Age of Information in Mobile Networks: Fundamental Limits and Tradeoffs

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ABSTRACT

Age of information (AoI), defined for an information source as the time elapsed since the latest received update was generated, is a recently proposed metric that quantifies the timeliness of information delivery in a communication system. This paper studies a fundamental problem of how the achievable AoI scales in mobile networks. Specifically, we consider a network consisting of $n/2$ source-destination (S-D) pairs and employ the protocol model to characterize interference incurred by concurrent transmissions. We consider a general class of scheduling policies potentially with the multi-hop transmission and the duplication of packets to multiple nodes. The analysis of AoI faces significant challenges due to potential out-of-order packet delivery, the inherent tradeoffs between packet-centric metrics (throughput and delay), and their unexplored relation to AoI. We first show that the average per-node AoI in static settings scales as $\Omega\left(\sqrt{n\log(n)}\right)$. In the case of networks with i.i.d. mobility, where the node locations vary independently over time, we introduce an *episodic technique* that allows us to establish lower bounds and design and analyze scheduling policies as constructive upper bounds. Our analytical results reveal that the average per-node AoI scales as $\tilde{\Theta}(n^{1/4})$ under i.i.d. mobility, which highlights that mobility can enhance timeliness. Finally, we show that, in a more general class of wireless network settings, one can design the age-minimal scheduling policy by balancing throughput and delay.

CCS CONCEPTS

• **Networks** → *Network performance modeling*; **Network performance analysis**; **Mobile ad hoc networks**.

KEYWORDS

Age of Information, Mobile ad hoc Networks

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1 INTRODUCTION

1.1 Motivations

Timeliness is an ever-increasing requirement in various real-time applications, ranging from healthcare, autonomous driving, and finance, to the Internet of Things (IoT), virtual reality, and metaverse. IoT, for instance, involves a plethora of sensors and devices that continuously generate and transmit data, which must be processed and analyzed in real time to enable intelligent decisions and actions. Similarly, autonomous driving relies on the timely communication of data between vehicles to ensure safe and efficient transportation.

These real-time systems have been raising significant research interest in *timeliness of status updates*, demanding a new network performance metric in addition to conventional network metrics including throughput and delay. This is because delay and throughput are insufficient to gauge the timeliness of updates since they are packet-centric measures that fail to capture the timeliness of information. To this end, researchers have proposed *Age of Information (AoI)* as a novel fundamental metric to quantify the freshness of data collected [1]. This metric provides an end-to-end characterization of latency in real-time systems and captures the information freshness from the *perspective of a destination.*

Real-time applications in which age is an important metric involve wireless networks, and interference constraints pose a primary limitation to system performance. Therefore, effective interference mitigation and efficient resource allocation and transmission scheduling are necessary to ensure continuous monitoring and timely updates in wireless networks. With the increasing complexity of wireless networks, it is crucial to improve our understanding of the fundamental performance and theoretical guidance on age-minimal design and operation. To this end, this paper tries to answer a fundamental question remaining in large-scale wireless networks:

Question 1. *How fresh can information in a wireless network be, and how can the design of scheduling policies be optimized to enhance information freshness?*

To answer this question, this paper primarily aims to establish the connection between the analysis and optimization of AoI and the methodology and results from a considerably large body of literature focusing on the fundamental throughput and delay scaling in wireless networks, specifically *mobile ad hoc networks* (e.g., [2–18]). Gupta and Kumar first in their seminal paper [3] showed that in a static networks with n mobile nodes distributed independently and uniformly, the per-node maximal throughput scales as $\Theta\left(1/\sqrt{n\log(n)}\right)$.[1] In addition, their findings highlighted the importance of user cooperation (via multi-hop transmission) in mitigating interference.

Node mobility can significantly improve the capacity of wireless ad hoc networks. Grossglauser and Tse in [4] showed that the per-node capacity of $\Theta(1)$ is achievable, however, at the cost of excessive network delay. In the sequel, a large body of literature studied the optimal tradeoffs between the achievable throughput and the packet delay in mobile ad hoc networks (e.g., [4–16]). This line of work also motivates the following key question:

Question 2. *Can mobility improve age of information in wireless networks and, if so, by how much?*

1.2 Challenges

To understand the principal challenges of addressing the above questions, we highlight key gaps in directly applying existing results to provide an answer.

- *Interdependence of throughput and packet delay:* In large-scale wireless networks, the pursuit of high throughput and low delay poses an inherent conflict due to interference. Typically, achieving high throughput results in increased delay [4–16]. This interdependence between the frequency of update packet generation and the time required for their delivery significantly complicates the design of scheduling policies. In contrast, many existing works based on queueing theory assume the statistical independence between packet arrival rates and service times[2], omitting important details of the status update delivery for wireless networks.
- *Out-of-order and size-variant packets*: The literature on scaling laws for wireless networks primarily centered around packet-centric delays. However, node mobility may incur out-of-order and hence non-informative packets. In addition, many studies assumed variant packet sizes as the number of nodes increases (e.g., [2, 5–9, 11–14]). Both issues render the packet delay not directly relevant to AoI and complicates the analysis.

1.3 Our Approaches and Contributions

In this paper, we analyze the fundamental limits of AoI scaling in mobile ad hoc networks. We consider a network consisting of n mobile nodes and adopt the interference protocol model, for both the static and independent and identically distributed (i.i.d.) mobility models. We employ a general class of policies with potential multi-hop transmissions and packet duplication. We propose an episode technique for deriving the lower bounds and the upper bounds are constructive. Our main contributions are in the following:

- *Problem Formulation:* To the best of our knowledge, this is the first work studying the fundamental limits of AoI scaling in large mobile (ad hoc) networks considering the impact of interference and mobility.
- *Fundamental Limits:* In the *static model*, we show that the achievable AoI scales as $\Omega(\sqrt{n\log(n)})$. In the *i.i.d. mobility model*, we introduce an episodic technique that allows us to establish lower bounds and design and analyze scheduling policies as constructive upper bounds. Our analytical results reveal that the average per-node AoI scales under i.i.d. mobility as $\tilde{\Theta}(n^{1/4})$, where $\tilde{\Theta}$ ignores logarithmic terms. These bounds reveal that *mobility improves AoI in wireless systems.*
- *Three-way Tradeoffs:* For a more general network setting, we establish the connection between AoI and packet-centric metrics. Our results show that the design of age-minimal scheduling policies involves making a specific tradeoff between throughput and delay.

2 LITERATURE REVIEW

2.1 Optimal Throughput-Delay Scaling in Wireless Networks

Albeit a large body of literature have studied the fundamental limits of wireless network throughput and the optimal throughput-delay tradeoffs (see [2–17] and a survey in [2]).

Static Networks: Gupta and Kumar studied a static wireless network model in their seminal paper [3], showing that the throughput scales as $\Theta(1/\sqrt{n\log n})$; they also reveal the importance of cooperation (via relaying). Gamal *et al.* studied the optimal throughput-delay tradeoffs [5, 10].

Mobile Networks: Grossglauser and Tse in [4] showed that node mobility dramatically improves throughput scaling. Subsequent efforts have been made to comprehend the optimal throughput-delay tradeoffs in various mobile settings, including the i.i.d. mobility model (e.g., [6–9]) and other more sophisticated and practical mobility models, such as the random walk models (e.g., [5, 12–14]).

The above line of work did not consider AoI, and applying existing studies directly to our setting is difficult. Specifically, while most existing studies assume diminishing-size packets, our analysis focuses on the constant-size-packet scenario. Furthermore, we delve into the convergence error to establish a more stringent bound.

2.2 Age of Information

In recent years, many works on AoI conducted analysis and designed optimization algorithms for queueing systems (e.g., [20–24, 46, 47]) and wireless networks (see references [19, 25–40, 43–45] and a survey in [1]). In the wireless network settings, various studies either considered two nodes (e.g., [27, 31, 36, 37]) or adopted interference-free assumptions (e.g., [32, 35, 39, 45, 47]) or simplified interference assumptions (e.g., no two nodes can transmit simultaneously [25, 44]). Notable examples include the scheduling policy design for wireless networks with throughput constraints (e.g., [30]), general interference constraints (e.g., [29]), and broadcast networks (e.g., [38]). *These existing studies did not consider the fundamental problems of the scaling limits of AoI in wireless networks with interference. In addition, while existing scheduling policies mainly*

[1]Recall the following notation: i) $f(n) = O(g(n))$ indicates that there exists a positive constant c and an integer N such that $f(n) \le cg(n)$ for $n > N$; ii) $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$; iii) $f(n) = \Theta(g(n))$ means that $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$.

[2]Although it is widely acknowledged that minimizing AoI in queueing systems also requires a balance between queueing delay and throughput (see [1, Sec. III-A]), it is important to note that in context of queueing theory, the service time is equivalent to the packet delay in our wireless network setting, rather than the queueing delay.

concentrated on single-hop settings (e.g., [25, 30–32, 36, 38–41, 44]), it is essential to exploit the multi-hop transmissions in order to minimize AoI.

In a closely related study [43], Buyukates *et al.* employed hierarchical cooperation [16] and conducted an analysis of AoI scaling in wireless networks. We would like to highlight one of the key distinctions between our work and this aforementioned study: Our main objective is to study the impacts of interference and mobility without relying on complex signal processing algorithms for multiple-input multiple-output (MIMO) communications. In addition, the study in [43] adopted the i.i.d. phase assumption initially introduced by [16]. This assumption has been identified as strong and may violate the laws of physics, as discussed in [17].

3 MODEL, METRICS, AND MAIN RESULTS

In this section, we present the wireless network models, define the metrics, and summarize the main results.

3.1 Model Overview

Mobile *ad hoc* networks: Consider a mobile *ad hoc* network, in which an even number of n single-antenna mobile nodes are positioned in a unit torus. Mobile nodes constitute $n/2$ distinct source-destination (S-D) pairs at random. We assume the area of the square is one unit size without loss of generality. Sources generate status updates that are carried by packets, and send to their destinations. One packet consists of L bits, which is scale-invariant (independent of n). Each node potentially serves as a *relay* for other S-D pairs.

We slot time for packetized transmission and consider an infinite-horizon model, in which we index each time slot by t and the system starts at $t = 0$.

We study the two mobility models as follows to characterize the dynamics of mobile nodes:

- *Static Model:* The locations of mobile nodes are positioned on the unit square uniformly at random and stay unchanged for the entire time horizon.
- *I.I.D. Mobility Model [6–9]:* At each time slot, mobile nodes are uniformly and randomly positioned on the unit square. Node positions vary *independently* across time slots, i.e., we reshuffle node locations completely after each time slot.

As the first work aims to understand the impact of mobility on age scaling in wireless networks with interference, we also stick to the i.i.d. mobility model, as this model has been widely adopted to acquire insights for more realistic and practical mobility models (i.e., random walk mobility [5, 12] and Levy mobility [14]). Additionally, we note that several key results do not rely on the assumption of i.i.d. mobility, which demonstrates the potential for extending towards the general mobility settings.

Status update models: We consider a *generate-at-will model* (as in, e.g., [27, 28, 31, 32, 36–42]), in which each source is able to generate a (status update) packet and send a packet to the dedicated destination. Let $t_{i,k}$ denote the generation time of the k-th packet for S-D pair i. Let $t'_{i,k}$ denote the delivery time when the k-th packet is received for S-D pair i. Notably, these packets may not arrive at their intended destinations in the order they were generated.

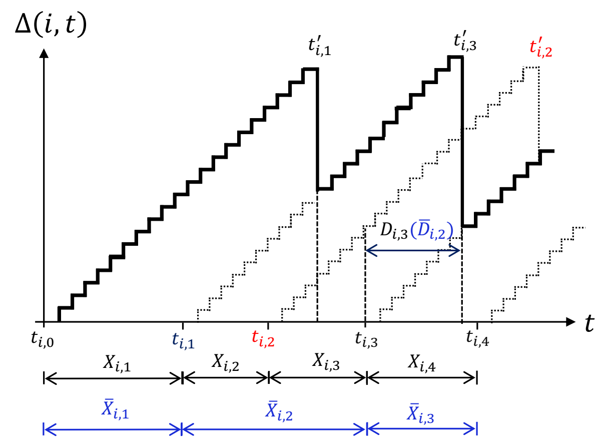


Figure 1: Age evolution in time. Update packets are generated at times $t_{i,k}$ and delivered at times $t'_{i,k}$. Packet 2 is out of order.

Communication models: We employ the *protocol interference* model (as in [3, 5–14]). Specifically, a transmission from source i to destination j in a time slot is successful if, for any other node k that is transmitting simultaneously, the following is true:

$$\text{dist}(k, j) \geq (1 + \gamma)\text{dist}(i, j), \quad (1)$$

for some coefficient $\gamma > 1$, where $\text{dist}(i, j)$ stands for the distance between nodes i and j. During a successful transmission, data is sent at a constant rate of W bits (or W/L packets) per time slot.[3]

Scheduling policies: We focus on *causal* scheduling policies, making decisions based only on current and past information. A source can generate and transmit packets directly to its destination if the destination is in proximity; otherwise, packets will go through *multiple hops*, with other nodes serving as relays. In addition, for mobile networks, one can *duplicate* packets to multiple nodes so that some can take advantage of their proximity and forward packets to the destination. This class of policies is considered very general [7].

Definition 1 (Policy). *A policy Π_n is a sequence of communication policies, $\{\Pi_n\}$, where policy Π_n determines how communication takes place in a network of n nodes.*

3.2 Performance Metrics

Throughput and Delay: We consider the following two conventional metrics to evaluate the network performance:

Definition 2 (Throughput). *Let $B_{\Pi_n}(i, t)$ be the total number of bits transmitted over S-D pair i up to time t. Policy Π achieves a per-node throughput of $\lambda_\Pi(n)$ (almost surely) if*

$$\Pr\left[\sum_{i=1}^{n/2} \liminf_{t\to\infty} \frac{1}{t} B_{\Pi_n}(i, t) \geq n\lambda_\Pi(n)\right] = 1. \quad (2)$$

Definition 3 (Delay). *The delay of a packet is the time it takes for the packet to reach its destination since its transmission (i.e., upon its*

[3]Another other commonly used model is the Physical model (e.g., [3, 4]), in which a transmission is successful if the signal-to-interference-and-noise ratio (SINR) exceeds some constant. The protocol model and the physical model are equivalent that if signal decays with distance and sources use the same transmit power [3]. We will thus stick to the Protocol model for the rest of this paper.

generation under our considered generate-at-will model). Let $D_{i,k} \triangleq t'_{i,k} - t_{i,k}$ denote the delay of packet k of S-D pair i. We define the average delay as

$$D_{\Pi}(n) = \limsup_{K\to\infty} \mathbb{E}\left[\frac{2}{nK}\sum_{i=1}^{n/2}\sum_{k=1}^{K} D_{i,k}\right]. \tag{3}$$

The throughput and delay metrics may exhibit randomness due to the inherent uncertainty associated with node locations and the utilization of randomized policies.

Age of Information: Age of packet k is defined as the time since it was generated: $A_i(k,t) = (t - t_{i,k})$ for all $t \geq t_{i,k}$. The AoI of each S-D pair i at time t is defined as [1, 20]:

$$\Delta(i,t) \triangleq \min_{k\in\mathcal{P}_i(t)} A_i(k,t),\ \forall i \in [n/2], \tag{4}$$

where $\mathcal{P}_i(t)$ denotes the set of packets received by the destination of S-D pair i, up to time t. Note that the AoI increases linearly and only drops at the times of certain packet receptions. Each S-D pair's age drops only when the destination receives an update packet fresher than all packets received thus far. As depicted in Fig. 1, the AoI drops at $t'_{i,1}$ and $t'_{i,3}$ but not at $t'_{i,2}$. We refer to these packets contributing to age drops as *informative packets* [20].

The following definitions characterize the overall timeliness performance of status updates in a wireless network.

Definition 4 (Time-Average Age). *Under Policy Π, the time-average age for each S-D pair i is given by*

$$\Delta_{\Pi,i}^{(ave)} \triangleq \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T} \mathbb{E}_{\Pi}\left[\Delta(i,t)\right],\ \forall i \in [n/2], \tag{5}$$

and the network's (long-term) average age is

$$\Delta_{\Pi}^{(ave)}(n) \triangleq \frac{2}{n}\sum_{i=1}^{n/2} \Delta_{\Pi,i}^{(ave)}. \tag{6}$$

Note that AoI $\Delta(i,t)$ and the average age $\Delta_{\Pi}^{(ave)}(n)$ are defined from the perspective of destinations. Hence, it is only the delays of *informative* packets that are crucial for computing the average age. This differs substantially from classical packet-centric average delay that accounts for every packet in the networks equally [20].

We further introduce the notation related to informative packets. For each S-D pair i, let $\bar{X}_{i,k}$ be the inter-arrival time between the $(k-1)$-th and k-th informative update packets and let $\bar{D}_{i,k}$ denote the transmission delay for the k-th informative packet. The average age for pair i can be expressed as [1][4]

$$\Delta_{\Pi,i}^{(ave)} = \liminf_{K\to\infty} \frac{\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}[\bar{X}_{i,k}\bar{D}_{i,k} + \bar{X}_{i,k}^2/2]}{\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}[\bar{X}_{i,k}]}. \tag{7}$$

We further use $\lambda_{\Pi}^{I}(n)$ and $D_{\Pi}^{I}(n)$ to denote *the throughput and the delay of informative packets*, respectively, defined in a similar way as Definitions 2 and 3.

[4] We drop a term $-1/2$ that arises from the difference between the discrete-time AoI and the continuous-time AoI.

3.3 Main Results

In this paper, we aim to understand the minimal average age in both static networks and i.i.d. mobility models

$$\Delta^{(ave),*}(n) \triangleq \min_{\Pi\in\Phi} \Delta_{\Pi}^{(ave)}(n), \tag{8}$$

where Φ stands for the feasible set of policies (satisfying (1)).

We will present several key results of this paper that capture the optimal age scaling and its connection to various tradeoffs inherent in mobile ad hoc networks.

In the static networks, we can show the following lower bound, with the sketch of proof in Appendix A.

Theorem 1. *In a static random network with $n/2$ pairs of S-R nodes, the minimal average age satisfies*

$$\Delta^{(ave),*}(n) = \Omega\left(\sqrt{n\log(n)}\right). \tag{9}$$

In this paper, our primary focus will be on examining the fundamental limit of the average age under the i.i.d. mobility model:

Theorem 2. *In a wireless network with i.i.d. mobility, the minimal average age satisfies*

$$\Delta^{(ave),*}(n) = \tilde{\Theta}\left(n^{\frac{1}{4}}\right). \tag{10}$$

The notation $\tilde{\Theta}(\cdot)$ ignores logarithmic terms in comparison with $\Theta(\cdot)$.[5] Theorem 2 implies that mobility improves AoI.

To generalize our results and garner more insights to a wider range of network settings, we study the age-throughput-delay tradeoffs via the following result:

Theorem 3 (Age-Throughput-Delay Relation (Informal)). *In a general class of wireless networks with $n/2$ pairs of S-R nodes, the optimal age-throughput-delay tradeoff satisfies*

$$\Delta^{(ave)}(n) = \Theta\left(\frac{1}{\lambda(n)} + D(n)\right). \tag{11}$$

Theorem 3 indicates that the design of age-minimal scheduling policies involves striking a specific balance between throughput and delay. The seemingly intuitive result in Theorem 3 is considered surprising, as it reveals the underlying connection between average age and packet-centric metrics in wireless networks, which is not directly observable due to the presence of the non-informative packet delivery.

4 MOBILE NETWORKS

In this section, we study the average age scaling in a random mobile network under the i.i.d. mobility model. The challenges in analysis main arise from potential non-informative packets.

[5] That is, $f(n) = \tilde{\Theta}(g(n))$ if there exist real constants E_1, E_2 such that $f(n) = O(g(n)\log^{E_1}(n))$ and $f(n) = \Omega(g(n)\log^{E_2}(n))$

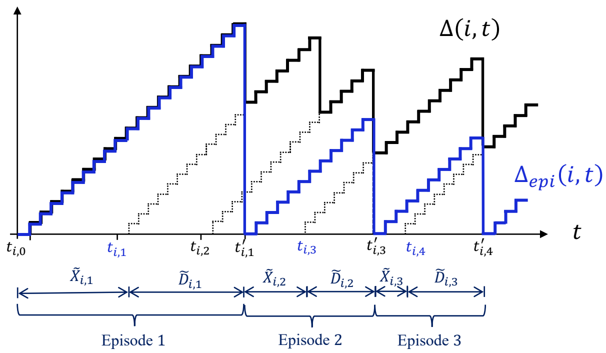


Figure 2: Illustration of the notion of episodes. The episode AoI $\Delta_{\text{epi}}(i,t)$ is below the actual AoI $\Delta(i,t)$, demonstrating a method to derive the lower bound of the average age.

4.1 Lower Bound

Establishing the lower bound of $\Delta^{(ave),*}(n)$ under the i.i.d. mobility model involves several key steps, including showing the optimality of a stationary policy, proposing an episode technique, and analyzing a submartingale. For the ease of exposition, we introduce the following notation.

4.1.1 Notation. **Episode:** We consider partitioning the entire time horizon into episodes, each of which is associated with the first informative packet generated during that episode. In particular, each episode e starts immediately after episode $e-1$ ends, and continues until the first informative packet generated within episode e is delivered. We therefore term the first informative packet generated within episode e as *the principal packet.* Let $\tilde{X}_{i,e}$ denote the time between the start of episode e and the generation of its principal packet and $\tilde{D}_{i,e}$ denote the delay of the principal packet of episode e, as illustrated in Fig. 2. Let $\tilde{L}_{i,e} = \tilde{X}_{i,e} + \tilde{D}_{i,e}$ denote the duration of episode e. We will prove that the average age of S-D pair i can be bounded from below by $\mathbb{E}_{\Pi}[\tilde{L}_{i,e}]$.

Due to the wireless resource constraint, each source may need to wait for a few time slots before it starts generating, transmitting, and duplicating packets. Let $G(i,e)$ be the time stamp when source i starts the generation of update packets for each episode; let $\mathcal{P}_{i,e}$ denote the set of all updates generated and transmitted by node i during episode e.

History: Let $I_p(j,t)$ be an indicator function, where $I_p(j,t)=1$ if node j has a copy of packet p at the beginning of time t, and $I_p(j,t)=0$ otherwise. Let $\mathcal{F}_t$ be the σ-algebra generated by the random variables $X(i,s)$, $I_p(i,s)$, $\Delta(i,s)$, $\{A_i(k,s)\}_k$ for all $s \le t$. Hence, $\{\mathcal{F}_t, t=0,1,\cdots\}$ is a filtration [49, Chap. 4.2] and $\mathcal{F}_t$ captures all historic information up to time t.

4.1.2 Markov Decision Process. The i.i.d. mobility nature enables us to formulate the average age minimization problem in (8) as a *Markov decision process (MDP).* The policy Π is a mapping from the history space to the action space. Further, a stationary policy is a policy that only depends on the *current state* $\mathcal{F}_t$. A desirable property is the existence of an optimal policy that is stationary (e.g., [50]), which is true for many important classes of MDPs.

Lemma 1 (Optimality of a Stationary Policy). *In the i.i.d. mobility model, there exists a stationary policy Π that is an optimal policy to the minimal average age problem in* (8).

Proof. Using the instantaneous AoI in (5) as rewards, we can see (8) is an average-cost MDP problem. From [48, Theorem 8.1.2], for infinite horizon average-cost MDP with state space that is countable and rewards and transition probabilities that are independent of the state, there always exists a stationary policy that is an optimal policy. □

We will see that it is useful to apply Lemma 1 to simplify the expression of the average age, for deriving its lower bound.

4.1.3 Episode Technique. To derive a lower bound of $\Delta(i,t)$, we consider the following definition of the *episode AoI*:

$$\Delta_{\text{epi}}(i,t) \triangleq t - \tilde{u}_i(t), \tag{12}$$

where $\tilde{u}_i(t)$ is time stamp of the most recent episode that starts before time t.

Intuitively, the episode AoI $\Delta_{\text{epi}}(i,t)$ can be understood as the AoI experienced when the destination assumes it receives a fresh update at the start of each episode, resulting in AoI drops to zero. Therefore, one immediate observation is that, for any realization of the generation times $\{t_{i,k}\}$ and the delivery times $\{t'_{i,k}\}$ generated by an arbitrary Π, we have the following:

$$\Delta(i,t) \ge \Delta_{\text{epi}}(i,t), \quad \forall i \in [n/2], \forall t, \tag{13}$$

as illustrated in Fig. 2. We are really to introduce the following principle proposition of the episode technique:

Proposition 1. *Under the optimal policy Π^*, the average age for each S-D pair $i \in [n/2]$ satisfies*

$$\Delta^{(ave)}_{i,\Pi^*} \ge \frac{1}{2}\mathbb{E}_{\Pi^*}\left[\tilde{L}_{i,e}\right]. \tag{14}$$

Proof. Since (13) holds for any realization $\{t_{i,k}\}_{i,k}$ and $\{t'_{i,k}\}_{i,k}$, it also holds when we take the expected values of both sides. Specifically, for all S-D pairs $i \in [n/2]$,

$$\begin{aligned}
\Delta^{(ave)}_{i,\Pi^*} &= \lim_{K\to\infty} \frac{\mathbb{E}_{\Pi^*}\left[\sum_{k=1}^{K}\left(\frac{1}{2}X_{i,k}^2 + X_{i,k}D_{i,k}\right)\right]}{\mathbb{E}_{\Pi^*}\left[\sum_{k=1}^{K}X_{i,k}\right]} \\
&= \frac{\mathbb{E}_{\Pi^*}\left[\left(\frac{1}{2}X_{i,k}^2 + X_{i,k}D_{i,k}\right)\right]}{\mathbb{E}_{\Pi^*}\left[X_{i,k}\right]} \text{ (Lemma 1)} \\
&\ge \frac{\mathbb{E}_{\Pi^*}\left[\frac{1}{2}(\tilde{X}_{i,e}+\tilde{D}_{i,e})^2\right]}{\mathbb{E}_{\Pi^*}\left[\tilde{X}_{i,e}+\tilde{D}_{i,e}\right]} \\
&\ge \frac{1}{2}\mathbb{E}_{\Pi^*}\left[\tilde{X}_{i,e}+\tilde{D}_{i,e}\right] \text{ (Jensen's Inequality)} \\
&= \frac{1}{2}\mathbb{E}_{\Pi^*}\left[\tilde{L}_{i,e}\right].
\end{aligned} \tag{15}$$

□

The inequality in (15) implies that the average age for each S-D pair i is bounded by the expected duration of an episode.

Remark 1. *The proof of Proposition 1 only requires the condition of the optimality of a stationary policy, and hence is not limited to the i.i.d. mobile networks. It implies that the potential of analyzing* *the optimal AoI scaling in other settings (e.g., the Brownian mobility [12]) based on Proposition 1.*

4.1.4 Submartingale Analysis. Let $C_{i,e}(t)$ denote the indicator representing whether a packet generated in episode e is delivered for S-D pair i. We define the *stopping time* with respect to the filtration $\{\mathcal{F}_t, t = 0, 1, \cdots\}$ as

$$s_{i,e} \triangleq \{t : t \geq G(i,e) \text{ and } C_{i,e}(t) = 1\}, \tag{16}$$

which stands for the time when episode e of the S-D pair i terminates. Let $R_p \triangleq r_p(s_{i,e})$ denote the number of mobile relays holding packet p at the time of capture. Let $Y_{i,e} \triangleq G(i,e) - s_{i,e}$ denote the *active duration* of episode e for S-D pair i. Let $l_{i,e}$ denote the shortest distance from any of the mobile relays carrying any packet in $\mathcal{P}_{i,e}$ to the destination at $s_{i,e}$. We now present the essential tradeoffs among these notions.

Lemma 2. *Under the i.i.d. mobility model, the following inequality holds for any policy* $\Pi \in \Phi$ *when* $n \geq 3$,

$$8\pi \log(n)\mathbb{E}_\Pi[Y_{i,e}] \geq \frac{1}{(\mathbb{E}_\Pi[l_{i,e}] + \frac{1}{n^2})^2 \sum_{p\in\mathcal{P}_{i,e}} \mathbb{E}_\Pi[R_p]}. \tag{17}$$

Sketch of Proof. Let $L_e(t)$ denote the minimum distance of any packet $p \in \mathcal{P}_{i,e}$ at time t. Generalizing the argument in [7, Lemma 10], we can show that, for all $t \geq G(i,e)$,

$$\mathbb{E}\left[\frac{1}{\max\{\frac{1}{n^2}, L_e(t)\}^2 \sum_{p\in\mathcal{P}_{i,e}} r_p(t)}\middle|\mathcal{F}_{t-1}\right] \leq 8\pi\log(n). \tag{18}$$

We consider the following quantity:

$$V_t = 8\pi \log(n)[t - G(i,e)] \tag{19}$$
$$- \sum_{s=G(i,e)+1}^{t} \frac{1}{L_e(t)^2 \sum_{p\in\mathcal{P}_{i,e}} r_p(t)} \mathbb{I}\{C_{i,e}(t) = 1\}.$$

Eq. (18) implies V_t is a *sub-martingale*, i.e., $\mathbb{E}[V_t|\mathcal{F}_{t-1}] \geq V_{t-1}$. Invoking the Optional Stopping Theorem leads to $\mathbb{E}[V_{s_{i,e}}] \geq 0$ [49, Theorem 4.1]. Finally, applying the Hölder's Inequality completes the proof. □

Based on Lemma 2, duplicating packets to multiple nodes could be advantageous for attaining the lower bound of $\mathbb{E}_\Pi[Y_{i,e}]$, as it increases the chances of opportunistic packet relay by at least one of the nodes. It follows that:

Proposition 2. *Under the i.i.d. mobility model, the average age satisfies*

$$\Delta^{(ave),*}(n) = \Omega\left(n^{1/4}\log^{-3/4}(n)\right). \tag{20}$$

Sketch of Proof. The protocol interference model requires that the ideal disks of radius $\gamma/2$ times the transmission range centered at the transmitter are disjoint from each other. Therefore, based on a similar technique as in [7], We can then compute the total area covered by all the disks as follows.

$$\sum_{e=1}^{E}\sum_{p\in\mathcal{P}_{i,e}} \frac{\gamma^2}{4}\left(\frac{\mathbb{E}[R_p]-1}{n} + \sum_{h=1}^{h_p}\frac{\mathbb{E}[(r(p,h)^2]-1}{n}\right) \leq \log(n)Tc, \tag{21}$$

for some constant c and for a large enough n and a sufficiently long duration of T, where $E = \sum_{i=1}^{n/2} 1/\mathbb{E}[\tilde{L}_{i,e}]$ stands for the numbers of episodes for all S-D pairs, and h_p stands for the number of hops for each packet p. Applying the Cauchy-Schwartz inequality and Jensen's inequality to (18) and (21) leads to $1/E^3 = O\left(\mathbb{E}_\Pi[Y_{i,e}]\log^3(n)/n\right)$, combining which with Lemma 1 and the fact that $\mathbb{E}_\Pi[\tilde{L}_{i,e}] \geq \mathbb{E}_\Pi[Y_{i,e}]$ for any Π completes the proof. □

Proposition 2 completes the first half of our analysis under the i.i.d. mobility model.

4.2 Upper Bound

(Order-Optimal) Policy 1: Mobile Networks

- We divide every time slot into c_1 *frames*, where c_1 is from Lemma 3.
- We partition the unit torus into square cells and consider an asymmetric cell partition structure as described in Subsection 4.2.2.
- For each time slot t,

(1) *Timeout:* The age of all carried packets increases by 1. Any packet that exceeds the age threshold, $D_{\text{to}}(n)$ (to be specified) time slots, will trigger a timeout and subsequently be discarded.

(2) *Source-to-Relay Mode:* In every odd time slot, each sending cell becomes active for one frame according to Lemma 3. When a sending cell is scheduled to be active, for a probability of

$$p(n) = \frac{D_{\text{to}}(n)^{1/3}n^{-1/3}c_2}{\sqrt{\log(n)}}, \tag{22}$$

where c_2 is a tunable parameter (independent of n) to be determined later. Each source node generates a packet and broadcasts it to all other nodes in the same cell. The nodes within the same cell coordinate themselves to broadcast sequentially.

(3) *Relay-to-Destination Mode:* In every even time slot, each receiving cell becomes active for one frame. Each mobile relay carrying informative packets is scheduled to the intended destinations whenever they meet in a receiving cell. If there are multiple mobile relays, we select the one with the freshest information; a tie is broken randomly. The packet from each designated mobile relay first forwards towards neighboring mini-cells along the X-axis, then to their destination nodes along the Y-aixs.

In this subsection, we present a heuristic policy aimed at achieving the minimal average age $\Delta^{(ave),*}(n)$. While a notable scheme [4] achieves a constant throughput $\lambda(n) = \Theta(1)$ for networks with mobility, it comes at the expense of potentially unbounded delay. Instead, we propose a scheme inspired by [7] and [8] that incorporates deadlines and cell structures, as outlined in Policy 1.

There are several significant distinctions from existing mobile networks. First, we do not partition a packet into smaller sizes, unlike schemes proposed by [7, 8] which require scaling down

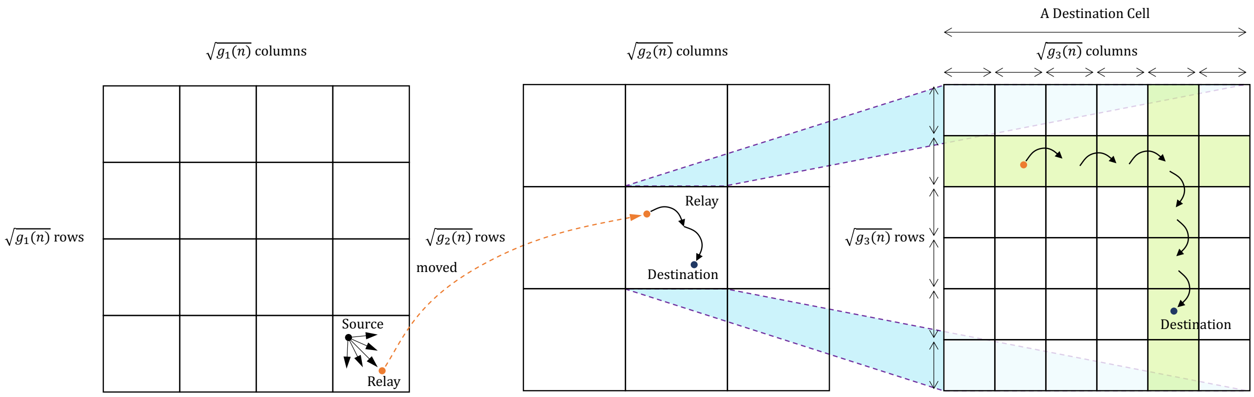


Figure 3: The asymmetric cell partitioning structure for Policy 1 under the i.i.d. mobility. There are $g_1(n)$ sending cells in the odd-numbered time slots for the source-to-relay mode (left); there are $g_2(n)$ receiving cells in the even-numbered time slots for the relay-to-destination mode (middle); each receiving cell consists of $g_3(c)$ mini-cells (right).

the packet size. It is mainly because the delay of a packet is determined by the time it takes to deliver its last subpacket. Hence, their approach of partitioning each packet into an increasing number of subpackets may introduce additional delays for complete informative packets.

The protocol interference model in (1) suggests for us to use the following lemma:

Lemma 3 (Bounded Packets [7, 8]). *There exists a scheduling policy such that each cell can be active for at least $1/c_1$ amount of time, where $c_1 = \lfloor 2\gamma + 6 \rfloor^2$ is a constant independent of n.*

Lemma 3 suggests that the number of cells that interfere with any given cell is bounded above by a constant c_1, independent of the cell size. Therefore, each cell can become active at a regular interval of every c_1 time slots without interfering with other cells.

4.2.1 Timeouts. We characterize Policy 1 by a timeout threshold $D_{\text{to}}(n)$, which determines the maximum number of time slots a packet can wait for delivery. If a packet duration exceeds $D_{\text{to}}(n)$ time slots, it is considered expired and will be discarded.

4.2.2 Cell Structures. We consider an asymmetric cell partition structure for establishing the transmission in Policy 1. In each odd time slot, we schedule transmissions from sources to relays. In each even time slot, we schedule transmissions from relays to destinations. We use different cell-partitioning for source-to-relay and relay-to-destination transmission, motivated by [7]. Specifically, in odd time slots, divide the unit torus using a square grid into $g_1(n) = \left\lfloor \sqrt{D_{\text{to}}(n)^{\frac{1}{3}} n^{\frac{2}{3}}} \right\rfloor^2$ *sending cells*, each with area $1/g_1(n)$. In even time slots, divide the unit torus using a square grid into$g_2(n) = \left\lfloor \sqrt{c_3 D_{\text{to}}(n)^{\frac{2}{3}} n^{\frac{1}{3}}} \right\rfloor^2$ *receiving cells*, each with area $1/g_2(n)$, and c_2 is a tunable parameter. Moreover, we first divide each receiving cell into $g_3(n)$ mini-cells (in $\sqrt{g_3(n)}$ rows and $\sqrt{g_3(n)}$ columns), given by $g_3(n) = \left\lfloor \sqrt{\frac{1}{c_3} D_{\text{to}}(n)^{-\frac{2}{3}} n^{\frac{2}{3}} \log^{-1}(n)} \right\rfloor^2$.

4.2.3 Packet Error. Recall that Lemma 3 states that the total number of packets transmitted within a cell is bounded by a constant. As a result, there may be instances where certain packets in the network are not transmitted or delivered to the destination nodes due to the following three types of errors:

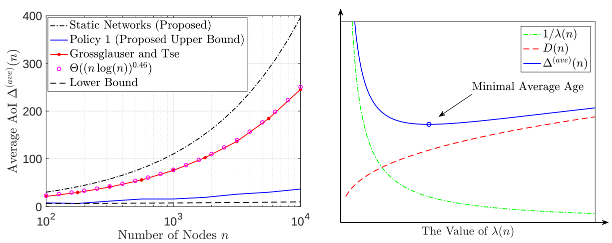
- *Type-I error (Sending Cells Overcrowded)*: There are more than $W/(Lc_1)$ packets scheduled to transmit within a sending cell.
- *Type-II error (Timeout)*: For a packet, its mobile relays never reach the destination (i.e., meets it within the same receiving cell) before it expires.
- *Type-III error (Receiving Cells Overcrowded)*: In each even time slot, when a packet carried by a relay meets its destination in any receiving cell, the path between the designated relay and the destination in Policy 1 either does not exist or any of the minicells along the path is overcrowded (more than $W/(Lc_1)$ packets).

In order to make Policy 1 scalable such that the probabilities of all errors converge to a constant less than 1 as $n \longrightarrow \infty$.

Proposition 3. *As $n \longrightarrow \infty$, the overall probability of all types is less than a constant $\epsilon = 2\exp\left(-\frac{1.1c_2}{2c_3^{3/2}}\zeta\left(\frac{Wc_3^{3/2}}{1.1Lc_1c_2} - 1\right)\right) + \exp(-1/(2c_3))$, where $\zeta(\delta) = (1+\delta)\log(1+\delta) - \delta$.*

Note that c_2 and c_3 are tunable parameters, allowing us to select suitable values to guarantee $\epsilon < 1$. The proof mainly makes uses of the Chernoff's bounds and is presented in Appendix B. As Proposition 3 implies, as $n \longrightarrow \infty$, all packets will be successfully delivered at a constant probability and hence the throughput of successful packets is given by $\lambda_\Pi(n) > (1-\epsilon)p(n)$ and the delay satisfies $D_\Pi(n) \leq D_{\text{to}}(n)$.

We highlight two key distinctions in our analysis compared to [7, 8]. First, while [7, 8] examined errors in the transmission of all packets at a given time slot from a global perspective, the three types of errors we consider focus on the microscopic perspective of each individual packet. Second, while [7, 8] proved that the probability of all errors converges to 0 as $n \longrightarrow \infty$, our analysis only requires the probability of all errors to converge to a constant less than 1. These distinctions enable us to establish a tighter bound.



(a) Performance comparison (b) Illustration of Theorem 4

Figure 4: Numerical examples.

4.2.4 Geo/Geo/∞ Queueing Model. We now consider the update process for each S-D pair. Due to the inherent i.i.d. mobility characteristics of mobile networks, combined with the transmission scheme described in Policy 1, this update process for each S-D pair is asymptotically equivalent to a discrete-time Geo/Geo/∞ queueing model, which is justified as follows:

- Since each activated packet will be successfully delivered with a probability of $1-\epsilon$ according to Lemma 3, the arrival satisfies the geometric distribution with a arrival rate of $(1-\epsilon)p(n)$, where ϵ is from Proposition 3.
- Furthermore, in the case of a successfully delivered packet, the time it takes for any of its relays to reach its destination follows a geometric distribution. This corresponds to a geometrically distributed service time for the packet with a mean service time is given by $g_1(n)g_2(n)/n = c_3 D_{\text{to}}(n)$.
- Finally, upon generation, each packet undergoes no queuing delay as it is promptly transmitted to and served by mobile relays. We can thus model this scenario by a queueing system with an infinite number of servers.

From [20, Lemma 3], the average age of a discrete-time Geo/Geo/∞ queue satisfies

$$\Delta^{(ave)}_{\Pi,i} = \frac{1}{2}\frac{\mathbb{E}[X_{i,k}^2]}{\mathbb{E}[X_{i,k}]} + \mathbb{E}\left[\min_{l\geq 0}\left(\sum_{k=1}^{l} X_{i,k} + D_{i,l+1}\right)\right] - \frac{1}{2}$$
$$\leq \frac{1}{(1-\epsilon)p(n)} + c_3 D_{\text{to}}(n), \forall i \in [n/2], \tag{23}$$

as n tends to ∞. It follows that

$$\Delta^{(ave),*}(n) \leq \frac{2}{n}\sum_{i=1}^{n/2}\Delta^{(ave)}_{\Pi,i} \leq \frac{1}{(1-\epsilon)p(n)} + c_3 D_{\text{to}}(n). \tag{24}$$

Choosing $D_{\text{to}}(n) = \Theta(n^{1/4}\log^{3/8}(n))$ in Policy 2 leads to the following upper bound of the average age:

Proposition 4. *Under the i.i.d. mobility model, the average age satisfies*

$$\Delta^{(ave),*}(n) = O(n^{1/4}\log^{3/8}(n)). \tag{25}$$

4.3 Summary

In Proposition 2 and (25), we have shown that

$$\Delta^{(ave),*}(n) = \tilde{\Theta}(n^{1/4}), \tag{26}$$

in the i.i.d. mobility model and completed the proof of Theorem 2. In a nutshell, we conclude that mobility improves AoI, which is partially because node mobility leads to a better throughput-delay tradeoff for mobile networks.

Fig. 4(a) illustrates numerical examples that compare the performances of different schemes. Our results demonstrate the significant benefits of Policy 1, compared to the scheme proposed in [4] (that is approximately the same as $\Theta((n\log(n))^{0.46})$). Moreover, the schemes under the i.i.d. mobility model outperform Policy 1 for static networks, highlighting that node mobility improves AoI. Fig. 4(b) shows the importance of striking the balance between throughput and delay to minimize the average age, to be further discussed.

5 DISCUSSIONS

5.1 Three-Way Tradeoffs

This section aims to extend our analysis of the fundamental limits of age and provide guidance on the age-minimal scheduling policy design for a broader range of environments. The main idea is to explore the potential tradeoffs among AoI, throughput, and delay. The following definition of Pareto optimality describes the tradeoffs:

Definition 5. *A pair $(\lambda(n), D(n))$ is **Pareto optimal** if there exists a scheme Π with*

$$\lambda_\Pi(n) = \Theta(\lambda(n)),\ D_\Pi(n) = \Theta(D(n)), \tag{27}$$

and no scheme Π' can improve one metric without worsening the other.

Our analysis also demonstrates the potential of the episode technique, which further enables us to derive the following result to characterize the three-way tradeoffs:

Theorem 4 (Age-Throughput-Delay Relation). *In a general wireless network under the protocol interference model with $n/2$ pairs of S-D nodes, if the following conditions hold:*

1. *there exists a stationary optimal policy to achieve Pareto optimal pairs $(\lambda(n), D(n))$;*
2. *there exists a Pareto optimal scheme Π whose inter-generation time of packets is either equal or follows a geometric distribution;*
3. *the packet generation processes and the packet delivery processes generated by the Pareto optimal scheme Π are independent;*
4. *there exists an optimal-tradeoff function $f(\cdot)$ such that $\lambda(n) = \Theta(f(D(n)))$, and*

$$\frac{1}{\sum_{i=1}^{n/2} 1/\mathbb{E}_\Pi[\tilde{L}_{i,e}]} = \Theta\left(f\left(\frac{2}{n}\sum_{i=1}^{n/2}\mathbb{E}_\Pi[Y_{i,e}]\right)\right). \tag{28}$$

Then, there exists a Pareto optimal pair $(\lambda(n), D(n))$ such that the minimal average age satisfies

$$\Delta^{(ave),*}(n) = \Theta\left(\frac{1}{\lambda(n)} + D(n)\right). \tag{29}$$

Sketch of Proof. The upper bound in (29) is from Conditions (2)-(3). Establishing the lower bound in (29) involves combining (28), Proposition 1, and the Jensen's inequality. By choosing the Pareto optimal $(\lambda(n), D(n))$, making the tradeoffs between $\mathbb{E}[\tilde{L}_{i,e}]$ and $\mathbb{E}[\tilde{Y}_{i,e}]$ is the same as choosing those between $(\lambda(n), D(n))$. □

Many schemes proposed in the literature satisfy the conditions in Theorem 4, including [4–6, 8–14, 16]. On the other hand, in the i.i.d. mobility model, an optimal-tradeoff function $f(\cdot)$ satisfying the condition 2) remains unknown, but can achieve $\lambda(n) = \tilde{\Theta}(f(D(n)))$, as shown in [7]. In this case, the result still holds if we replace the Θ-type bound in (29) by a $\tilde{\Theta}$-type bound.[6]

The significance of Theorem 4 is three-fold. First, it suggests that, to design an age-minimal scheduling policy in a wide range of wireless networks, one can seek for the schemes that achieve the optimal throughput-delay tradeoffs (e.g.,[2, 4–16]) and then strike a specific balance between $\lambda(n)$ and $D(n)$, as depicted in in Fig. 4(b). Second, as [20] showed that the variance of the service times can improve the AoI, our result in (29) also implies that the optimal throughput-delay tradeoff in fact limits the variance of packet delays. For instance, under the (order) optimal Policy 1 in static networks, the delay for each S-D pair is deterministic. Third, Theorem 4 characterizes the conditions of the wide range of wireless networks. Other settings potentially satisfy Condition 1) include the wireless networks with potential Markovian mobility (e.g., Brownian mobility [12] and Lévy flights [14]). To analyze whether (29) is satisfied, one can potentially exploit the martingale analysis similar to that used in the proof of Lemma 2.

6 CONCLUSIONS

We have presented fundamental limits of information freshness in large-scale mobile networks with interference and mobility for the first time. We have revealed that i) the achievable AoI scales in the *static model* as $\Omega(\sqrt{n\log(n)})$; ii) the achievable average age in the *i.i.d. mobility model* scales as $\tilde{\Theta}(n^{1/4})$, indicating that mobility improves AoI. Our proposed episode technique has further facilitated the characterization of a general class of wireless networks, in which the age-minimal scheduling policy design involves striking a balance between throughput and delay.

Future directions include the extension of the results on the large-scale vehicular networks, the study of the impacts of enqueued packets, more practical mobility settings (e.g., Brownian mobility and Lévy flights), and the inclusion of wired infrastructure nodes (e.g., base stations).

A PROOF SKETCH OF THEOREM 1

It follows from (7) that

$$\Delta^{(ave)}_{\Pi,i} \geq \liminf_{K\to\infty} \frac{\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}\left[\bar{X}^2_{i,k}/2\right]}{\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}[\bar{X}_{i,k}]} \geq \liminf_{K\to\infty}\frac{1}{2K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}[\bar{X}_{i,k}], \tag{30}$$

where the second inequality is from the Cauchy–Schwarz inequality. Note that $\frac{1}{\lambda_{\Pi,i}} = \liminf_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}_{\Pi}[\bar{X}_{i,k}]$. Applying the fact that the per-node maximum throughput scales as $\Theta(1/\sqrt{n\log(n)})$ in static networks [3, 5], we complete the proof.

B PROOF OF PROPOSITION 3

We start with the Chernoff's bound:

[6] We note that, for the i.i.d. mobility model, considering a slightly different setting as in [9] may still potentially lead to a Θ-type bound.

Lemma 4 (Chernoff's Bound). *Let X be a random variable following a binomial distribution with parameters k (representing the number of Bernoulli experiments) and p (indicating the probability of success for each Bernoulli experiment). For any $\delta > 0$, the Chernoff's bound is*

$$\Pr[X > (1+\delta)kp] < \exp\left(-kp\zeta(\delta)\right), \tag{31}$$

where $\zeta(\delta) = (1+\delta)\log(1+\delta) - \delta$. Note that $\zeta(\delta) > 0$ for any $\delta > 0$.

Let p_{I}, p_{II}, and p_{III} denote the probabilities of type-I, type-II, and type-III errors, respectively.

B.1 Type-I Error

Let x_j denote the number of packets to be scheduled to broadcast in the sending cell j. From Lemma 3, each sending cell can be active for $1/c_1$ and therefore can transmit a total amount of $W/(Lc_1)$ packets. Applying the Chernoff's bound (31), we have

$$\begin{aligned} p_{\mathrm{I}} &= \Pr\left[x_j > \frac{W}{Lc_1}\right] = \Pr\left[x_j > \frac{Wp(n)\sqrt{\log(n)}}{Lc_1c_2g_1(n)}n\right] \\ &< \exp\left(-\frac{np(n)}{g_1(n)}\zeta\left(\frac{\sqrt{\log(n)}W}{Lc_1c_2}-1\right)\right) \\ &= O\left(\exp\left(-\frac{\zeta(\sqrt{\log(n)})}{\sqrt{\log(n)}}\right)\right) \to 0, \text{ as } n\to\infty. \end{aligned} \tag{32}$$

Here, $\zeta(\delta)$ is defined in Lemma 4.

B.2 Type-II Error

To evaluate p_{II}, we can consider a binomial distribution with $n\cdot D_{\mathrm{to}}(n)/2$ independent experiments and success probability $1/(g_1(n)g_2(n))$. Therefore, the probability of a packet timeout (relays do not the destination in $D_{\mathrm{to}}(n)/2$ time slots) is given by,

$$\begin{aligned} p_{\mathrm{II}} &= \left(1-\frac{1}{g_1(n)g_2(n)}\right)^{nD_{\mathrm{to}}(n)/2} \leq \exp\left(-\frac{nD_{\mathrm{to}}(n)}{2g_1(n)g_2(n)}\right) \\ &= \exp(-1/(2c_3)), \end{aligned} \tag{33}$$

where the inequality is because of $(1-1/x)^x \leq \exp(-1)$ for any $x > 1$. Hence, within the $D_{\mathrm{to}}(n)/2$ odd-numbered time slots, each destination node can find a mobile relay that holds the intended message and resides in the same receiving cell.

B.3 A Proof Sketch for the Type-III Error

To examine the probability of Type-III error, p_{III}, we need to establish a specific method for scheduling the hop-by-hop transmissions from the designated mobile relays to the destination nodes within each receiving cell. Lemma 3 suggests that we can implement a scheduling scheme in which each mini-cell can be active for a duration of $1/c_1$. During the half active state of each mini-cell, it forwards a message (or a portion of a message) to a neighboring mini-cell. The messages from the designated mobile relays are initially directed towards neighboring cells along the X-axis, using a duration of $1/(2c_1)$, and subsequently towards their respective destination nodes along the Y-axis using another duration of $1/(2c_1)$ (as depicted in Figure 3).

Each destination needs to receive only at most one packet with the freshest information that need to be scheduled. The aforementioned scheduling scheme can successfully transmit all messages from the designated mobile relays to their respective destinations, given the following conditions:

- Each mini-cell must include at least one node, ensuring that each node can always find another node in the neighboring cell to act as a static relay.
- The number of messages passing through any mini-cell remains bounded by $W/(Lc_1)$.

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