Energy Harvesting Two-Way Channel with Decoding Costs

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***Abstract*—We consider an energy harvesting two-way channel with decoding costs. In this system, each node spends energy to transmit data to the other user, and also to decode data coming from the other user; that is, each user divides its harvested energy for transmission and reception. The power needed for decoding the incoming data is a function of the incoming data rate. We determine the optimal offline power scheduling policies for both users that maximize the sum throughput of the system by a given deadline. We first consider the case with a single energy arrival at each user. We show that the transmission is limited by the user with the smaller energy. In this case, the user with larger energy may not consume all of its energy. We next consider the case with multiple energy arrivals at both users. We show that the optimal power allocations are non-decreasing over time, and they increase synchronously at both users. We then develop an iterative algorithm based on two-slot updates to obtain the optimal power allocations for both users.**

I. Introduction

We consider an energy harvesting two-way channel with decoding costs, see Fig. 1. In this system, users spend energy to transmit data as well as to decode the incoming data. Each user depends solely on energy harvested from nature to transmit and receive data. In this paper, we characterize the optimal offline power allocation policies for both users that maximize the sum throughput of the system by a given deadline. In these policies, each user divides its harvested energy optimally for transmission and reception powers, and also schedules available energy usage over upcoming slots.

Energy harvesting communication systems have been studied extensively in recent literature, see e.g., [1]–[29]. References [1]–[23] focus on energy harvesting at the transmitter side, and consider the single-user setting [1]–[4], broadcast, multiple access, and interference channels [5]–[10], two-hop and relay channels [11]–[13], two-way channels [14], [15], cooperative multiple access channels [16], diamond channels [17], energy sharing and energy cooperation concepts [18]–[20], battery imperfections [21], [22], and temperature constrained sensor operations [23]. These references optimize the transmit power schedules of the users over time, using concave rate-power relationships, to minimize the transmission completion time or maximize the throughput by a deadline.

References [24]–[29] focus on energy harvesting at the receivers. In these references, the energy needed for receiving

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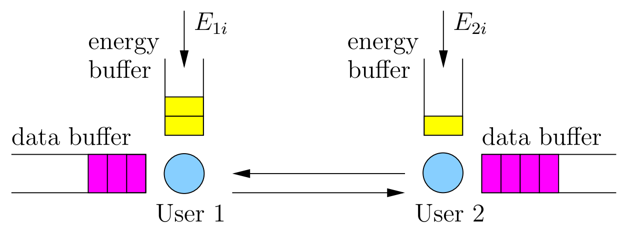

Fig. 1. Two-way channel with an energy harvesting transmitter and receiver.

incoming data is modeled as a monotone increasing convex function of the incoming rate (see also [30], [31]). In this case, the receivers need to optimally allocate their harvested energy for decoding, and the transmitters need to optimize their transmit powers and therefore rates such that the receivers can handle, i.e., decode and process, the incoming data with their available energies. In the above references, each energy harvesting node is either a transmitter or a receiver, i.e., each node either needs to optimize its transmit power over time slots or needs to optimize its decoding power over time slots.

In this paper, we consider an energy harvesting two-way channel. Each node in this two-way channel transmits data to the other user, and receives data from the other user. Therefore, each node is simultaneously an energy harvesting transmitter and an energy harvesting receiver, and needs to optimize its power schedule over time slots by optimally dividing its energy for transmission and decoding. The power used for transmission is modeled through a concave rate-power relationship as in the Shannon formula; and the power used for decoding is modeled as a convex increasing function of the incoming rate. In particular, throughout this paper, we focus on decoding costs that are exponential in the incoming rate.

Even in the case of energy harvesting transmitters only and energy harvesting receivers only, the energy availability of one side limits the transmission and reception abilities of the other side; energy harvesting introduces coupling between transmitters and receivers. In the energy harvesting two-way channel, this coupling is even stronger. We first consider the case with a single energy arrival at each user. We show that the transmission is limited by the user with smaller energy; the user with larger energy may not consume all of its energy. We next consider the case with multiple energy arrivals at both users. We show that the optimal power allocations are non-decreasing over time, and they increase synchronously at both users. We develop an iterative algorithm based on two-slot updates to obtain the optimal power allocations for both users. We prove the convergence and the optimality of the proposed algorithm, and provide simulation results on its performance.

II. Single Energy Arrival

In this section, we consider the case where both users have a single energy arrival each. In particular, users 1 and 2 have E_1 and E_2 amounts of energy available at the beginning of communication. Without loss of generality, the communication takes place over a time slot of unit length. The physical layer is Gaussian with unit-variance noise at both users. The sum rate is given by the sum of the single-user rates in the full-duplex Gaussian two-way channel [32]. Therefore, the rate per user is the single-user Shannon rate of $\frac{1}{2}\log(1+p)$, where p is the transmit power and $\log$ is the natural logarithm. The decoding power at a receiver is exponential in the incoming rate r, i.e., $\phi(r) = a(e^{br} + c)$. In particular, we take $b = 2$ and $c = -1$ for convenience, without of loss of generality; any other such exponential decoding power can be handled by appropriately modifying the incoming energy. Then, if the first user transmits with power p, the incoming rate is $\frac{1}{2}\log(1 + p)$, and the second user spends a power of ap to decode the incoming data. Therefore, the throughput maximization problem is:

$$\begin{aligned} \max_{p_1,p_2} \quad & \frac{1}{2}\log(1+p_1) + \frac{1}{2}\log(1+p_2) \\ \text{s.t.} \quad & p_1 + ap_2 \leq E_1 \\ & p_2 + ap_1 \leq E_2 \end{aligned} \tag{1}$$

where p_1 and p_2 are the powers of the users. We assume $a \neq 1$, for if $a = 1$, by concavity of the $\log$, the optimal solution will be given by $p_1^* = p_2^* = \min\{E_1, E_2\}/2$.

We have the following lemma regarding this problem.

Lemma 1 *In the optimal policy, at least one user consumes all of its energy in transmission and decoding. This is the user with the smaller energy.*

Proof: We prove this by contradiction. Assume that in the optimal policy $\{p_1^*, p_2^*\}$, neither user consumes its energy fully, i.e., both constraints in (1) are strict. Then, we can increase p_1^* until one of them holds with equality. This strictly increases the objective function, and thus, contradicts the optimality of the original policy. This proves the first part of the lemma.

Now, assume without loss of generality that $E_1 \leq E_2$, but only the second user consumes all of its energy, i.e.,

$$p_2^* + ap_1^* = E_2 \geq E_1 > p_1^* + ap_2^* \tag{2}$$

which further leads to having

$$p_1^* < p_2^*, \quad \text{if } a < 1 \tag{3}$$

$$p_1^* > p_2^*, \quad \text{if } a > 1 \tag{4}$$

Let us consider the case in (3) (similar arguments follow for (4)), choose some $\epsilon > 0$, and define the following new policy:

$$\tilde{p}_1 = p_1^* + \epsilon, \quad \tilde{p}_2 = p_2^* - \epsilon \tag{5}$$

The new policy consumes the following amounts of energy

$$\tilde{p}_2 + a\tilde{p}_1 = p_2^* + ap_1^* - (1-a)\epsilon < E_2 \tag{6}$$

$$\tilde{p}_1 + a\tilde{p}_2 = p_1^* + ap_2^* + (1-a)\epsilon \tag{7}$$

Since the first user did not consume all of its energy, we can choose ϵ small enough such that the new policy is feasible with respect to the first user. By concavity of the $\log$, this new policy strictly increases the sum rate, and therefore, the original policy cannot be optimal, i.e., the first user has to consume all of its energy. ■

The above lemma states that, in the presence of decoding costs, one user may not be able to use up all of its energy. This is because each user now needs to adapt its power (and rate) to both its own energy and to the energy of the other user, in order to guarantee decodability. This makes the user with smaller energy a bottleneck for the system.

Without loss of generality, we continue assuming $E_1 \leq E_2$. Therefore, by Lemma 1, we have $p_1^* + ap_2^* = E_1$. Substituting this condition in (1), we get the following problem for $a < 1$:

$$\begin{aligned} \max_{p_2} \quad & \frac{1}{2}\log(1+E_1-ap_2) + \frac{1}{2}\log(1+p_2) \\ \text{s.t.} \quad & 0 \leq p_2 \leq \frac{E_2 - aE_1}{1-a^2} \end{aligned} \tag{8}$$

Alternatively, we get the following problem for $a > 1$:

$$\begin{aligned} \max_{p_1} \quad & \frac{1}{2}\log(1+p_1) + \frac{1}{2}\log\left(1 + \frac{E_1 - p_1}{a}\right) \\ \text{s.t.} \quad & 0 \leq p_1 \leq \frac{aE_2 - E_1}{a^2-1} \end{aligned} \tag{9}$$

Let us focus on problem (8). By a first derivative analysis, we obtain the optimal second user power as:

$$p_2^* = \min\left\{\left[\frac{1+E_1-a}{2a}\right]^+, \frac{E_2-aE_1}{1-a^2}\right\} \tag{10}$$

where $[x]^+ = \max(x, 0)$. Then, we find the optimal first user power by substituting $p_1^* = E_1 - ap_2^*$. Similar arguments follow for problem (9). In the next section, we use insights from the solution of the single energy arrival case to solve the multiple energy arrival case.

III. Multiple Energy Arrivals

We now consider the case of multiple energy arrivals. Energies arrive at the beginning of time slot i with amounts E_{1i} and E_{2i} at the first and the second user, respectively, ready to be used in the same slot. Unused energies are saved in batteries to be used in later slots. The problem becomes:

$$\begin{aligned} \max_{\mathbf{p}_1,\mathbf{p}_2} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_{1i}) + \frac{1}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} + ap_{2i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} p_{2i} + ap_{1i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \end{aligned} \tag{11}$$

which is a convex optimization problem. The Lagrangian is:

$$\begin{aligned}\mathcal{L} = &-\sum_{i=1}^{N}\frac{1}{2}\log\left(1+p_{1i}\right)-\sum_{i=1}^{N}\frac{1}{2}\log\left(1+p_{2i}\right)\\ &+\sum_{k=1}^{N}\lambda_{1k}\left(\sum_{i=1}^{k}p_{1i}+ap_{2i}-\sum_{i=1}^{k}E_{1i}\right)\\ &+\sum_{k=1}^{N}\lambda_{2k}\left(\sum_{i=1}^{k}p_{2i}+ap_{1i}-\sum_{i=1}^{k}E_{2i}\right) \qquad (12)\end{aligned}$$

where $\{\lambda_{1k}\}$ and $\{\lambda_{2k}\}$ are the non-negative Lagrange multipliers associated with the energy causality constraints of the first and the second user, respectively. The KKT optimality conditions are:

$$p_{1i} = \frac{1}{\sum_{k=i}^{N}(\lambda_{1k}+a\lambda_{2k})}-1 \qquad (13)$$

$$p_{2i} = \frac{1}{\sum_{k=i}^{N}(\lambda_{2k}+a\lambda_{1k})}-1 \qquad (14)$$

along with the complementary slackness conditions:

$$\lambda_{1k}\left(\sum_{i=1}^{k}p_{1i}+ap_{2i}-\sum_{i=1}^{k}E_{1i}\right)=0, \quad \forall k \qquad (15)$$

$$\lambda_{2k}\left(\sum_{i=1}^{k}p_{2i}+ap_{1i}-\sum_{i=1}^{k}E_{2i}\right)=0, \quad \forall k \qquad (16)$$

In the following lemmas, we characterize the properties of the optimal power control policies for this problem.

Lemma 2 *In the optimal policy, both users' powers are non-decreasing in time, i.e.,* $p_{1(i+1)} \geq p_{1i}$ *and* $p_{2(i+1)} \geq p_{2i}$.

Proof: The proof follows from (13) and (14) since the denominators are non-negative and non-increasing as $\lambda_{1k}, \lambda_{2k} \geq 0$. ■

Lemma 3 *In the optimal policy, the power of user* $j \in \{1,2\}$ *increases in a time slot only if at least one of the two users consumes all of its available energy in transmission/decoding in the previous time slot.*

Proof: From (13) and (14), we see that powers can only increase from slot i to slot $i+1$ if at least λ_{1i} or λ_{2i} is strictly positive, or else powers will stay the same. By complementary slackness conditions in (15) and (16), we see that the first (resp., second) user's energies must all be consumed by slot i if $\lambda_{1i} > 0$ (resp., $\lambda_{2i} > 0$). ■

Lemma 4 *In the optimal policy, powers of both users increase synchronously.*

Proof: Let us assume that we have $p_{1i} < p_{1(i+1)}$. By Lemma 3, we must have at least $\lambda_{1i} > 0$ or $\lambda_{2i} > 0$. This in turn makes $p_{2i} < p_{2(i+1)}$ from (14). Similarly, if we have $p_{2i} < p_{2(i+1)}$, then we must also have $p_{1i} < p_{1(i+1)}$ from (13). Thus, the two users' powers increase synchronously. ■

A. The Case of Two Arrivals

We now solve the case of two energy arrivals at each user explicitly. We will provide an iterative algorithm to solve the general multiple energy arrivals case by using the two-slot case. In a two-slot setting, it is optimal to have at least one user consume all of its energy in the second slot. It is not clear, however, if this is the case in the first slot. Towards that, we check the feasible energy consumption strategies and choose the one that gives the maximum sum rate. In each strategy, we find the optimal residual energy transferred from the first to the second slot for a given user. We begin by checking a constant-power strategy which, by concavity of the objective function, is optimal if it is feasible [1]. This occurs when neither user consumes all of its energy in the first slot, and hence, by Lemma 3, the powers of each user in the two slots are equal, i.e., $p_{11} = p_{12} \triangleq p_1$, and $p_{21} = p_{22} \triangleq p_2$. This leaves us with solving a *single-arrival* problem, as discussed in Section II, with the average energy $E_1 = \frac{E_{11}+E_{12}}{2}$ and $E_2 = \frac{E_{21}+E_{22}}{2}$, at the first and the second user, respectively. There can be four more consumption strategies to check if the above is infeasible. We highlight one of them in the following analysis. The remaining ones follow similarly.

We consider the strategy in which the first user consumes all of its energy in the first slot, and the second user consumes all of its energy in the second slot. The second user may have some residual energy left from the first slot to be used in the second slot. Denoting this energy residual by r, we have: $p_{11}+ap_{21} = E_{11}$, and $p_{21}+ap_{11} = E_{21}-r$. Solving these two equations for p_{11} and p_{21}, we obtain: $p_{11} = \frac{E_{11}-a(E_{21}-r)}{1-a^2}$, and $p_{21} = \frac{E_{21}-r-aE_{11}}{1-a^2}$. Since, the second user consumes all of its energy in the second slot we have: $p_{21}+ap_{11}+p_{22}+ap_{12} = E_{21}+E_{22}$, which leads to: $p_{22}+ap_{12} = E_{22}+r$. Next, we divide the energy consumption in the second slot between the two users as: $p_{12} = \frac{\delta}{a}$ and $p_{22} = E_{22}+r-\delta$, for some $\delta \geq 0$. Finding the optimal sum rate in this strategy is tantamount to solving for the optimal values of r and δ. Thus, problem (11) in this case can be rewritten as:

$$\begin{aligned}\max_{r,\delta} \quad & \frac{1}{2}\log\left(1+\frac{\delta}{a}\right)+\frac{1}{2}\log\left(1+E_{22}+r-\delta\right)\\ &+\frac{1}{2}\log\left(1+\frac{E_{11}-a(E_{21}-r)}{1-a^2}\right)\\ &+\frac{1}{2}\log\left(1+\frac{E_{21}-r-aE_{11}}{1-a^2}\right)\\ \text{s.t.} \quad & 0 \leq \delta \leq E_{22}+r\\ & \left(E_{21}-\frac{E_{11}}{a}\right)^{+} \leq r \leq E_{21}-aE_{11}\\ & \delta \leq \frac{1}{1-a^2}\left(E_{12}-a\left(E_{22}+r\right)\right) \qquad (17)\end{aligned}$$

which is a convex optimization problem in (r,δ) [33]. Note that for the above problem to be feasible, we need to have: $E_{21} \geq aE_{11}$, and $E_{12} \geq aE_{22}$. Other consumption strategies will have similar conditions.

To solve the above problem, we first assume that the

Lagrange multiplier associated with the last constraint is zero, i.e., the constraint is not binding (this is the energy causality constraint of the first user in the second time slot), and obtain a solution. The solution is optimal if it satisfies that constraint with strict inequality. Otherwise, the constraint is binding, and needs to be satisfied with equality. In the latter case, we substitute $\delta = \frac{1}{1-a^2}\left(E_{12} - a\left(E_{22}+r\right)\right)$ in the objective function and solve a problem of only one variable, r, which can be solved directly by first derivative analysis over the feasible region of r. We now characterize the solution after removing that last constraint. We define $r_1 \triangleq \left(E_{21} - \frac{E_{11}}{a}\right)^+$, $r_2 \triangleq E_{21} - aE_{11}$ and introduce the following Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\log\left(1+\frac{\delta}{a}\right) - \frac{1}{2}\log\left(1+E_{22}+r-\delta\right)\\ &-\frac{1}{2}\log\left(1+\frac{E_{11}-a(E_{21}-r)}{1-a^2}\right)\\ &-\frac{1}{2}\log\left(1+\frac{E_{21}-r-aE_{11}}{1-a^2}\right)+\lambda_\delta(\delta-E_{22}-r)\\ &-\eta_\delta\delta+\lambda_r(r-r_1)+\eta_r(r_2-r)\end{aligned} \tag{18}$$

where λ_δ, η_δ, λ_r, and η_r are the non-negative Lagrange multipliers. Taking the derivatives with respect to δ, r, and equating to 0, we get the following KKT conditions

$$\frac{1}{a+\delta}+\eta_\delta = \frac{1}{1+E_{22}+r-\delta}+\lambda_\delta \tag{19}$$

$$\begin{aligned}&\frac{1}{1+E_{22}+r-\delta}+\frac{a}{1-a^2+E_{11}-a(E_{21}-r)}+\eta_r\\ &\qquad = \frac{1}{1-a^2+E_{21}-r-aE_{11}}+\lambda_r\end{aligned} \tag{20}$$

along with the usual complementary slackness conditions. From (19), we solve for δ in terms of r as follows:

$$\delta(r) = \begin{cases} 0, & a > 1+E_{22}+r\\ \frac{1+E_{22}+r-a}{2}, & 1-(E_{22}+r)\le a\le 1+E_{22}+r\\ E_{22}+r, & a < 1-(E_{22}+r)\end{cases} \tag{21}$$

Next, we find the optimal value of r. For that, we substitute by $\delta(r)$ in (20). Assuming that the middle expression in (21) holds, we have:

$$\eta_r + f_1(r) = \lambda_r + f_2(r) \tag{22}$$

where f_1 and f_2 are given by

$$f_1(r) = \frac{2}{1+E_{22}+a+r}+\frac{a}{1-a^2+E_{11}-aE_{21}+ar} \tag{23}$$

$$f_2(r) = \frac{1}{1-a^2+E_{21}-aE_{11}-r} \tag{24}$$

To solve this, we first assume $\lambda_r = \eta_r = 0$, and equate both sides of (22). The existence of a feasible solution of r in this case depends on the extreme values of f_1 and f_2. In particular, since $f_1(r)$ is decreasing in r, while $f_2(r)$ is increasing in r, the solution exists if and only if $f_1(r_2) \le f_2(r_2)$ and $f_1(r_1) \ge f_2(r_1)$. Note that such solution can be found, for example, by a bisection search. If this condition is not satisfied, then one of the Lagrange multipliers needs to be strictly positive in order to equate both sides. In particular, if $f_1(r_2) > f_2(r_2)$, then we need $\lambda_r > 0$, which implies by complementary slackness that $r = r_2$. On the other hand, if $f_1(r_1) < f_2(r_1)$, then we need $\eta_r > 0$, which implies by complementary slackness that $r = r_1$. After solving for r, we check if it is consistent with $\delta(r)$ by checking the conditions in (21). If not, then we check the other two cases: $\delta(r) = 0$ and $\delta(r) = E_{22}+r$, and re-solve for r. The analysis in these cases follows similarly as above. This concludes the solution of the two-slot case.

In the next section, we use the above analysis to find the optimal solution in the general case of multiple energy arrivals.

B. Iterative Solution for the General Case

We solve problem (11) iteratively in a two-slot by two-slot manner, starting from the last two slots and going backwards. Once we reach the first the two slots, we re-iterate starting from the last two slots, and go backwards again. These iterations stop if the policy does not change after we reach the first two slots. The details are as follows.

We first initialize the energy state of each slot of both users by $\mathbf{S}_1 = \mathbf{E}_1$ and $\mathbf{S}_2 = \mathbf{E}_2$, and solve each slot independently, as discussed in Section II, to get an initial feasible power policy $\{\mathbf{p}_1^{(0)}, \mathbf{p}_2^{(0)}\}$. We then start by examining slots $N-1$ and N. We solve the throughput maximization problem for these two slots with energies $\{S_{1(N-1)}, S_{1N}\}$ and $\{S_{2(N-1)}, S_{2N}\}$ at the first and second users, respectively, as discussed in Section III-A. After we solve this problem, we update the energy state vectors $\mathbf{S}_1$ and $\mathbf{S}_2$, and move back one slot to examine slots $N-2$ and $N-1$. We solve the throughput maximization problem for these two slots using the updated energy state $\{S_{1(N-2)}, S_{1(N-1)}\}$ and $\{S_{2(N-2)}, S_{2(N-1)}\}$ at the first and second users, respectively. We update the energy state vector after solving this problem, and continue moving backwards until we solve for slots 1 and 2. After that, we get another feasible power policy $\{\mathbf{p}_1^{(1)}, \mathbf{p}_2^{(1)}\}$, where the superscript stands for the iteration index. We then compare this power policy with the initial one. If they are the same, we stop. If not, we perform this process again starting from the last two slots, going backwards, until we get an updated power policy $\{\mathbf{p}_1^{(2)}, \mathbf{p}_2^{(2)}\}$. We stop after the kth iteration if $\mathbf{p}_1^{(k-1)} = \mathbf{p}_1^k$ and $\mathbf{p}_2^{(k-1)} = \mathbf{p}_2^k$. Since the sum throughput can only increase with the iterations, and since it is also upper bounded due to the energy constraints, the convergence of the above two-slot iterations is guaranteed.

Next, we check whether the limit point satisfies the KKT optimality conditions. Namely, we solve for the Lagrange multipliers in (13) and (14). If they are all non-negative, then the KKT conditions are satisfied and by the convexity of the problem, the limit point is optimal [33]. If not, then the energy state vectors need to be updated. This might be the case for instance if while updating some given two slots, more than necessary amount of energy is transferred forward. While this may be optimal with respect to these two slots, it does not

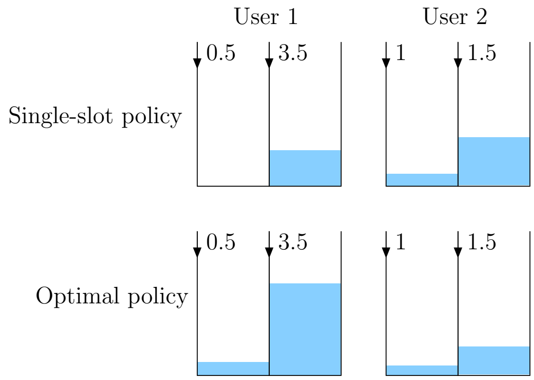

Fig. 2. Two-slot numerical example.

take into consideration the energy arrival vectors in the entire N slots. Therefore, in such cases, we perform another round of iterations where we take some of the energy back if this increases the objective function. Taking energy back without violating causality can be done, e.g., via putting measuring meters in between the slots during the two-slot update phase to record the amount of energy moving forward [18]. Since the problem feasibility is maintained with each update, and by the convexity of the problem, if we cycle through all the slots infinitely often, iterations converge to the optimal policy.

IV. Numerical Results

In this section, we provide numerical examples to illustrate the previous analysis. First, we consider a two-slot system with energy arrivals $\mathbf{E}_1 = [0.5, 3.5]$ and $\mathbf{E}_2 = [1, 1.5]$, for the first and the second user, respectively. The decoding power factor is equal to $a = 0.5$. We first solve for each slot independently using the single arrival result to get $\mathbf{p}_1 = [0, 1]$ and $\mathbf{p}_2 = [0.33, 1.33]$. Then, we find the optimal solution as discussed in Section III-A. First, we check the constant-power strategy, where neither user consumes its energy in the first slot, and solve a single arrival problem with average energy arrivals $\bar{E}_1 = 2$ and $\bar{E}_2 = 1.25$ to get $\bar{p}_1 = 1.75$ and $\bar{p}_2 = 0.375$, which are found infeasible. Thus, we move to check on the second consumption strategy: the first user consumes all energy in the first slot while the second user consumes all energy in the second slot, i.e., we solve problem (17). We first remove the last constraint, and take $\delta(r) = \frac{1+E_{22}+r-a}{2}$, the middle term of (21), and solve for r using (22). This gives $r = 0.55$, which satisfies the middle constraint in (21), thus the assumed $\delta(r)$ is correct, and gives $\delta = 1.27$. Finally, we check the relaxed (last) constraint of (17); we find that it is satisfied with strict inequality. Therefore, $(r^* = 0.55, \delta^* = 1.27)$ is the optimal solution for this consumption strategy. The corresponding powers are given by $\mathbf{p}_1 = [0.36, 2.55]$ and $\mathbf{p}_2 = [0.26, 0.77]$. Next, we check the other strategies. Among the feasible ones, we find that the maximum throughput is given by that of the second strategy above, and is therefore the optimal solution of this two-slot system.

In Fig. 2, we show the single-slot solution on the top and the optimal solution on the bottom of the figure. The height of the water in blue represents the power level of a user in a given slot. We note that the first user's optimal power in the first slot is larger than the corresponding single-slot power allocation. That is because the second user's optimal power is smaller than the single-slot power allocation, which gives more room for the first user to transmit. This shows how decoding costs closely couple the performance of the two users.

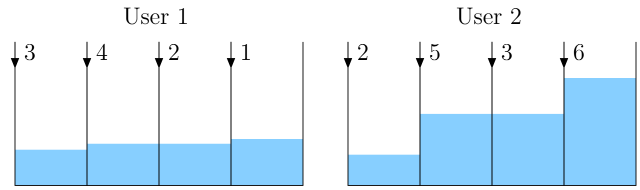

Fig. 3. Optimal power allocation for a four-slot system.

Next, we consider a four-slot system with $\mathbf{E}_1 = [3, 4, 2, 1]$, $\mathbf{E}_2 = [2, 5, 3, 6]$, and $a = 0.1$. We begin by solving the optimal powers for slots 3 and 4. Then, fixing slot 4, we solve for slots 2 and 3, followed by slots 1 and 2. This is considered one iteration of the algorithm. Whenever there is a residual energy transferred from a slot to the next we perform another iteration. The process is terminated when we converge to a point, i.e., further iterations do not change the power allocations. At the convergence point, the limit powers are given by $\mathbf{p}_1^* = [2, 2.14, 2.14, 2.2]$ and $\mathbf{p}_2^* = [1.8, 3.79, 3.79, 5.78]$, which admit a non-negative solution for the Lagrange multipliers in (13) and (14), and are therefore optimal. We plot the optimal power allocations in Fig. 3. The powers of the two users are non-decreasing as shown in Lemma 2, and they increase simultaneously, as shown in Lemma 4. Also, since some energy is transferred during the iterations from slot 2 to 3 at both users, the powers in these slots are equal. We plot the squared error $\|\mathbf{p}^{(k)} - \mathbf{p}^*\|_2^2$ in Fig. 4, and the the sum throughput in Fig. 5 over the iterations for this case. We observe that the error decreases monotonically and the sum throughput increases monotonically with iterations.

To show the possible necessity of taking some of the energy back after the two-slot updates, we consider another four-slot system with energies $\mathbf{E}_1 = [0.9, 0.1, 3, 0.8]$ and $\mathbf{E}_2 = [0.8, 1.5, 2, 2]$; decoding cost parameter $a = 0.7$. The two-slot updates converge to powers $\mathbf{p}_1 = [0.24, 0.24, 0.82, 0.84]$ and $\mathbf{p}_2 = [0.31, 0.31, 1.56, 1.63]$. However, this does not admit non-negative Lagrange multipliers solution in (13) and (14), and therefore we go through another round of iterations where energy can be taken back to the extent allowed by the meters. Iterations converge to the optimal powers $\mathbf{p}_1^* = [0.1, 0.1, 0.8, 0.8]$ and $\mathbf{p}_2^* = [0.57, 0.57, 1.57, 1.57]$.

V. Conclusions

We considered throughput maximization for an energy harvesting two-way channel with decoding costs. In this system, each user is an energy harvesting transmitter and an energy harvesting receiver. The users need to allocate their harvested energies for transmission power and decoding power. We determined the optimal power control policies for both users

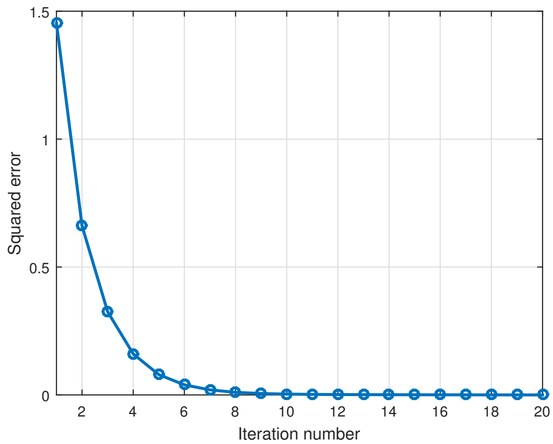

Fig. 4. Convergence of the squared error from the optimal powers.

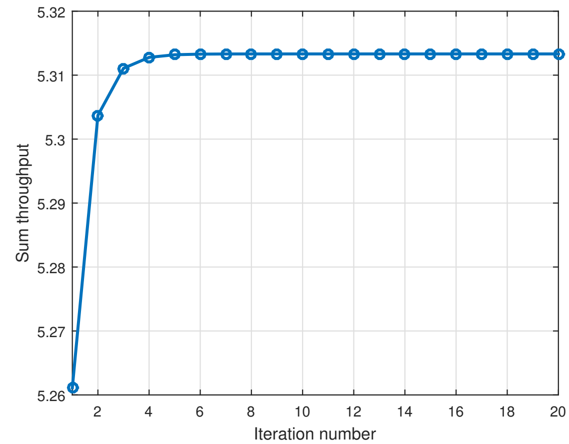

Fig. 5. Convergence of the sum throughput.

under energy causality constraints. We showed that for the single energy arrival case, the communication is limited by the user with smaller energy. In the multiple energy arrival case, we showed that the transmission powers of both users are non-decreasing over time and increase synchronously. We provided an iterative algorithm to obtain the optimal power allocations based on two-slot updates.

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