Delay Minimal Policies in Energy Harvesting Communication Systems

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***Abstract*— We characterize delay minimal power scheduling policies in energy harvesting communication systems. We consider a continuous-time system, where the delay experienced by each bit is given by the time spent by the bit in the queue waiting to be transmitted to its receiver. We first consider a single-user channel, where the transmitter has a finite-sized battery to save its harvested energy. Data arrives during the course of communication and are saved in a finite data buffer as well. We find the optimal power policy that minimizes the average delay experienced by the bits subject to energy and data causality constraints. We characterize the optimal solution in terms of Lagrange multipliers, and calculate their values in a recursive manner. We show that, different from the existing literature, the optimum transmission power is not constant between the energy and data arrival events; the transmission power starts high, decreases linearly, and potentially reaches zero between energy and data arrivals. Intuitively, untransmitted bits experience cumulative delay due to the bits to be transmitted ahead of them, and hence the reason for transmission power starting high and decreasing over time. Next, we study a multiuser version of this problem, namely, a two-user broadcast channel, and characterize the optimal transmission policies that minimize the *sum* delay. For this setting, we consider the case, where the transmitter has an infinite-sized battery, and that all data packets intended for the receivers are available at the beginning of the communication session. We characterize the optimal solution in terms of Lagrange multipliers, and present an iterative solution that calculates their values. Our results show that in the optimal policy, both users may not be served simultaneously all the time; there may be times, where only one of the two users is served alone. We also show that the optimal policy may have *gaps* in transmission in between energy arrivals, where none of the users is served, echoing the results of the single-user setting.**

***Index Terms*— Energy harvesting, delay minimization, single-user channel, broadcast channel, finite battery, finite buffer.**

I. Introduction

We CONSIDER energy harvesting communication systems where the transmitter relies solely on energy harvested from nature to maintain the power necessary for data transmission. According to a specific data demand, the transmitter needs to schedule the transmission of data packets using the available energy such that the average delay experienced by the data is minimal.

Optimal resource allocation and scheduling policies have been considered for various energy harvesting communication models. Earlier works focus on throughput maximization and transmission completion time minimization policies for single-user settings [3]–[6], broadcast channels [7]–[9], multiple access channels [10], interference channels [11], relay channels [12]–[15], and settings with energy cooperation [16]. In addition, energy leakage from battery over time [17], battery inefficiency at the time of charging [18], systems with hybrid energy storage [19], processing costs [20], [21], and receiver decoding costs [22], [23] have been considered.

In [3], the problem of minimizing the transmission completion time is considered. Reference [3] and the subsequent literature showed that, due to the concavity of the rate-power relationship, the transmit power must be kept constant between energy harvesting and data arrival events, and the transmitter must schedule data transmissions using longest possible stretches of constant power, subject to energy and data causality. While [3] minimizes the time by which all of the data packets are transmitted, different data packets experience different delays, and the average delay of the system is not minimized. In particular, when the earlier-arriving data packets are transmitted slowly, the later-arriving data packets experience not only the delay in their own transmissions, but a portion of the delay experienced by the earlier-arriving packets, as they have to wait extra time in the data queue while those packets are being transmitted. This compounds the delays that the later-arriving data packets experience. The delay minimization problem was considered previously in [24] for a non energy harvesting system.

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In this paper, we consider the problem of average delay minimization in an energy harvesting system. First, we consider a single-user channel where the transmitter is equipped with a finite-sized battery and a finite-sized data buffer. We show that, unlike the previous literature, the transmission power should not be kept constant between energy harvesting and data arrival events. We let the power (and therefore the rate) vary even during the transmission of a single packet. We show that the optimal packet scheduling is such that the transmit power starts with a high value and decreases linearly over time possibly reaching zero before the arrival of the next energy or data packet into the system. The high initial transmit power values ensure that earlier bits are transmitted faster, decreasing their own delay and also the delays of the later-arriving data packets. We develop a recursive solution that finds the optimal transmit power over time by determining the optimal Lagrange multipliers.

Next, we consider a two-user energy harvesting broadcast channel where the transmitter is equipped with an infinite-sized battery, and data packets intended for both users are available before the transmission starts. In this system, there is a tradeoff between the delays experienced by both users; as more resources (power) is allocated to a user, its delay decreases while the delay of the other user increases. We consider the minimization of the *sum* delay in the system. We formulate the problem using a Lagrangian framework, and express the optimal solution in terms of Lagrange multipliers. We develop an iterative solution that solves the optimum Lagrange multipliers by enforcing the KKT optimality conditions. Similar to the single-user setting, we show that the optimal transmission power decreases between energy harvests, and may possibly hit zero before the next energy harvest, yielding communication *gaps*, where no data is transmitted. During active communication, data may be sent to both users, or only to the stronger user, or only to the weaker user, depending on the energy harvesting profile. We contrast our work with [7] which developed an algorithm that minimizes the transmission completion time, i.e., a time by which all data is delivered to users. To that end, [7] studies the throughput maximization problem, and shows that, for general user priorities, there exists a *cut-off power* level such that only the total power above this level is used to serve the weaker user. In particular, for sum throughput maximization, this cut-off is infinity, and all power is allocated to packets sent to the stronger user. In contrast, in our *sum delay minimization* problem, the weaker user always gets a share of the transmitted power, as otherwise, its delay becomes unbounded, and the sum delay will not be minimized. In our work, we show that there exists a *cut-off time*, beyond which data is sent only to the weaker user.

II. Single-User Channel

In this section we consider a single-user AWGN channel, see Fig. 1, where at arrival time t_m, $m = 0, 1, \ldots, M-1$, with $t_0 = 0$, energy is harvested at the transmitter with amount E_m and data intended for the receiver arrives with amount B_m. The transmitter saves energy and data in a battery with finite capacity $E_{\max}$ and in a data buffer with finite capacity $B_{\max}$,

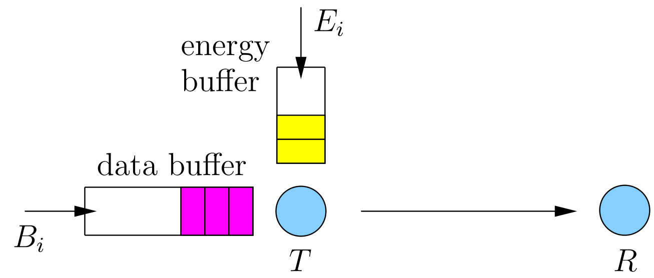

Fig. 1. Single-user energy harvesting channel with finite-sized battery and data buffer.

respectively. We denote the cumulative harvested energy and the total amounts of received data at time t by

$$E_a(t) = \sum_{i=0}^{m-1} E_i, \quad t_{m-1} < t \le t_m, \ m = 1, \ldots, M \tag{1}$$

$$B_a(t) = \sum_{i=0}^{m-1} B_i, \quad t_{m-1} < t \le t_m, \ m = 1, \ldots, M \tag{2}$$

where we define $t_M = \infty$. For a power policy $p(t)$ at time t, the cumulative consumed energy and the total departed data to the receiver at time t are given by

$$E(t) = \int_0^t p(\tau)d\tau \tag{3}$$

$$B(t) = \int_0^t \frac{1}{2}\log(1 + p(\tau))d\tau \tag{4}$$

where log is the natural logarithm throughout this paper. We call a policy feasible if the following is satisfied

$$E_a(t) - E_{\max} \le E(t) \le E_a(t), \quad \forall t \tag{5}$$

$$B_a(t) - B_{\max} \le B(t) \le B_a(t), \quad \forall t \tag{6}$$

The above conditions assure that the policy conforms to energy and data causality constraints, and that energy and data buffers are not overflown.

The delay experienced by each bit is the time interval from its arrival time to its actual transmission time. The total average delay for the system is given by

$$\bar{D} = \int_0^\infty t dB(t) - \int_0^\infty t dB_a(t) \tag{7}$$

which represents the area in between the data arrival and departure curves, see Fig. 2. Our objective is to characterize the optimal power policy that minimizes the total average delay in (7) subject to feasibility conditions in (5) and (6). For a given data arrival profile, the second term in (7) is constant, and therefore minimizing $\bar{D}$ is equivalent to minimizing the *gross* delay defined as

$$D = \int_0^\infty t dB(t) = \int_0^\infty \frac{t}{2}\log(1 + p(t))dt \tag{8}$$

We note that the maximum data buffer constraint can model strict delay requirements for data packets in this setting.

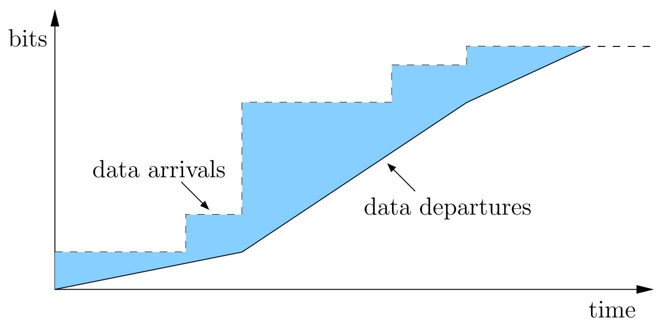

Fig. 2. Example of data arrival and departure curves. The area in between represents the total average delay to be minimized.

Our optimization problem is now formulated as

$$\begin{aligned}
\min_{p} \quad & \int_0^\infty t\log(1+p(t))dt \\
\text{s.t.} \quad & E_a(t_m) - E_{\max} \le \int_0^{t_m} p(t)dt \le E_a(t_m), \\
& \qquad\qquad\qquad m = 1,\dots,M \\
& B_a(t_m) - B_{\max} \le \int_0^{t_m} \log(1+p(t))dt \le B_a(t_m), \\
& \qquad\qquad\qquad m = 1,\dots,M-1 \\
& \int_0^\infty \log(1+p(t))dt = B_a(t_M) \\
& p(t) \ge 0, \quad \forall t
\end{aligned} \tag{9}$$

where for convenience we dropped the half term of the rate-power function.[1] We solve problem (9) in the remainder of this section.

A. Properties of the Optimal Solution

We note that problem (9) is not a convex optimization problem. However, our analysis will show that the KKT optimality conditions[2] admit a unique, and therefore the optimal, solution. We introduce the following Lagrangian

$$\begin{aligned}
\mathcal{L} = & \int_0^\infty t\log(1+p(t))dt - \int_0^\infty \eta(t)p(t)dt \\
& + \sum_{m=1}^{M} \lambda_{1m}\left(\int_0^{t_m} p(t)dt - E_a(t_m)\right) \\
& + \sum_{m=1}^{M} \lambda_{2m}\left(E_a(t_m) - E_{\max} - \int_0^{t_m} p(t)dt\right) \\
& + \sum_{m=1}^{M-1} \mu_{1m}\left(\int_0^{t_m} \log(1+p(t))dt - B_a(t_m)\right) \\
& + \sum_{m=1}^{M-1} \mu_{2m}\left(B_a(t_m) - B_{\max} - \int_0^{t_m} \log(1+p(t))dt\right) \\
& - \nu\left(\int_0^\infty \log(1+p(t))dt - B_a(t_M)\right)
\end{aligned} \tag{10}$$

[1] This is indeed without loss of optimality as the objective function and the data constraints can both be multiplied by 2.

[2] Please refer to Appendix A for justification of the necessity of KKT conditions for optimality in this non-convex problem. We also refer the reader to [25] for general treatments of optimality conditions of constrained optimization problems.

where $\{\lambda_{1m}, \lambda_{2m}, \mu_{1m}, \mu_{2m}\}$, ν, and $\eta(t)$ are Lagrange multipliers. Taking the (functional) derivative with respect to $p(t)$ and equating to 0, and using the fact that $\eta(t)$ is non-negative, we get the following KKT optimality conditions

$$p(t) = \left(\frac{\mu(t) - t}{\lambda(t)} - 1\right)^+ \tag{11}$$

where we have defined

$$\lambda(t) = \sum_{\{m:\ t_m \ge t\}} \lambda_{1m} - \lambda_{2m} \tag{12}$$

$$\mu(t) = \nu - \sum_{\{m:\ t_m \ge t\}} \mu_{1m} - \mu_{2m} \tag{13}$$

along with the complementary slackness conditions

$$\lambda_{1m}\left(\int_0^{t_m} p(t)dt - E_a(t_m)\right) = 0, \quad \forall m \tag{14}$$

$$\lambda_{2m}\left(E_a(t_m) - E_{\max} - \int_0^{t_m} p(t)dt\right) = 0, \quad \forall m \tag{15}$$

$$\mu_{1m}\left(\int_0^{t_m} \log(1+p(t))dt - B_a(t_m)\right) = 0, \quad m = 1,\dots,M-1 \tag{16}$$

$$\mu_{2m}\left(B_a(t_m) - B_{\max} - \int_0^{t_m} \log(1+p(t))dt\right) = 0, \quad m = 1,\dots,M-1 \tag{17}$$

We now state the following lemma.

Lemma 1: The optimal $\lambda(t)$ (resp. $\mu(t)$) is a piece wise constant function, with possible changes only if the energy (resp. data) buffer is either depleted or full.

Proof: By the complementary slackness conditions we have

$$\lambda_{1m} = \lambda_{2m} = 0, \quad \text{if } E_a(t_m) - E_{\max} < E(t_m) < E_a(t_m) \tag{18}$$

$$E(t_m) = E_a(t_m), \quad \text{if } \lambda_{1m} > 0 \tag{19}$$

$$E(t_m) = E_a(t_m) - E_{\max}, \quad \text{if } \lambda_{2m} > 0 \tag{20}$$

Therefore, $\lambda(t)$ stays constant between arrival times, and can only change when $\lambda_{1m} > 0$ or $\lambda_{2m} > 0$ for some m, which occurs only if the energy buffer is either depleted or full at t_m. Similar arguments follow for $\mu(t)$. ■

By Lemma 1, both $\lambda(t)$ and $\mu(t)$ are sequences rather than continuous functions of time. We denote by $\{s_1, s_2, \dots, s_L\} \subseteq \{t_0, t_1, \dots, t_{M-1}\}$ the change times of $\lambda(t)$ and $\mu(t)$, with $s_1 = 0$. Therefore we have

$$\lambda(t) = \begin{cases} \lambda_k^c, & t \in [s_k, s_{k+1}) \\ \lambda_L^c, & t \in [s_L, \infty) \end{cases} \tag{21}$$

$$\mu(t) = \begin{cases} \mu_k^c, & t \in [s_k, s_{k+1}) \\ \mu_L^c, & t \in [s_L, \infty) \end{cases} \tag{22}$$

where the superscript c is short for change. Therefore, by definition of $\{s_k\}$, at least one constraint is met with equality at s_k, $\forall k$, and no constraint is met with equality during the interval (s_{k-1}, s_k). The following lemma provides the necessary conditions for the two sequences $\{\lambda_k^c\}$ and $\{\mu_k^c\}$ to increase/decrease.

Lemma 2: In the optimal policy: 1) $\lambda_k^c > \lambda_{k-1}^c$ *(resp.* $\lambda_k^c < \lambda_{k-1}^c$*) only if the battery is full (resp. depleted) at time* s_{k-1}*; and 2)* $\mu_k^c > \mu_{k-1}^c$ *(resp.* $\mu_k^c < \mu_{k-1}^c$*) only if the data buffer is depleted (resp. full) at time* s_{k-1}.

Proof: By definition of $\lambda(t)$ in (12), the function can only increase (resp. decrease) after time s_{k-1} if $\lambda_{2m} > 0$ (resp. $\lambda_{1m} > 0$) for m such that $t_m = s_{k-1}$. By complementary slackness, the battery must be full (resp. depleted) at time s_{k-1}. The second statement of the lemma follows using similar arguments. ∎

We conclude the optimality conditions by the following lemma.

Lemma 3: Whenever the optimal power $p(t) > 0$ *on some open interval in between arrival times, it is monotonically decreasing.*

Proof: Let us have $p(t) > 0\ \forall t \in (l_1, l_2)$ where (l_1, l_2) lies in between arrival times. By Lemma 1, we know that both $\lambda(t)$ and $\mu(t)$ are constants during that interval (say λ_l and μ_l). Hence, from (11), $p(t)$ is either monotonically increasing or decreasing (depending on the sign of λ_l). Now assume it is increasing during this interval, i.e., $\lambda_l < 0$, and denote $\lambda_l' = -\lambda_l$, and $\mu_l' = l_2 - \mu_l + l_1$. Now define a new power policy $p'(t) = (\mu_l' - t)/\lambda_l' - 1$, for $t \in (l_1, l_2)$. It is direct to see that both $p(t)$ and $p'(t)$ use the same energy and deliver the same data amount during (l_1, l_2), as what we did is merely flipping the curve of $p(t)$ in (l_1, l_2) around $\frac{l_1+l_2}{2}$. However, the (now decreasing) new policy $p'(t)$ does so with a strictly less delay. This is due to the multiplicative term t in the objective function; it is strictly better to use higher powers at the beginning and lower powers at the end, so that data arriving earlier in time are delivered faster. ∎

By Lemma 3, we conclude that the optimal $\lambda(t)$ is non-negative for all t, and that it is necessary, from (11), to have $\mu(t) > t$ for all t before the total amount of data is delivered. Lemma 3 also shows that power can reach 0 in between arrivals, where the communication stops until the next energy or data arrival instant.

B. Recursive Formulas

In this section, we show how to find λ_k^c, μ_k^c, and s_k in a recursive manner. We will use these recursive formulas to construct the optimal solution in the next section. First, assume s_k, $E(s_k)$, $B(s_k)$, and μ_k^c are known, and define the following values for all $\{m : t_m > s_k\}$

$$\lambda_m^{eu} : E(s_k) + \int_{s_k}^{t_m} \left(\frac{\mu_k^c - t}{\lambda_m^{eu}} - 1\right)^+ dt = E_a(t_m) \tag{23}$$

$$\lambda_m^{bu} : B(s_k) + \int_{s_k}^{t_m} \log\left(1 + \left(\frac{\mu_k^c - t}{\lambda_m^{bu}} - 1\right)^+\right) dt = B_a(t_m) \tag{24}$$

$$\lambda_m^u = \max\{\lambda_m^{eu}, \lambda_m^{bu}\} \tag{25}$$

$$\lambda_m^{el} : E(s_k) + \int_{s_k}^{t_m} \left(\frac{\mu_k^c - t}{\lambda_m^{el}} - 1\right)^+ dt = E_a(t_m) - E_{\max} \tag{26}$$

$$\lambda_m^{bl} : B(s_k) + \int_{s_k}^{t_m} \log\left(1 + \left(\frac{\mu_k^c - t}{\lambda_m^{bl}} - 1\right)^+\right) dt = B_a(t_m) - B_{\max} \tag{27}$$

$$\lambda_m^l = \min\{\lambda_m^{el}, \lambda_m^{bl}\} \tag{28}$$

Therefore, λ_m^u is the minimum value of λ such that either the energy or the data buffer is depleted by time t_m, i.e., an upper bound is met with equality. On the other hand, λ_m^l is the maximum value of λ such that either the energy or the data buffer is full by time t_m, i.e., a lower bound is met with equality. Observe that these values are unique, by monotonicity of the integrands on the left hand side of the above equations. Let us denote $\Lambda(m) = [\lambda_m^u, \lambda_m^l]$. Hence, to maintain feasibility, we need to have $\lambda_k^c \in \Lambda(m)$ if $s_{k+1} \geq t_m$. Now define the following integers

$$m_1^{\max}(k) = \max\left\{m : \bigcap_{i:\ t_i > s_k}^{m} \Lambda(i) \neq \emptyset\right\} \tag{29}$$

$$m_1^u(k) = \max\left\{m : \lambda_m^u \in \bigcap_{i:\ t_i > s_k}^{m} \Lambda(i)\right\} \tag{30}$$

$$m_1^l(k) = \max\left\{m : \lambda_m^l \in \bigcap_{i:\ t_i > s_k}^{m} \Lambda(i)\right\} \tag{31}$$

We now have the following lemma; with the assumption that the optimal solution of the problem is only partially revealed up to a given time, it provides a method to proceed forward and find the optimal solution up to some specific future time.

Lemma 4: Assume that one has the optimal solution up to time s_k*, along with* μ_k^c*. Then,* λ_k^c *and* s_{k+1} *are found as follows:*

$$\begin{aligned} &\textit{If } \Lambda\left(m_1^{\max}(k) + 1\right) > \bigcap_{i:\ t_i > s_k}^{m_1^{\max}(k)} \Lambda(i) \\ &\qquad \Rightarrow \lambda_k^c = \lambda_{m_1^l(k)}^l, \quad s_{k+1} = t_{m_1^l(k)} \\ &\textit{Else, if } \Lambda\left(m_1^{\max}(k) + 1\right) < \bigcap_{i:\ t_i > s_k}^{m_1^{\max}(k)} \Lambda(i) \\ &\qquad \Rightarrow \lambda_k^c = \lambda_{m_1^u(k)}^u, \quad s_{k+1} = t_{m_1^u(k)} \end{aligned}$$

where the comparisons of the intervals above are pointwise.[3]

Proof: Let us assume that $\Lambda\left(m_1^{\max}(k) + 1\right) > \bigcap_{i:\ t_i > s_k}^{m_1^{\max}(k)} \Lambda(i)$ and consider two different possibilities. First, if $\lambda_k^c > \lambda_{m_1^l(k)}^l$, then a lower bound will be met before $t_{m_1^l(k)}$. By Lemma 2, we know that $\lambda(t)$ can only increase if a lower bound is met with equality. This means that eventually the lower bound at $t_{m_1^l(k)}$ will be breached. On the other hand, if $\lambda_k^c < \lambda_{m_1^l(k)}^l$, then by definition of $m_1^l(k)$, we know that $\lambda_m^l \geq \lambda_{m_1^l(k)}^l$ for all $m : s_k < t_m < m_1^l(k)$. This means that only an upper bound can be met before or at $t_{m_1^l(k)}$.

[3]By $[a_1, a_2] > [b_1, b_2]$ we mathematically mean that $a_1 > b_2$, and inversely by $[a_1, a_2] < [b_1, b_2]$ we mathematically mean that $a_2 < b_1$. Note that by definition of $m_1^{\max}(k)$, the two intervals $\Lambda\left(m_1^{\max}(k) + 1\right)$ and $\bigcap_{i:\ t_i > s_k}^{m_1^{\max}(k)} \Lambda(i)$ have no intersection, and thus the two cases considered in the lemma are mutually exclusive.

By Lemma 2, we know that $\lambda(t)$ can only decrease if an upper bound is met with equality. Therefore, $\lambda(t)$ will not increase to have a value inside $\Lambda\left(m_1^{\max}(k)+1\right)$ (which lies above $\bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i)$ by assumption) at $t_{m_1^{\max}(k)+1}$, i.e., the upper bound at $t_{m_1^{\max}(k)+1}$ will be breached. Thus, we must have $\lambda_k^c = \lambda_{m_1^l(k)}^l$, $s_{k+1} = t_{m_1^l(k)}$ in this case. Similar arguments follow for the other case when $\Lambda\left(m_1^{\max}(k)+1\right) < \bigcap_{i:\ t_i>s_k}^{m_1^{\max}(k)} \Lambda(i)$. ■

Similarly to what we did above, we can define the quantities $\{\mu_m^{eu}, \mu_m^{bu}, \mu_m^u, \mu_m^{el}, \mu_m^{bl}, \mu_m^l\}$ as we did in (23)-(28) with fixed (known) λ_k^c. Further, we can also define the set $U(m) = [\mu_m^l, \mu_m^u]$, which gives rise to the following integers

$$m_2^{\max}(k) = \max\left\{m : \bigcap_{i:\ t_i>s_k}^{m} U(i) \neq \emptyset\right\} \tag{32}$$

$$m_2^u(k) = \max\left\{m : \mu_m^u \in \bigcap_{i:\ t_i>s_k}^{m} U(i)\right\} \tag{33}$$

$$m_2^l(k) = \max\left\{m : \mu_m^l \in \bigcap_{i:\ t_i>s_k}^{m} U(i)\right\} \tag{34}$$

We now have the following lemma, complementing and serving the same purpose as Lemma 4. The proof follows using similar arguments as in that of Lemma 4, and is therefore omitted for brevity.

Lemma 5: Assume that one has the optimal solution up to time s_k, along with λ_k^c. Then, μ_k^c and s_{k+1} are found as follows

$$\text{If } U\left(m_2^{\max}(k)+1\right) > \bigcap_{i:\ t_i>s_k}^{m_2^{\max}(k)} U(i) \Rightarrow \mu_k^c = \mu_{m_2^u(k)}^u, \quad s_{k+1} = t_{m_2^u(k)}$$

$$\text{Else, if } U\left(m_2^{\max}(k)+1\right) < \bigcap_{i:\ t_i>s_k}^{m_2^{\max}(k)} U(i) \Rightarrow \mu_k^c = \mu_{m_2^l(k)}^l, \quad s_{k+1} = t_{m_2^l(k)}$$

where the comparisons of the intervals above are pointwise.

Lemmas 4 and 5 show how to optimally construct λ_k^c and μ_k^c, along with s_{k+1}, given μ_k^c and λ_k^c, respectively, along with the optimal solution up to s_k. In solving our problem, we neither know the optimal value of λ_1^c or μ_1^c in order to apply those lemmas, and hence, we need to assume some initialization values for either of them in order to start computing the remaining ones recursively. It then remains to find out if such initializations were erroneous, and how to adjust them if this were the case. In addition to that issue, we also note that Lemmas 4 and 5 only give the value of s_{k+1}. One needs either λ_{k+1}^c or μ_{k+1}^c along the way in order to reapply the results of the lemmas and move forward to find s_{k+2}. We address these issues formally through the next series of lemmas. Throughout the lemmas, we first assume a value for μ_k^c and find the corresponding values of λ_k^c and s_{k+1} by Lemma 4. We then assess the optimality of the assumed μ_k^c according to the constraints met at s_{k+1}. The next lemma will help in that assessment. We provide its proof in Appendix B.

Lemma 6: Given a time interval $[\underline{s}, \bar{s}]$, and a decreasing power policy $p_0(t)$, if we define another decreasing power policy $p_1(t)$ that consumes the same amount of energy during $[\underline{s}, \bar{s}]$, and has a slower decline, i.e., has a larger slope, then the policy $p_1(t)$ departs more data during that interval. Similarly, if we define another power policy $p_2(t)$ that departs the same amount of data during $[\underline{s}, \bar{s}]$, and has a slower decline, i.e., has a larger slope, then the policy $p_2(t)$ consumes less energy during that interval.

Next, we use the results in Lemma 6 to prove the statements in the following lemmas. Throughout the lemmas, as mentioned previously, we assess the optimality of an assumed value of μ_k^c based on how the constraints at s_{k+1} are met/violated. We provide proofs of these lemmas in Appendices C through F.

Lemma 7: If an energy constraint is binding at s_{k+1}, while data constraints are not, and if $\mu_k^c > s_{k+1}$, then we have $\mu_{k+1}^c = \mu_k^c$. Otherwise, μ_k^c is not optimal, and needs to increase. Similarly, if a data constraint is binding at s_{k+1}, while energy constraints are not, and if $s_{k+1} < t_M = \infty$, then we have $\lambda_{k+1}^c = \lambda_k^c$. Otherwise, μ_k^c is not optimal, and needs to decrease.

Lemma 8: If the battery is empty at s_{k+1}, and the data buffer is overflown, then μ_k^c is not optimal and needs to increase. Similarly, if the data buffer is empty at s_{k+1}, and the battery is overflown, then μ_k^c is not optimal and needs to decrease.

The next two lemmas deal with the cases where both data and energy constraints are binding at s_{k+1}. In such cases, we re-solve a shifted problem starting at s_{k+1} recursively using the above analysis, with initial conditions as indicated by the binding constraints at s_{k+1}, e.g., a full/empty data/energy buffer, and denote the optimal Lagrange multipliers of this shifted problem by $\{\bar{\lambda}_i, \bar{\mu}_i\}_{i=k+1}^M$. We then compare the values of those Lagrange multipliers obtained from the shifted problem to λ_k^c and μ_k^c and examine their optimality as follows.

Lemma 9: If the battery is empty (resp. full) and the data buffer is full (resp. empty) at s_{k+1}, and the solution of the shifted problem satisfies: $\bar{\lambda}_{k+1} \leq \lambda_k^c$ and $\bar{\mu}_{k+1} \leq \mu_k^c$ (resp. $\bar{\lambda}_{k+1} \geq \lambda_k^c$ and $\bar{\mu}_{k+1} \geq \mu_k^c$), then the solution of the shifted problem, as well as the pair $\{\lambda_k^c, \mu_k^c\}$, is optimal. Otherwise, μ_k^c is not optimal and needs to increase (resp. decrease).

Lemma 10: If both the battery and the data buffer are empty (resp. full) at s_{k+1}, and the solution of the shifted problem satisfies: $\bar{\lambda}_{k+1} \leq \lambda_k^c$ and $\bar{\mu}_{k+1} \geq \mu_k^c$ (resp. $\bar{\lambda}_{k+1} \geq \lambda_k^c$ and $\bar{\mu}_{k+1} \leq \mu_k^c$), then the solution of the shifted problem, as well as the pair $\{\lambda_k^c, \mu_k^c\}$, is optimal. Otherwise, if $\bar{\lambda}_{k+1} > \lambda_k^c$ (resp. $\bar{\mu}_{k+1} > \mu_k^c$), then μ_k^c is not optimal and needs to increase. On the other hand, if $\bar{\mu}_{k+1} < \mu_k^c$ (resp. $\bar{\lambda}_{k+1} < \lambda_k^c$), then μ_k^c is not optimal and needs to decrease.

It is clear from the above recursive formulas that the optimal Lagrange multipliers can only have one unique set of values. For instance, equations (23)-(28) constitute the method of computing the Lagrange multipliers from one epoch to the next. In there, we note that the left hand sides are all monotone in λ_m given μ_k^c is fixed, and vice

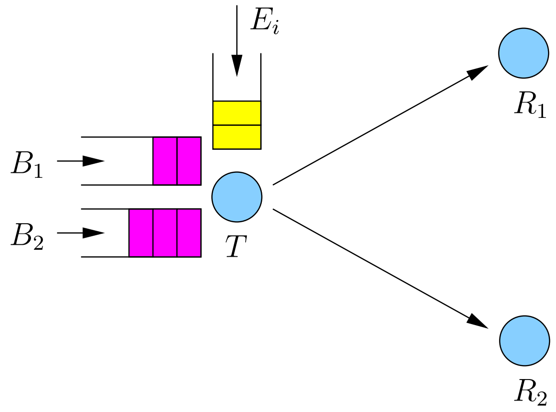

Fig. 3. Two-user energy harvesting broadcast channel.

versa. Since our solution approach is based on fixing one parameter and finding the other one through these equations in Lemma 4 (and their complements in Lemma 5), we conclude that the KKT conditions have a unique solution for this problem, as mentioned in the beginning of the analysis in Section II-A. We summarize the proposed algorithmic solution next.

C. Constructing the Optimal Solution

In this section, we summarize the solution of the single-user problem. We first initialize by setting $s_1 = 0$, and choosing a value for μ_1^c. We then find the value of λ_1^c and s_1 by Lemma 4. Next, we check the constraints at s_1 and use Lemmas 7, 8, 9, and 10 to assess the optimality of the initialized μ_1^c. This results into one of the following cases: 1) the value of μ_2^c or λ_2^c is given because μ_1^c is optimal; 2) μ_1^c is not optimal and needs to increase or decrease; 3) the optimal solution of the problem is obtained according to Lemmas 9 and 10. In case 3, we need to solve a shifted problem starting at s_2; we do so by initializing a value of μ_2^c and continue as discussed above. In case 2, one can find the optimal μ_1^c by using, e.g., a bisection search. In case 1, we either use Lemma 4 to find λ_2^c and s_3 if μ_2^c was given, or use Lemma 5 to find μ_2^c and s_3 if λ_2^c was given; we then repeat the above constraints' checks at s_3, and so on. We stop when all data is transmitted under the above conditions.

III. BROADCAST CHANNEL

In this section, we consider an energy harvesting two-user broadcast channel, see Fig. 3, where energy is harvested at times $\{t_0, t_1, \ldots, t_{M-1}\}$ in amounts $\{E_0, E_1, \ldots, E_{M-1}\}$, respectively, with $t_0 = 0$. Unlike the single-user problem, the data packets in this broadcast setting are available before the communication starts, in amounts B_1 and B_2, for the first and the second user, respectively.

The physical layer is a degraded broadcast channel,

$$Y_j = X + Z_j, \quad j = 1, 2 \tag{35}$$

where X is the transmitted signal, Y_j is the received signal of user j, and Z_j is the Gaussian noise at receiver j with variance σ_j^2. We assume $\sigma_1^2 = 1 < \sigma_2^2 \triangleq \sigma^2$, i.e., the first user is stronger. The capacity region for this channel is [26]

$$r_1 \leq \frac{1}{2}\log\left(1+\alpha P\right), \quad r_2 \leq \frac{1}{2}\log\left(1+\frac{(1-\alpha)\,P}{\alpha P+\sigma^2}\right) \tag{36}$$

where α is the fraction of the total power assigned to the first (stronger) user, and log is the natural logarithm. Working on the boundary of the capacity region we have,

$$P = e^{2(r_1+r_2)} + \left(\sigma^2-1\right)e^{2r_2} - \sigma^2 \triangleq g\left(r_1, r_2\right) \tag{37}$$

which is the minimum power needed to achieve rates r_1 and r_2, at the first and the second user, respectively. Note that $g(r_1, r_2)$ is strictly convex in (r_1, r_2) [27]. We call a policy feasible if the following are satisfied:

$$\int_0^t g\left(r_1(\tau), r_2(\tau)\right) d\tau \leq E_a(t), \quad \forall t \tag{38}$$

$$\int_0^\infty r_1(t)dt = B_1 \tag{39}$$

$$\int_0^\infty r_2(t)dt = B_2 \tag{40}$$

where the first constraint is the energy causality constraint with $E_a(t)$ as defined in (1), and the remaining two are to ensure data delivery to both users.

As discussed in the single-user scenario, the average gross delay experienced by each user is given by

$$D_1 = \int_0^\infty r_1\left(t\right) t dt \tag{41}$$

$$D_2 = \int_0^\infty r_2\left(t\right) t dt \tag{42}$$

Note that, unlike the single-user scenario, in this two-user setting, there is a tradeoff between the delays experienced by the two users. This tradeoff can be characterized by developing the *delay region*, similar to *departure region* in [7], where all achievable (D_1, D_2) can be plotted. It can be shown that this region is strictly convex, and in order to achieve pareto-optimum delay points, one needs to solve *weighted sum delay minimization* problems in the form of $\min \mu_1 \; D_1 + \mu_2 \; D_2$ subject to energy causality constraints. We focus on the *sum* delay minimization problem by taking $\mu_1 = \mu_2 = 1$. Therefore, in this section, we consider the following optimization problem:

$$\begin{aligned} \min_{r_1, r_2} \quad & \int_0^\infty r_1\left(\tau\right)\tau d\tau + \int_0^\infty r_2\left(\tau\right)\tau d\tau \\ \text{s.t.} \quad & \int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau \leq E_a(t_m), \quad m = 1, \ldots, M \\ & \int_0^\infty r_1(\tau)d\tau = B_1 \\ & \int_0^\infty r_2(\tau)d\tau = B_2 \\ & r_1(t) \geq 0, \quad r_2(t) \geq 0, \quad \forall t \end{aligned} \tag{43}$$

A. Minimum Sum Delay Policy

We note that (43) is a convex optimization problem [27]. We solve using a Lagrangian approach:

$$\begin{aligned}\mathcal{L} = &\int_0^\infty r_1(\tau)\,\tau d\tau + \int_0^\infty r_2(\tau)\,\tau d\tau \\ &+ \sum_{m=1}^{M} \lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau - E_a(t_m)\right) \\ &- \nu_1 \left(\int_0^\infty r_1(\tau) d\tau - B_1\right) - \nu_2 \left(\int_0^\infty r_2(\tau) d\tau - B_2\right) \\ &- \int_0^\infty \gamma_1(\tau) r_1(\tau) d\tau - \int_0^\infty \gamma_2(\tau) r_2(\tau) d\tau \end{aligned} \tag{44}$$

where $\{\lambda_m\}$, ν_1, ν_2, $\gamma_1(t)$, and $\gamma_2(t)$ are Lagrange multipliers. KKT optimality conditions are:

$$t + \lambda(t)\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} - \nu_1 - \gamma_1(t) = 0 \tag{45}$$

$$t + \lambda(t)\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} - \nu_2 - \gamma_2(t) = 0 \tag{46}$$

where we have:

$$\lambda(t) = \sum_{\{m: t_m \ge t\}} \lambda_m \tag{47}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_1(t)} = 2e^{2(r_1(t)+r_2(t))} \tag{48}$$

$$\frac{\partial g\left(r_1(t), r_2(t)\right)}{\partial r_2(t)} = 2e^{2(r_1(t)+r_2(t))} + 2\left(\sigma^2 - 1\right) e^{2r_2(t)} \tag{49}$$

along with the complementary slackness conditions:

$$\lambda_m \left(\int_0^{t_m} g\left(r_1(\tau), r_2(\tau)\right) d\tau - E_a(t_m)\right) = 0, \ \forall m \tag{50}$$

$$\nu_1 \left(\int_0^\infty r_1(\tau) d\tau - B_1\right) = 0, \quad \gamma_1(t) r_1(t) = 0 \ \forall t \tag{51}$$

$$\nu_2 \left(\int_0^\infty r_2(\tau) d\tau - B_2\right) = 0, \quad \gamma_2(t) r_2(t) = 0 \ \forall t \tag{52}$$

From the above KKT conditions, we can write the rates and total power expressions in terms of the Lagrange multipliers. First, we write the rate expressions as:

$$r_1(t) = \frac{1}{2}\log\left(\frac{\left(\sigma^2 - 1\right)\left(\gamma_1(t) + \nu_1 - t\right)}{\gamma_2(t) - \gamma_1(t) + \nu_2 - \nu_1}\right) \tag{53}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\gamma_2(t) - \gamma_1(t) + \nu_2 - \nu_1}{\lambda(t)\left(\sigma^2 - 1\right)}\right) \tag{54}$$

We now state the following result.

Lemma 11: The optimal Lagrange multipliers (ν_1^*, ν_2^*) *satisfy:* $\nu_1^* < \nu_2^* < \sigma^2 \nu_1^*$.

Proof: We show this by contradiction. Assume $\nu_2^* \le \nu_1^*$. Then, by (54), the value of $r_2(t)$ is well-defined only if $\gamma_2(t) > 0 \ \forall t$, which means by complementary slackness that $r_2(t) = 0 \ \forall t$. Therefore, assuming $B_2 > 0$, the weak user will never get to receive any of its data. This proves the first inequality.

To show the second inequality, assume $\sigma^2 \nu_1^* \le \nu_2^*$. Thus,

$$\frac{\left(\sigma^2 - 1\right)\left(\nu_1 - t\right)}{\gamma_2(t) + \nu_2 - \nu_1} \le 1, \quad \forall t, \gamma_2(t) \ge 0 \tag{55}$$

Therefore, the right hand side of (53) can only be positive if $\gamma_1(t) > 0$, but this means, by complementary slackness, that $r_1(t) = 0$, which is a contradiction. Hence, $r_1(t) = 0 \ \forall t$, and, assuming $B_1 > 0$, the strong user will never get to receive any of its data. ■

Next, we characterize the optimal total transmit power $g\left(r_1(t), r_2(t)\right)$ by the following lemma. The proof is in Appendix G.

Lemma 12: In the optimal policy, the total transmit power $g\left(r_1(t), r_2(t)\right)$ *is given by*

$$g(r_1(t), r_2(t)) = \max\left\{\frac{\nu_2 - t}{\lambda(t)} - \sigma^2, \frac{\nu_1 - t}{\lambda(t)} - 1\right\}^+ \tag{56}$$

The above lemma shows that the optimal power in the broadcast channel decreases with time between energy harvests, and can reach zero before increasing again with the next energy harvest, similar to the results of the single-user channel in Section II. The following lemmas characterize the structure of the optimal policy.

Lemma 13: In the optimal policy, the transmission starts by sending data to the strong user, and finishes by sending data to the weak user.

Proof: We show this by contradiction. Assume that the transmission starts by sending data to the weak user only, i.e., $r_2(0) > r_1(0) = 0$.[4] By complementary slackness, we have $\gamma_2(0) = 0$. By Lemma 11, since $\sigma^2 \nu_1 > \nu_2$, we have

$$\frac{\left(\sigma^2 - 1\right)\left(\gamma_1(0) + \nu_1\right)}{\nu_2 - \nu_1 - \gamma_1(0)} > 1, \quad \forall \gamma_1(0) \ge 0 \tag{57}$$

which implies, by (53), that $r_1(0) > 0$, which is a contradiction. For the second part of the lemma, assume that the transmission ends at some time t_f with $r_1(t_f) > r_2(t_f) = 0$. By Lemma 12, we know that this can only occur if $\lambda(t_f) > \frac{\nu_2 - \nu_1}{\sigma^2 - 1} \triangleq \lambda_{th}$. Since $\lambda(t)$ is non-increasing, we have $\lambda(t) \ge \lambda(t_f)$, $\forall t \le t_f$. This means that $\lambda(t)$ does not fall below λ_{th} throughout the transmission, which is equivalent to saying, again by Lemma 12, that the weak user does not receive any of its data, which is a contradiction. ■

Lemma 14: For $t < t_{th} \triangleq \frac{\sigma^2 \nu_1 - \nu_2}{\sigma^2 - 1}$*, if the transmitter is sending data, then it is sending to the strong user.*

Proof: We show this by contradiction. Assume that for some $t < t_{th}$ data is sent only to the weak user, i.e., we have $r_1(t) = 0$ and $r_2(t) > 0$. By complementary slackness, we have $\gamma_2(t) = 0$. Since $t < t_{th}$, it follows by simple manipulations that the numerator of the term inside the log in (53) is strictly larger than its denominator $\forall \gamma_1(t) \ge 0$, i.e., $r_1(t) > 0$, which is a contradiction. The only case where $r_1(t) = 0$ for some $t < t_{th}$ is when $\gamma_2(t) > 0$, which means by complementary slackness that $r_2(t) = 0$. ■

1) Modes of Operation: There can be four different modes of operation at a given time, depending on which user is receiving data. The first mode is when only the strong user is receiving data, i.e., $r_1(t) > 0$ and $r_2(t) = 0$. By Lemma 12, this can be the case only if $\lambda(t) \ge \lambda_{th} = \frac{\nu_2 - \nu_1}{\sigma^2 - 1}$. In this mode,

[4]Extension of the contradiction arguments in this lemma to an ϵ-length interval, $\epsilon > 0$, follows directly.

we have the total power and the strong user's rate given by

$$g(r_1(t), 0) = \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{58}$$

$$r_1(t) = \frac{1}{2}\log\left(\frac{\nu_1 - t}{\lambda(t)}\right) \tag{59}$$

The second mode of operation is when both users are receiving data, i.e., $r_1(t) > 0$ and $r_2(t) > 0$. Again by Lemma 12, this can be the case only if $\lambda(t) < \lambda_{th}$. Moreover, by (53), we also need $t < t_{th} = \frac{\sigma^2\nu_1 - \nu_2}{\sigma^2 - 1}$. In this mode, the total power and the users' rates are given by

$$g(r_1(t), r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{60}$$

$$r_1(t) = \frac{1}{2}\log\left(\frac{(\sigma^2 - 1)(\nu_1 - t)}{\nu_2 - \nu_1}\right) \tag{61}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2 - \nu_1}{\lambda(t)(\sigma^2 - 1)}\right) \tag{62}$$

The third mode of operation is when only the weak user is receiving data, i.e., $r_1(t) = 0$ and $r_2(t) > 0$. For this to occur we need both $\lambda(t) < \lambda_{th}$ and $t \geq t_{th}$. The total power and the weak user's rate are then given by

$$g(0, r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{63}$$

$$r_2(t) = \frac{1}{2}\log\left(\frac{\nu_2 - t}{\lambda(t)\sigma^2}\right) \tag{64}$$

The fourth mode is when both rates (and the power) are zero. We denote this mode as a communication *gap*. These gaps may occur, for instance, if there is a small amount of energy in the battery that is insufficient to deliver all the data, and a large amount of energy arrives later. The transmitter may then finish up this small amount of energy to send some bits out and wait for additional energy to send the remaining bits.

2) Finding the Value of λ(t): We next characterize the rates and powers. The following lemma shows that $\lambda(t)$ is a piecewise constant function. The proof follows by complementary slackness as in the proof of Lemma 1, and is omitted for brevity.

Lemma 15: In the optimal policy, the Lagrange multiplier function $\lambda(t)$ is piecewise constant, with possible changes only when energy is depleted.

By Lemma 15, $\lambda(t)$ is a sequence rather than a continuous function of time. Following the same notation as in the single-user channel, we denote the times of change of $\lambda(t)$ by $\{s_1, s_2, \ldots, s_L\}$ with $s_1 = 0$, and the values of $\lambda(t)$ between such times by

$$\lambda(t) = \begin{cases} \lambda_k^c, & t \in [s_k, s_{k+1}) \\ \lambda_L^c, & t \in [s_L, \infty) \end{cases} \tag{65}$$

Next, we characterize the optimal $\{\lambda_k^c\}$ sequentially. Determining the value of λ_k^c requires the knowledge of ν_1^* and ν_2^*, and also which mode of operation is active during the interval $[s_k, s_{k+1})$. Let us define $B_j(t)$ as the total amount of bits transmitted to user j by time t. The next lemma shows how to compute λ_k^c given the mode of operation. The proof uses similar steps as in the proof of Lemma 4 in the single-user setting and is omitted for brevity.

Lemma 16: Given a mode of operation, with the optimal ν_1^*, ν_2^*, λ_l^c, s_l, $\forall l < k$, define the following quantities $\forall m$: $t_m > s_k$

$$\bar{\lambda}_m : E^*(s_k) + \int_{s_k}^{t_m} g(r_1(\tau), r_2(\tau))^+ d\tau = E_a(t_m) \tag{66}$$

$$\tilde{\lambda}_1 : B_1^*(s_k) + \int_{s_k}^{\infty} r_1(\tau)^+ d\tau = B_1 \tag{67}$$

$$\tilde{\lambda}_2 : B_2^*(s_k) + \int_{s_k}^{\infty} r_2(\tau)^+ d\tau = B_2 \tag{68}$$

where r_1, r_2, and $g(r_1, r_2)$ are defined by the mode of operation in Section III-A.1, with the convention that $\tilde{\lambda}_j = 0$ whenever a mode of operation has $r_j = 0$, $j = 1, 2$. Then, the optimal λ_k^c for this mode of operation is given by

$$\lambda_k^c = \max\{\bar{\lambda}_m, \tilde{\lambda}_1, \tilde{\lambda}_2\}, \quad \forall m : t_m > s_k \tag{69}$$

The results in Lemma 16 imply that one has to know the mode of operation before computing the optimal values of the Lagrange multipliers. Note that communication gaps occur naturally due to the $(\cdot)^+$ operation in these expressions. In the next section, we develop an iterative solution that computes $\{\lambda_k^c\}$ based on an initial assignment of the mode of operation and the values of ν_1, ν_2. The solution is based on the necessary conditions stated in the previous lemmas. By Lemma 11, we know that the optimal values of ν_1, ν_2 lie in a cone in $\mathbb{R}_{++}^2$. We also know, by Lemmas 12 and 13, that the communication stops if $t > \nu_2$. Therefore, we find an upper bound on the value of ν_2^* as follows. First, we move all of the energy to t_{M-1}, the arrival time of the last energy packet, and start the communication from there. Second, we solve this single energy arrival problem and find its optimal ν_2^* which we denote by ν_2^{single}. Therefore, an upper bound on ν_2^* of the multiple energy arrival problem is

$$\nu_2^* \leq \nu_2^{\text{single}} + t_{M-1} \triangleq \nu^{\text{ub}} \tag{70}$$

Once this upper bound is found, one can perform a two-dimensional grid search over the feasible region of ν_1, ν_2:

$$\mathcal{R}_{\nu_1\nu_2} = \left\{\nu_1, \nu_2 : 0 < \nu_1 < \nu_2 < \sigma^2\nu_1, \nu_2 \leq \nu^{\text{ub}}\right\} \tag{71}$$

Next, we analyze the single energy arrival case to characterize the upper bound on ν_2^*.

3) Single Energy Arrival: For the single energy arrival case, we first note that there can be no communication gaps, as this can only increase the delay. We also note that since there is only one value of λ, corresponding to only one energy arrival constraint, the optimal power is given by the first term in (56). If not, then the weak user will never receive its data. Hence, the first mode of operation where only the strong user is receiving data never occurs. Thus, the optimal total power is given by

$$p_s(t) = \frac{\nu_2 - t}{\lambda} - \sigma^2, \quad \forall t \leq t_f \triangleq \nu_2 - \lambda\sigma^2 \tag{72}$$

where the subscript s denotes single arrival, and t_f is such that $p_s(t)$ is non-negative. From the above, we also note that

λ cannot be 0, or else the power is infinitely large. Since $\lambda > 0$, by complementary slackness, the transmitter has to consume all of its energy by the end of transmission. This simplifies the single energy arrival problem, as in this case, we have all the three constraints, both users' data and transmitter's energy, met with equality. Therefore, we can solve for the optimal values of the Lagrange multipliers satisfying the following:

$$\int_0^{t_{th}} \frac{1}{2} \log\left(\frac{(\sigma^2-1)(\nu_1-t)}{\nu_2-\nu_1}\right) dt = B_1 \quad (73)$$

$$\frac{t_{th}}{2} \log\left(\frac{\nu_2-\nu_1}{\lambda(\sigma^2-1)}\right) + \int_{t_{th}}^{t_f} \frac{1}{2} \log\left(\frac{\nu_2-t}{\lambda\sigma^2}\right) dt = B_2 \quad (74)$$

$$\int_0^{t_f} p_s(t) dt = E \quad (75)$$

The above three equations are direct consequences of the modes of operation analysis in Section III-A.1. These can be further simplified into:

$$\frac{\nu_1}{2} \log\left(\frac{(\sigma^2-1)\nu_1}{\nu_2-\nu_1}\right) = B_1 \quad (76)$$

$$\frac{\nu_2}{2} \log\left(\frac{\nu_2-\nu_1}{\lambda(\sigma^2-1)}\right) = B_2 \quad (77)$$

$$\frac{\left(\nu_2-\lambda\sigma^2\right)^2}{2\lambda} = E \quad (78)$$

Note that (76)-(78) are three equations in three unknowns, and can be solved numerically for the values of λ^*, ν_1^*, and ν_2^*. Note from the above analysis that, since we always start with the second mode of operation, where both users receive data, in this setting, we have $\lambda < \lambda_{th}$. This implies that $t_f > t_{th}$, and enables the following stronger version of Lemma 13.

Lemma 17: In the optimal policy solving (43), transmission always ends by sending data only to the weak user.

Proof: In the single energy arrival case, since $t_f > t_{th}$, we always end transmission by sending data only to the weak user. In the multiple arrival case, the last energy arrival can be viewed as a single energy arrival problem with the remaining data in the data buffers as modified constraints. Then the single energy arrival result applies, yielding the stated result. ∎

We have now characterized how to get the upper bound ν^{ub} in (70). In the next section we present an iterative method to find the optimal Lagrange multipliers solving problem (43).

B. Iterative Solution

The analysis presented in Lemma 16 describes an optimal method of finding $\{\lambda_k^c\}$ given ν_1^* and ν_2^*. To find the latter two, we perform a grid search over the region $\mathcal{R}_{\nu_1\nu_2}$, which is fully characterized by the single arrival analysis. We perform the search as follows. We fix $(\nu_1, \nu_2) \in \mathcal{R}_{\nu_1\nu_2}$, and solve for $\{\lambda_k^c\}$ to acquire a transmission policy accordingly. We denote by Mode 1, Mode 2, and Mode 3, the mode of operation where data is sent only to the strong user, both users, and only to the weak user, respectively. Since Mode 1 can only occur at the beginning, we assume that the transmission starts according to that mode, and compute the corresponding λs by Lemma 16. If these λs are all less than λ_{th}, then they are correct. We move to Mode 2 once we get a value of λ larger than λ_{th}. We stay at Mode 2 until the time passes t_{th}, then move to Mode 3 till the end of communication. By Lemma 17, we know that Mode 3 always exists. The transmission then ends whenever the weak user's data or the transmission energy is finished.

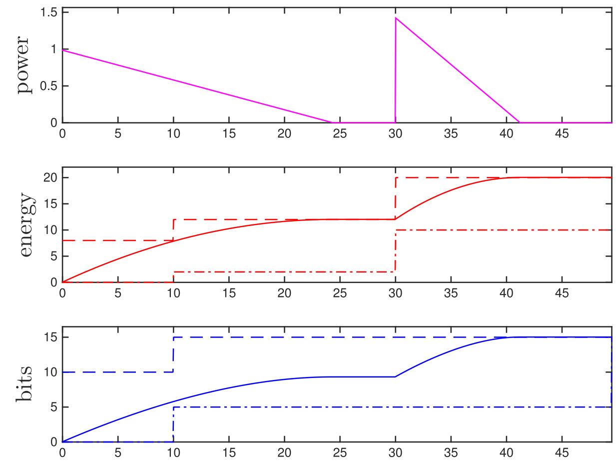

Fig. 4. Optimal solution for a single-user system with 3 energy arrivals and 2 data arrivals.

After we find the transmission policy, we check whether the data buffers of both users are empty. If this is the case, then by the convexity of the problem, this policy is optimal as we have thus found a feasible policy satisfying the KKT conditions [27]. Note that we might end up with a policy that either does not finish up all the users' data, or even transmits more than the available. If either is the case, we re-solve using another (ν_1, ν_2) point. We summarize how to find the optimal (ν_1, ν_2) iteratively as follows. We initialize by setting $\nu_1 = \epsilon$ and $\nu_2 = \nu_1 + \epsilon$ for some $\epsilon > 0$ small enough. We then solve for $\{\lambda_k^c\}$ as described above. If we do not reach a feasible KKT point, we increase ν_2 by another ϵ and repeat. We keep doing this until we reach a feasible KKT point, or ν_2 becomes larger than $\min\{\sigma^2\nu_1, \nu^{\text{ub}}\}$. In the latter case, we increase ν_1 by ϵ and repeat the whole procedure again. Since the region $\mathcal{R}_{\nu_1\nu_2}$ is bounded, iterations are guaranteed to find the optimal solution.

IV. Numerical Results

In this section, we present some numerical examples to further illustrate the results in this paper. We begin by considering a single use channel with $E_{\max} = B_{\max} = 10$ units. Energy arrives with amounts of $[8, 12, 20]$ at times $t = [0, 10, 30]$, while data arrives with amounts of $[10, 15]$ at times $t = [0, 10]$. In Fig. 4 we show the delay minimal solution in this setting. We see that the power is monotonically decreasing between arrival times, and actually drops to 0 before the last energy arrival. The optimal energy and data profiles are also shown in the figure. The upper and lower dotted lines represent the upper and lower constraints, respectively, as dictated by the arrival profile and the value of the finite-sized battery or data buffer.

In Fig. 4, we see that the size of the buffers is not a bottleneck to the system. We therefore consider another example where energy arrives with amounts of $[10, 15, 20, 25]$ at times $t = [0, 15, 20, 40]$, while data arrives with amounts of $[10, 18, 22]$ at times $t = [0, 20, 40]$, and plot the optimal solution in Fig. 5. We see that the power in this case does

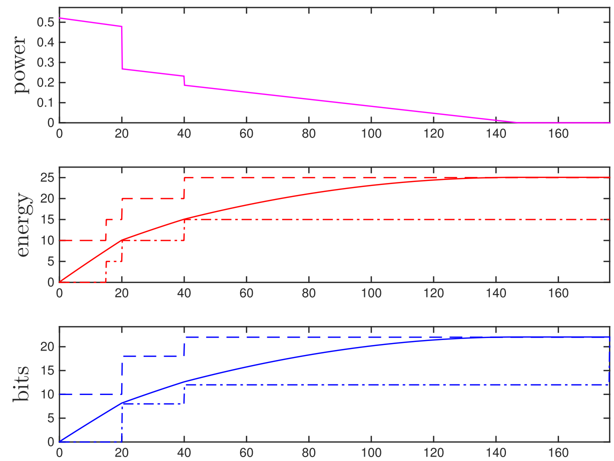

Fig. 5. Effects of having a finite-sized battery and data buffer in a single-user system.

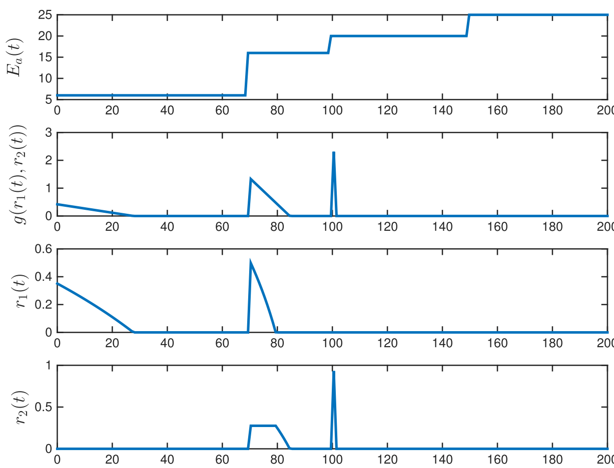

Fig. 6. Optimal power and rates for a system with four energy arrivals.

not drop down to 0 until at the end of communication, and that its slope changes when the optimal energy or data profiles hit the lower bounds indicated by the size of the buffers.

Next, we present a numerical example to illustrate the results of the broadcast setting. We consider a system where energy arrives with values $[6, 10, 4, 5]$ at times $t = [0, 70, 100, 150]$, with amounts of data $B_1 = 8$ and $B_2 = 4.25$ intended for the strong and the weak user, respectively. We first find the upper bound on ν_2^* by solving the single energy arrival case by setting $E = 25$ in (78) and finding the value of ν_2^{single}. Adding $t_{M-1} = 150$, we get $\nu^{\text{ub}} \simeq 170$. We then apply the iterative solution described in Section III-B to find the optimal total power allocation for the multiple arrival case and the corresponding users' rates. These are shown in Fig. 6 as a function of time. We see that all four modes of operation are present in this example: the transmitter begins by sending data only to the strong user (Mode 1) until it consumes the initial energy arrival, and stays silent until the next energy arrival, then it sends data to both users simultaneously (Mode 2) until all strong user's data is finished, which occurs at $t_{th} \simeq 79.4$. Then, it starts sending data only to the weak user (Mode 3), before keeping silent until the third energy arrival, and then finishes up the weak user's data. Note that the fourth energy

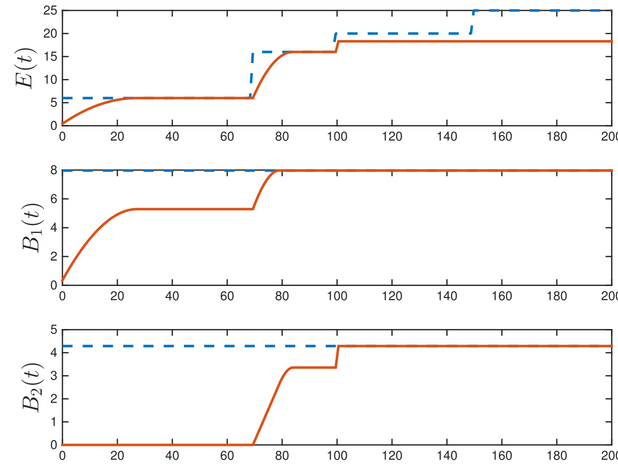

Fig. 7. Optimal energy and data consumption.

arrival is not used in this example. In Fig. 7, we show the corresponding optimal total energy and data consumption for this policy as a function of time.

Finally, we compare this to the transmission completion time minimization problem in [7] with the same data values and energy arrival profile. The optimal transmission completion time is equal to $T^* = 90$. Calculating the delay achieved by such policy gives $D \simeq 717.2$. On the other hand, our delay minimizing policy achieves a smaller delay of $D^* \simeq 593.3$, however, it takes a larger amount of time to finish $T \simeq 101.5$. This shows that there exists a tradeoff between delay minimization and transmission completion time minimization, and that the two problems are different, even when all data is available before the start of communication. That is, finishing data delivery by a minimum time, and having data experience minimum overall delay yield different optimum policies.

V. Conclusion and Discussion

We considered delay minimization in energy harvesting communication channels. First, we studied the single-user channel where the transmitter has a finite-sized battery and data buffer, and energy and data packets become available at the transmitter during the course of communication. We determined the optimum power control policy in terms of the Lagrange multiplier functions. We identified the properties of these functions and gave a method that evaluates them recursively. We proposed a solution which iteratively updates the initial value of a Lagrange multiplier, and obtains the optimum power allocation policy. The optimal power values start high, decrease linearly, potentially reaching zero between energy harvests and data arrivals. This policy is different from the piecewise constant power policies of the existing literature which focus on minimizing a deadline by which all packets are transmitted or maximizing the throughput before a fixed deadline. Initial high powers in our case make sure that the delay does not accumulate by transmitting data at faster rates first, then decreasing the rate gradually.

Next, we considered a two-user energy harvesting broadcast channel and characterized the minimal sum delay policy subject to energy harvesting constraints, when the transmitter has an infinite-sized battery, and all data intended for both users is available before transmission. We showed that the optimal power is decreasing between energy harvests, and that there can be times when data is sent only to the strong user, both users, or only to the weak user. We also showed that there can be communication gaps where the transmitter is silent between energy arrivals. We presented a method to find the optimal policy iteratively.

We note that there is another metric that is considered in current energy harvesting literature that resembles delay in certain aspects. That is, the *age of information* metric [28]–[37]. This metric measures the delay from the receiver's perspective, and is defined as the time spent since the freshest information packet has been received by the receiver. In our recent work [32], we have discussed the precise difference, mathematically, between delay and age minimization problems in energy harvesting systems; see [32, Sec. V]. In summary, the main difference lies in that the age metric is more sensitive to data packets' arrival times than the delay metric (which is mainly due to the difference in definitions of both metrics), and hence the solutions of both problems can be much different. It is therefore of interest to study the problems in this paper under an age metric and relate the solutions of these two lines of research.

Appendix

A. Necessity of KKT Conditions of Optimality for Problem (9)

It is known that the (extended) Fritz John conditions for optimality [38] are always necessary. If we apply such conditions for our problem, we get that any optimal solution should be a stationary point of a slightly modified Lagrangian compared to $\mathcal{L}$ in (10) where an extra Lagrange multiplier $\kappa \geq 0$ is multiplied by the objective function. We note that the Fritz John conditions are weaker than the KKT conditions since the optimal κ^* may turn out to be 0, and hence the objective function vanishes from the Lagrangian at the optimal solution. We also note that the KKT conditions will be directly implied if one can show that $\kappa^* > 0$, since one can divide all other Lagrange multipliers by κ^* in this case after differentiating and equating to 0, and hence any optimal solution would be a stationary point of the Lagrangian in (10) in this case, i.e., with $\kappa = 1$. We show that $\kappa^* > 0$ by the following contradiction argument.

Assume that $\kappa^* = 0$. Then, the optimal $p(t)$ should satisfy

$$p(t) = \left(\frac{\mu(t)}{\lambda(t)} - 1\right)^+$$

which means, by Lemma 1, that in any optimal solution, the power policy is a piece wise constant function. Now choose any interval (l_1, l_2) over which $p(t) = l$ for some constant $l > 0$. Using arguments similar to those in Lemma 3, we can define another policy $p'(t)$ to be monotonically decreasing on (l_1, l_2), consuming the same amount of energy, delivering the same amount of data, and achieving strictly less delay. Since this monotonically decreasing policy can only be satisfied if and only if $\kappa^* > 0$, and is better than any other policy with $\kappa^* = 0$, then the assumption of $\kappa^* = 0$ cannot be true, and KKT conditions are necessary for optimality.

B. Proof of Lemma 6

Assume without loss of generality that $E_i(\underline{s}) = B_i(\underline{s}) = 0$, for $i = 0, 1, 2$. Since we have $E_1(\bar{s}) = E_0(\bar{s})$, and that $p_1(t)$ declines slower than $p_0(t)$, therefore it must hold that $E_0(t) = \int_{\underline{s}}^{t} p_0(\tau)d\tau \geq \int_{\underline{s}}^{t} p_1(\tau)d\tau = E_1(t)\ \forall t \in [\underline{s}, \bar{s}]$, i.e., $p_0(t)$ majorizes $p_1(t)$ in the interval $[\underline{s}, \bar{s}]$. By concavity of the log, it then follows that $B_1(\bar{s}) = \int_{\underline{s}}^{\bar{s}} \frac{1}{2}\log(1 + p_1(t))dt > \int_{\underline{s}}^{\bar{s}} \frac{1}{2}\log(1 + p_0(t))dt = B_0(\bar{s})$ by the theory of continuous majorization [39]. This proves the first part of the lemma.

We prove the second part by contradiction. Assume $E_2(\bar{s}) \geq E_0(\bar{s})$. Since $p_2(t)$ declines slower than $p_0(t)$, therefore there must exist some point $t' \in (\underline{s}, \bar{s}]$ at which $E_2(t') = E_0(t')$ with $E_0(t) \geq E_2(t)\ \forall t \in [\underline{s}, t']$. Using the first assertion of the lemma, we have

$$B_2(t') > B_0(t') \tag{79}$$

Since $E_2(\bar{s}) \geq E_0(\bar{s})$, then we must have

$$B_2(\bar{s}) - B_2(t') > B_0(\bar{s}) - B_0(t') \tag{80}$$

From (79) and (80), we get $B_2(\bar{s}) > B_0(\bar{s})$, which contradicts the assumption that both policies depart the same amount of data. Therefore we must have $E_2(\bar{s}) < E_0(\bar{s})$.

C. Proof of Lemma 7

By complementary slackness, we know that we must have $\mu_{k+1}^c = \mu_k^c$ since the data constraints are not binding at s_{k+1}. However, if $\mu_k^c \leq s_{k+1}$, then by (11), $p(t) = 0\ \forall t \geq s_{k+1}$, and the transmitter will not be able to deliver the required amount of data to the receiver. Hence, μ_k^c needs to increase in order to maintain feasibility of the problem. This proves the first part of the lemma. To show the second part, we also note that by complementary slackness, we must have $\lambda_{k+1}^c = \lambda_k^c$ since the energy constraints are not binding at s_{k+1}. However, if $s_{k+1} = \infty$, i.e., we reached the end of the communication session, then we can use some extra amounts of energy to decrease the delay as follows: decrease the value of μ_k^c and decrease that of λ_k^c such that the amounts of departed bits in $[s_k, \infty)$ stays the same. This makes the power in the interval $[s_k, \infty)$ be of a faster decline, i.e., finish transmission faster, and in turn by Lemma 6 will consume more energy, which is feasible since the energy constraints are not binding.

D. Proof of Lemma 8

To show the first part, let us increase the value of μ_k^c and increase that of λ_k^c such that the consumed energy in the interval $[s_k, s_{k+1})$ stays the same. This means that the power in the interval $[s_k, s_{k+1})$ will have a slower decline. By Lemma 6, this new policy departs more bits, and prevents the overflow of the data buffer. Similarly, for the second part, let us decrease the value of μ_k^c and decrease that of λ_k^c such that the data delivered in the interval $[s_k, s_{k+1})$ stays the same. This means that the power in the interval $[s_k, s_{k+1})$ will have a faster decline, and in turn by Lemma 6 will consume more energy and prevent the overflow of the battery.

E. Proof of Lemma 9

We first note that the conditions of optimality stated in the lemma are those stated in Lemma 2. If these are not satisfied, and the battery is empty while the data buffer is full at s_{k+1}, then we can increase the value of μ_k^c and increase that of λ_k^c such that the consumed energy in $[s_k, s_{k+1})$ stays the same. This means that the power in the interval $[s_k, s_{k+1})$ will have a slower decline. By Lemma 6, this new policy departs more bits, which is feasible since the data buffer is full at s_{k+1}, and eventually achieves less delay. The proof of the other scenario stated in the lemma where the battery is full and the data buffer is empty at s_{k+1} follows using similar arguments as in the proof of the second part of Lemma 8.

F. Proof of Lemma 10

We first note that the conditions of optimality stated in the lemma are those stated in Lemma 2. If these are not satisfied, and both the battery and the data buffer are empty at s_{k+1}, and $\bar{\lambda}_{k+1} > \lambda_k^c$, then we can increase the value of μ_k^c and increase that of λ_k^c such that the amount of data delivered in $[s_k, s_{k+1})$ stays the same. This means that the power in the interval $[s_k, s_{k+1})$ will have a slower decline. By Lemma 6, this new policy consumes a smaller amount of energy, i.e., energy constraints will not be binding at s_{k+1}, and therefore we will have $\lambda_{k+1}^c = \lambda_k^c$. On the other hand if $\bar{\mu}_{k+1} < \mu_k^c$, then we can decrease the value of μ_k^c and decrease that of λ_k^c such that the amount of energy consumed in $[s_k, s_{k+1})$ stays the same. This means that the power in the interval $[s_k, s_{k+1})$ will have a faster decline. By Lemma 6, this new policy delivers a smaller amount of data, i.e., data constraints will not be binding at s_{k+1}, and therefore we will have $\mu_{k+1}^c = \mu_k^c$. The proof of the other scenario stated in the lemma where both the battery and the data buffer are full follows using similar arguments.

G. Proof of Lemma 12

From (46) and (49), we have

$$g(r_1(t), r_2(t)) = \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \tag{81}$$

Since from (48) and (49) we always have

$$\frac{\partial g(r_1(t), r_2(t))}{\partial r_2(t)} - \sigma^2 \geq \frac{\partial g(r_1(t), r_2(t))}{\partial r_1(t)} - 1 \tag{82}$$

with equality iff $r_2(t) = 0$, from (45) and (46), we have

$$\frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \geq \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 \tag{83}$$

Thus, if $r_2(t) > 0$, by complementary slackness $\gamma_2(t) = 0$, and the total power is given by

$$g(r_1(t), r_2(t)) = \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{84}$$

$$> \frac{\nu_1 + \gamma_1(t) - t}{\lambda(t)} - 1 \tag{85}$$

$$\geq \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{86}$$

On the other hand, if $r_2(t) = 0$ and $r_1(t) > 0$, we have

$$g(r_1(t), r_2(t)) = \frac{\nu_2 + \gamma_2(t) - t}{\lambda(t)} - \sigma^2 \tag{87}$$

$$= \frac{\nu_1 - t}{\lambda(t)} - 1 \tag{88}$$

$$\geq \frac{\nu_2 - t}{\lambda(t)} - \sigma^2 \tag{89}$$

Finally, if both rates are zero, then the total power is zero. Combining this with the above gives (56).

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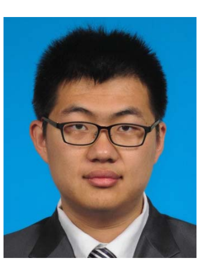

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