of time. We see that all four modes of operation are present in this example: the transmitter begins by sending data only to the strong user (Mode 1) until it consumes the initial energy arrival, and stays silent until the next energy arrival, then it sends data to both users simultaneously (Mode 2) until all strong user's data is finished, which occurs at $t_{th} \simeq 79.4$. Then, it starts sending data only to the weak user (Mode 3), before keeping silent until the third energy arrival, and then finishes up the weak user's data. Note that the fourth energy arrival is not used in this example. In Fig. 2, we show the corresponding optimal total energy and data consumption for this policy as a function of time.

Next, we compare this to the transmission completion time minimization problem in [6] with the same data values and energy arrival profile. The optimal transmission completion time is equal to $T^* = 90$. Calculating the delay achieved by such policy gives $D \simeq 717.2$. On the other hand, our delay minimizing policy achieves a smaller delay of $D^* \simeq 593.3$, however, it takes a larger amount of time to finish $T \simeq 101.5$. This shows that there exists a trade-off between delay minimization and transmission completion time minimization, and that the two problems are different, even when all data is available before the start of communication. That is, finishing data delivery by a minimum time, and having data experience minimum overall delay yield different optimum policies.

VI. Conclusion

We considered a two-user energy harvesting broadcast channel and characterized the minimal sum delay policy subject to energy harvesting constraints, when all data intended for both users is available before transmission. We showed that the optimal power is decreasing between energy harvests, and that there can be times when data is sent only to the strong user, both users, or only to the weak user. We also showed that there can be communication gaps where the transmitter is silent between energy arrivals. We presented an algorithm to find the optimal policy iteratively.

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