Block-Organized Topology Visualization for Visual Exploration of Signed Networks

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Abstract—**Many networks nowadays contain both positive and negative relationships, such as ratings and conflicts, which are often mixed in the layouts of network visualization represented by the layouts of node-link diagram and node indices of matrix representation. In this work, we present a visual analysis framework for visualizing signed networks through emphasizing different effects of signed edges on network topologies. The theoretical foundation of the visual analysis framework comes from the spectral analysis of data patterns in the high-dimensional spectral space. Based on the spectral analysis results, we present a block-organized visualization approach in the hybrid form of matrix, node-link, and arc diagrams with the focus on revealing topological structures of signed networks. We demonstrate with a detailed case study that block-organized visualization and spectral space exploration can be combined to analyze topologies of signed networks effectively.**

Keywords—*Hybrid network visualization; block-organized visualization; signed networks; spectral analysis; visual analytics;*

I. INTRODUCTION

Many networks in real-life applications are signed networks, which can reflect a wide range of relationships, such as like or dislike and friend and enemy. While signed networks can be treated as special cases of multivariate network visualization, such as coloring signed edges differently, the impacts of negative edges to the network topology should be studied and incorporated in the visual exploration process of signed networks [19].

Our approach is built upon a theoretical foundation of spectral network analysis, which studies spectral features of community structures. It is known that there are intimate relationships between the combinatorial characteristics of a graph and the algebraic properties of its adjacency matrix [8]; however, it is often not clear how to analyze a complex network visually with spectral analysis theories. In this work, we concentrate on addressing two research challenges of visual analytics: what are the important spectral patterns and how to use them to study signed networks.

Our visual analysis framework for studying signed networks contains two components. The spectral analysis component starts from two example signed networks, the k-block and k-partite networks, representing the internal and external relationships of communities respectively. We describe the spectral features of general signed networks for interactive exploration. The results provide essential information for visual analysis of community structures, including identifying the important parameter k, the k-dimensional subspace, and visual patterns related to the community structures.

The visualization component presents a block-organized topology visualization for signed networks. The block-organized visualization represents network topologies as multilevel block structures and separates the visual space to three sections for representing communities, positive, and negative relationships. Signed edges are visualized as curved splines oriented to two opposite directions, representing their opposite meanings. Taking a hybrid form of matrix, node-link, and arc diagrams, the block-organized visualization arranges the three representations as ordered layers consistently for building strong visual connections.

The interactive exploration of signed networks combines the two components - spectral space exploration and block-organized visualization. We have developed interaction functions to explore network topologies and adjust the block-organized visualization. We use a case study to demonstrate how complex network topologies can be interactively explored with our approach.

The remainder of this paper is organized as follows. We first review the related work of network visualization in Section II. Section III presents the spectral analysis results and Section IV describes block-organized visualization. The interactive exploration process and case study are described in section V. Section VI discusses the design of signed network visualization, performance, and exploration experiences. Finally, we conclude this paper and discuss future work in Section VII.

II. RELATED WORK

A. Designs of Network Visualization

Network visualization and visual analytics have been very active research areas for the last 30 years [13], [22], [27]. Due to the page limit, we concentrate on different designs of network visualization, although recent work has extended to multivariate and multimodal types [20].

Node-link diagram: The most popular network visualization approach has been the node-link diagram. Classical layout algorithms are force-directed approaches and spectrum-based approaches. Related to this work, Hu et al. [16] presented a node-link layout algorithm and demonstrated the quasi-orthogonal theorem on unsigned networks. Variations of

node-link diagrams include PivotGraph [29] with a grid-based approach and Hive plot [21] with radially oriented linear axes.

Matrix representation: Matrix representation has also been a popular approach to visualize networks. Ghoniem et al. [12] demonstrated the advantages of matrices and node-link diagrams experimentally. Variations of the matrix visualizations include the gestaltmatrix [7], where cells also contained small graphics or glyphs; the Zoomable Adjacency Matrix Explorer (ZAME) [10], which was designed for exploring large scale graphs; and the Compressed Adjacency Matrices [9], which achieved compact visualization by cutting open and rearranging an adjacency matrix.

Arc Diagrams: It is sometimes useful to layout the nodes of a network along a straight line and draw edges as circular arcs [28].

Circular layout: The circular layout is achieved by positioning nodes on the circumference of a circle. Drawing edges as curves rather than straight lines increases the readability of the drawings [11].

Hybrid approaches: Hybrid designs have been shown to be effective in many cases [3], [5]. For example, Topo-Layout [2] detected subgraphs with specific characteristics and applied an appropriate node-link layout algorithm to each subgraph. Nodetrix [15] took advantage of node-link diagrams for sparse networks and matrices for dense networks.

Other approaches: There are a number of interesting ways to visualize networks. For example, a canonical visual matrix visualization only depended on the topological information and nodes were positioned based on computed metrics and/or associated attributes of the nodes [13].

B. Network Visualization and Analysis of Signed Networks

According to our knowledge, signed networks have not been systematically studied in network visualization, while conflicts or controversy relationships in social or political networks have been visualized. For example, Brandes et al. [6] presented a visual summary method for bilateral conflict structures embodied in event data. Suh et al [25] described a model for identifying patterns of conflicts in Wikipedia articles based on users' editing history and relationships between user edits. Kermarrec and Moin [18] presented a Signed LinLog model for graph drawing.

III. SPECTRAL ANALYSIS OF SIGNED NETWORKS

We start with discussions of two example signed networks, the *k*-block network with only internal edges inside *k*-communities and the *k*-partite network with only external cross-community edges. For each example signed network, we provide formal definitions as well as descriptions and examples of spectral patterns for visual exploration.

For general signed networks, we describe how the results from the two example networks can be extended to explore general signed networks. According to the topology structure determined by communities, all the edges of a network, no matter their signs, can be divided to internal or external categories. Therefore, the two special signed networks represent the most important community structures of a signed network.
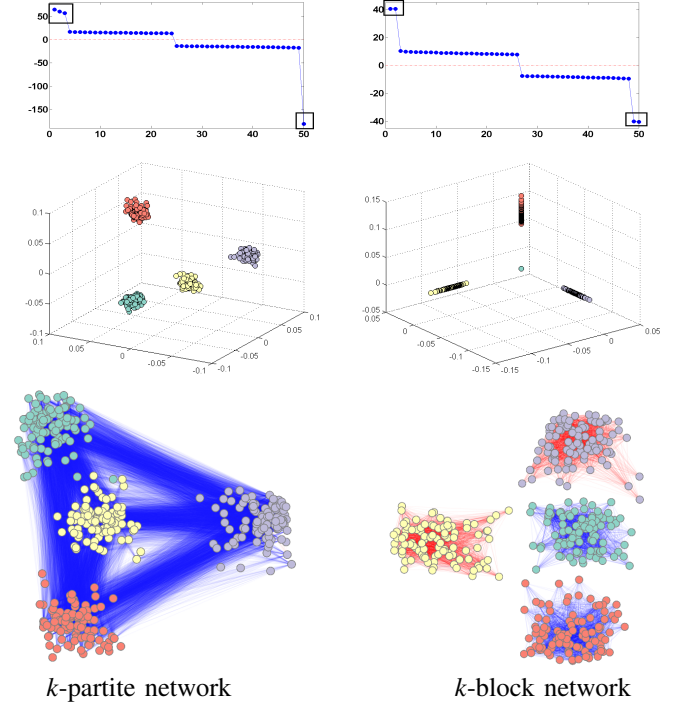

Fig. 1. Examples of *k*-partite and *k*-block signed networks. The top to bottom rows show the eigenvalue curves of 50 dimensions, spectral patterns in selected subspaces, and node-link network visualizations. Positive edges are in red and negative edges are in blue.

A. k-partite Network

The *k*-partite network describes the relationships of nodes between different communities. We first provide the definition and then summarize the study result in [30] for completeness of our framework for general signed networks.

Definition: A *k*-partite network represents a graph with *k* communities such that 1) there are no links inside the communities; and 2) nodes from different communities are densely connected with the same signs. The adjacency matrix A_p can be written in the following form with proper permutation of the nodes:

$$A_p = \begin{pmatrix} \mathbf{0} & B_{12} & \cdots & B_{1k} \\ B_{21} & \mathbf{0} & \cdots & B_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ B_{k1} & B_{k2} & \cdots & \mathbf{0} \end{pmatrix}, \qquad (1)$$

where B_{ij} is the $n_i \times n_j$ matrix to represent the relationships between community *i* and community *j*. We call A_p as a *k*-partite matrix.

Spectral Patterns: For k-partite matrix, Wu et al. [30] has showed the approximation forms of eigenvectors and spectral coordinates. They proved that such a matrix shows *k* orthogonal clusters when the communities have similar densities and the first eigenvalue has a different sign with the following *k* − 1 eigenvalues in magnitude. The *k*-partite network with *k* comparable communities shows *k* orthogonal clusters in the *k*-dimensional spectral subspace spanned by x_i's of the adjacency matrix with corresponding eigenvalues $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_k|$. Furthermore, $\|\lambda_1\|$ has a different sign with the rest *k* − 1 eigenvalues.

An Example: The left column of Figure 1 provides an example of the k-partite network. This network contains four communities with 400 nodes and 100 positive internal edges and total 36000 negative external edges added randomly. As shown in the curve of eigenvalues, there are three very high positive eigenvalues and one very low negative eigenvalue, representing the four communities in the network.

B. k-block Network

Definition: A k-block signed network represents a graph with k communities such that 1) inside each community, nodes are densely connected with the same signs; and 2) there are no links between different communities. The adjacency matrix A_b of a k-block signed network can be written in the following form with proper permutation of the nodes:

$$A_b = \begin{pmatrix} A_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_k \end{pmatrix}, \qquad (2)$$

where A_i is the $n_i \times n_i$ adjacency matrix of the i_{th} community with n_i nodes. We call A_b as a k-block matrix.

Spectral Patterns: For a k-block signed network with k_1 blocks non-negative and k_2 blocks non-positive, we can observe k eigenvalues with large absolute values. The number of large positive eigenvalues (k_1) indicates the number of communities with dense positive internal relationships and the number of negative eigenvalues (k_2) indicates the number of communities with dense negative internal relationships. Specifically, a k-block network with all non-negative entries has k large positive eigenvalues and node coordinates form k orthogonal lines in the subspace spanned by their corresponding eigenvectors. In contrast, for a k-block network with all non-positive entries, we have a similar conclusion: node coordinates form k orthogonal lines in the subspace spanned by eigenvectors corresponding to k large negative eigenvalues.

An Example: The right column of Figure 1 provides an example of the k-block network. This network contains four communities with 100 nodes each. Two communities have 2000 positive edges and the other two have 2000 negative edges. The curve of eigenvalues reveals two positive outstanding eigenvalues and two negative outstanding eigenvalues, which support our spectral analysis results.

C. Discussions for General Signed Networks

For general signed networks, communities are loosely defined as collections of network nodes that interact unusually frequently, including both positive and negative relationships. The adjacency matrix A can be written in the following form with proper permutation of the nodes:

$$A = \begin{pmatrix} A_1 & B_{12} & \cdots & | & \cdots & B_{1k} \\ B_{21} & A_2 & \cdots & | & \cdots & B_{2k} \\ \vdots & \vdots & \cdots & | & \cdots & \vdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \vdots & \vdots & \cdots & | & \cdots & \vdots \\ B_{k1} & B_{k2} & \cdots & | & \cdots & A_k \end{pmatrix}, \quad (3)$$

where the definitions of A_i and B_{ij} are the same as in Formulas 1 and 2.

We have performed a brutal experiment to explore the variations of spectral patterns. By varying the four parameters of edge ratios, internal positive, internal negative, external positive, and external negative, we generate synthetic networks ranging from the two example networks, to approximate k-block and k-partite networks for simulating general signed networks. Due to the space limit, only example results with the base network of 40% internal positive edges are shown in Figure 2. Both the first 3 dimensions of spectral space and eigenvalue curves of selected networks are presented as example patterns.

The examples in Figure 2 demonstrate several variations of the spectral patterns. The knowledge of the spectral patterns can help users to select communities in the spectral space.

1) By adding external negative edges, the network gradually changes to external dominated networks related to the k-partite network. The spectral patterns change from quasi-orthogonal lines or blocks to parallel lines along the third dimension. The eigenvalue curves always contain 3 outstanding absolute values, but they change from 3 positive values to 2 positive and 1 negative value.

2) By adding internal negative edges, the network gradually changes to the k-block networks with dominant negative edges. The spectral patterns change from quasi-orthogonal lines or blocks to 3 quasi-orthogonal lines crossing at the Origin of the spectral space. The 3 outstanding eigenvalues change from all positive to all negative.

In practice, general signed networks may contain both complex internal and external relationships. We argue that our spectral analysis results of the two example signed networks can still be used to study general signed networks from the following three aspects.

First, spectral analysis always presents the dominant community structures in the networks. Eigen-decomposition produces an indexed set of linearly independent eigenvectors, where the first eigenvector having the direction of the largest variance of the data. No matter how complex a network is, the dominant community structures are always revealed on the first several dimensions. This is consistent with the fact that the community relationships of complex networks can be represented as a hierarchical structure.

Second, we discuss general signed networks which contain majority positive or negative edges. While the global community structure may be complex, it can be decomposed to local communities with structures similar to one of the two example networks. For example, as shown in Formula 3, the first group of communities may appear as a k-partite network and the second group appears as k-block networks. Also, the edge densities between these local structures should be much smaller than the densities inside each local community. Otherwise, two communities with both strong internal and external connections, no matter their signs, should be treated as one community instead. Therefore, even though a general signed network may contain a complex hierarchical community structure, it can be decomposed to a number of k-block and k-partite networks.

Third, for general signed networks with various combinations of positive and negative edges. As shown in Figure 2,

External negative edges of 0%, 15%, 30%, 50%, 98% of the maximum external edges are added respectively.

Internal negative edges of 0%, 20%, 40%, 54%, 60% of the maximum internal edges are added respectively.
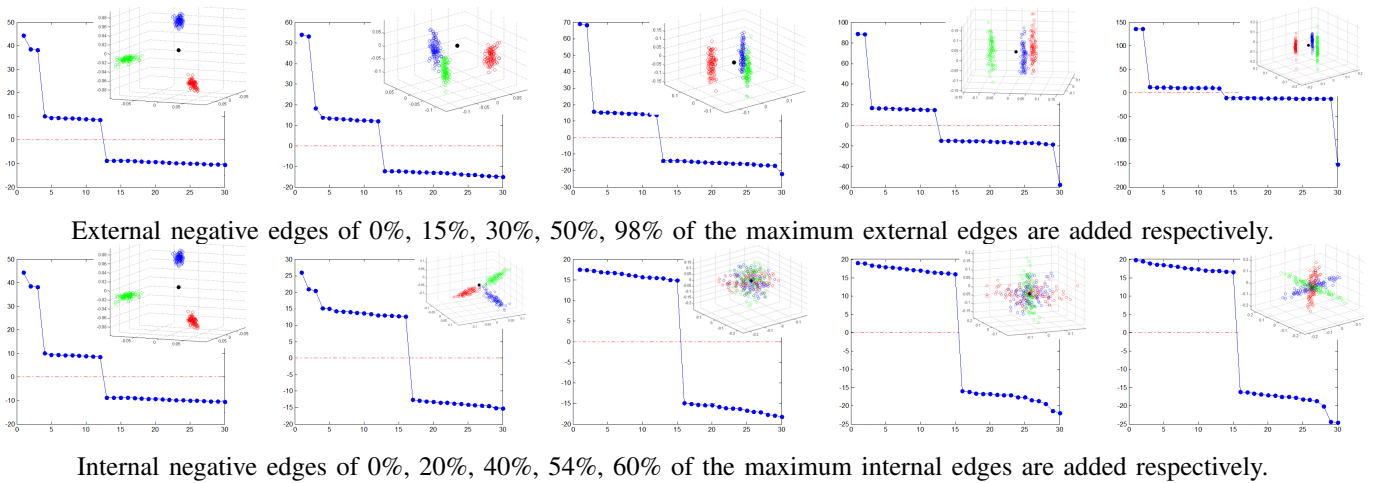
Fig. 2. Example variations of spectral patterns of signed networks with 40% internal positive edges and 2% external positive edges.

the patterns of node distributions in the spectral space adjust gradually when the ratios of negative to positive edges change. Even for the cases whose positive and negative edges are comparable, especially when both positive and negative edges are large enough, the spectral features of both signed and unsigned networks can be shown. The visualization of spectral patterns can also help users to identify similar patterns.

IV. BLOCK-ORGANIZED TOPOLOGY VISUALIZATION

The block-organized visualization is designed for general signed networks through revealing important topology structures and visualizing the positive and negative connections in different formats with different level-of-details.

A. Block-Organized Visualization Design

The block-organized visualization divides a 2D visual space to three sections: blocks on the diagonal line for internal relationships and two opposite regions for the two types of cross-community relationships. The basic design comes from the representation of adjacency matrix A in Formula 3, where the k-block and k-partite networks are well presented in this form. The block-organized visualization integrates the following three concepts.

Block Structures: Block-organized visualization uses a block structure to organize all the nodes in the node-link diagram and all the rectangles in the matrix simultaneously according to network topologies. The block structure is automatically generated during an interactive exploration process and can be adjusted for highlighting interesting communities and connection patterns. With the overlaying of visual elements in the same block structure, we emphasize the consistent visual connections among all the representations.

Signed Edges: We visualize signed edges with curved splines oriented to two opposite directions, representing their opposite meanings. The positive connections stretch out to the top right section and the negative connections to the bottom left section. The different orientations help to organize all the signed edges on the two opposite regions separated automatically by the blocks on the diagonal line. The curved

splines also fully utilize the block spaces and provide strong visual cues for external relationships between communities.

Ordered Layers: Block-organized visualization includes three layers separating all the visual elements, nodes, curved edges, and the matrix. The order of the three layers can be adjusted for different visualization purposes. For example, the nodes and edges can be on the top for visualizing direct connections between communities and the matrix can be on the top for showing the distributions of connections inside the communities. The ordered layers ensure that block-organized visualization can take advantage of all the visualization forms.

B. Multi-level Visualization

Multi-level visualization helps to abstract both internal and external relationships and visualize the network topologies. The block structure is very convenient for multi-level visualization, especially for building a hierarchical structure to visualize the nodes. The multi-level block-organized visualization can be achieved with two options.

First, we can emphasize selected communities and visualize them in large sizes by increasing their importance degrees. The block structure automatically adapts to the change and reorganize the node-link and matrix visualizations consistently.

Second, we can abstract the topology structures of communities at different level-of-details. The grids inside each block control the abstraction level and generate "super nodes" for simplifying the networks. The size of a super node is adjusted by the number of nodes it represents. It is also limited by the size of the block it belongs to.

C. Node Layout

The block-organized visualization organizes the nodes in the network according to their block locations. The layout algorithm can also be different or the same for all the communities in the network.

To assist the visual exploration with spectral patterns, we adopt a simple and efficient spectrum-based strategy which allows us to combine multiple spectral dimensions. While users browse the spectral space, they can identify dimensions

which spread out the nodes in the network. Often these are the spectral dimensions with small absolute eigenvalues and several dimensions can be involved for networks with multiple communities. Users can specify the contributions of the selected spectral dimensions to the X and Y node coordinates in the block-organized visualization by adjusting linear weights $w_x(x_i)$ for X coordinates and $w_y(x_i)$ for Y coordinates. The new coordinate (x', y') of a node is computed for the two dimensions respectively as follows:

$$x' = \sum w_x(x_i) \times x_i$$
$$y' = \sum w_y(x_i) \times x_i \qquad (4)$$

D. Curved Splines for Signed Edges

As shown in Figure 3, both the colors and orentations of curves are used to visualize signed edges. The shapes of the curved splines can be adjusted by the magnitudes of edges, as any of the examples in Figure 3 (left). The orentations can be one of the four cases as shown in Figure 3 (right), depending on the relative positions of two nodes and the sign of the edge. We have all the edges stretched out to the intersection point of the row of one node and the column of another. As the matrix is co-organized with the node-link diagram, the orientations of curved splines are also consistent with the directions of edge locations on the matrix.

The width of a curve can be adjusted with node sizes in the multi-level visualization. The transparency value is controlled by the importance degrees of one community for internal edges and two communities for external edges.
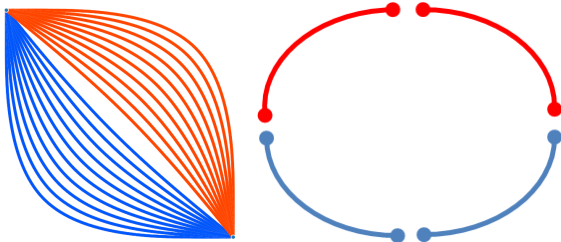

Fig. 3. Curved splines for signed edges. (left) The curve is controled by the absolute edge value. (right) The four cases of curved splines depending on the sign of the edge and relative positions of nodes.

For signed networks, as the sign of edges represents opposite relationships, we can choose any complementary colors. Generally we use a warm color for positive relationships and a cold color for negative relationships. For overlaying the curved edges and matrix blocks, we prefer to use two sets of colors in similar tones, so that each layer in the block-organized visualization can be shown well for building visual connection. Figure 4 shows the colors used in all the results of this paper. The node colors of communities are from the d3.js category20 function. The nodes are colored differently according to their communities.

V. Visual Exploration of Network Topology

For exploring topologies of signed networks, our visualization system combines two components, block-organized visualization and spectral space exploration. The visual exploration procedure includes a number of inter-actions between the two components. The block-organized
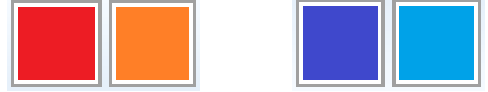

Fig. 4. Color sets for block-organized visualization. Warm colors for positive relationships and cold colors for negative relationships. The left color on each set is used for curved edges and the right color is used for the matrix.

visualization system is shown in Figure 5. We use a real-life dataset to show how the block-organized visualization system can solve several important problems, which are essential for understanding the topology structures of signed networks.
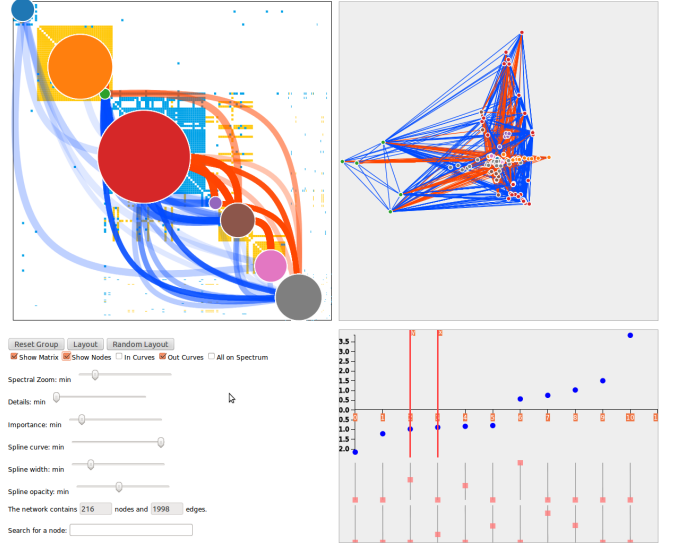

Fig. 5. Our visualization system encloses four panels: a block-organized visualization (left top) and its parameter panel (left bottom), and a spectral space projection (right top) and its parameter panel (right bottom). The parameter panel of the spectral space shows the most significant eigenvalues for the case study.

A. Case Study

We use the Correlates of War dataset [24] to demonstrate the interactive exploration process. The relationships of countries from 1993 to 2001 are accumulated as a signed network, as the relationships are relatively stable during this time range. The positive relationships range from 1 to 3 depending on the alliance relationships; and the negative relationships range from -5 to -1 depending the disputation types. There are total 216 nodes and 1998 edges in this network.

Figure 6 captures the snapshots of block-organized visualization and spectral patterns during the interactive exploration process. As the step 1 shows, we start with the two spectral dimensions with the largest absolute eigenvalues. We can adjust the weights of spectral dimensions for a better separated node layout than the spectral patterns on the first two dimensions. At this moment, the block-organized visualization only contains one block, which is the entire network, and neither the matrix nor the node-link shows much information of the network topology.

Steps 2-8 demonstrate the procedure of identifying new communities. Each time, we observe the spectral space and
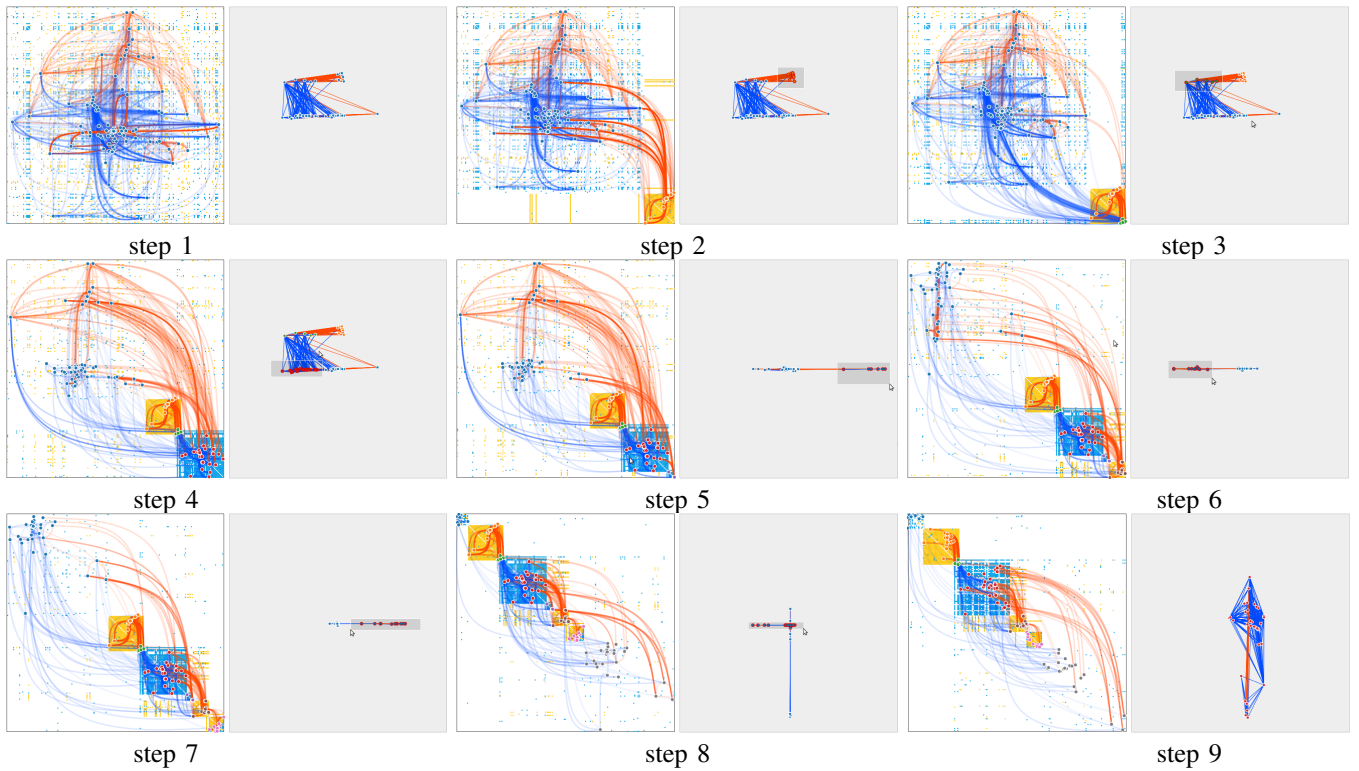
Fig. 6. Interactive exploration of the Correlates of War dataset. Our interface encloses the block-organized visualization on the left and a spectral space projection on the right (the parameter panels are provided and omitted in this Figure). The steps 1-8 show the selected node patterns and block re-organized visualizations. The step 9 shows the grouping of two similar communities, which reveals 6 communities for the final topology structure.

search for variations of patterns ranging from clusters to line structures. The first three communities (steps 2-4) are selected from the initial spectral dimensions. We also switch to other spectral dimensions for additional communities. The next four communities (steps 5-8) are selected from different sub-spaces. To search for these communities, we need to filter the patterns in the spectral space; otherwise the patterns are hidden among large community structures. Note that the spectral spaces for these steps only show the patterns of the nodes remaining in the blue groups. These new communities are communities with smaller sizes (steps 5-7) or loosely connected communities (step 8).

We can also adjust the hierarchical level of the topology visualization by searching for sub-communities or grouping small communities with similar and related connections. For each identified community, we can select the community and check the spectral patterns from different dimensions to ensure that there are no sub-communities. The steps 5-8 can be viewed as searching for sub-communities of the blue block in step 4. From the result of step 8, we have observed that the two communities with red and purple nodes are similar, since they are both densely-connected negative-dominant blocks and they both connect to the community with brown nodes with positive relationships. Step 9 shows the result of grouping these two communities.

We identify step 9 reaches the right topology structure for several reasons. First, the block-organized visualization with only internal edges in Figure 7 (left) shows a nicely ordered matrix visualization. Second, each community is shown as a positive-dominant or negative-dominant block with a homoge-

neous matrix pattern. Third, the external relationships appear to be consistent between different communities. Fourth, Figure 5 shows three large positive eigenvalues for the three communities with positive-dominant internal relationships and two negative eigenvalues for the two communities with negative-dominant internal relationships. Fifth, the sizes of most communities are comparable to each other.

To explore the external relationships, we have observed the connections between two groups. The first group is the communities with red and brown nodes. We can adjust the block visualization options to enlarge these communities at different detail levels. As Figure 7 (middle) shows, among the first group, there are both positive and negative external relationships. The matrix view shows positive-dominant external edges. This group mixes the k-block and k-partite structures. The second group is the communities with the orange, green, and red nodes. As Figure 7 (right) shows, All the connections between the communities with the orange and red nodes are through the green community. While the size of green community is small (only 5 nodes), the role of this community is special in the network for connecting two densely connected k-block communities.

VI. DISCUSSIONS

A. Design of Block-Organized Visualization

Compared to the hybrid design approaches between node-link diagrams and matrix representations [3]–[5], [14], [15], [23], block-organized visualization is different by emphasizing the simultaneous visualization of three representations. Note
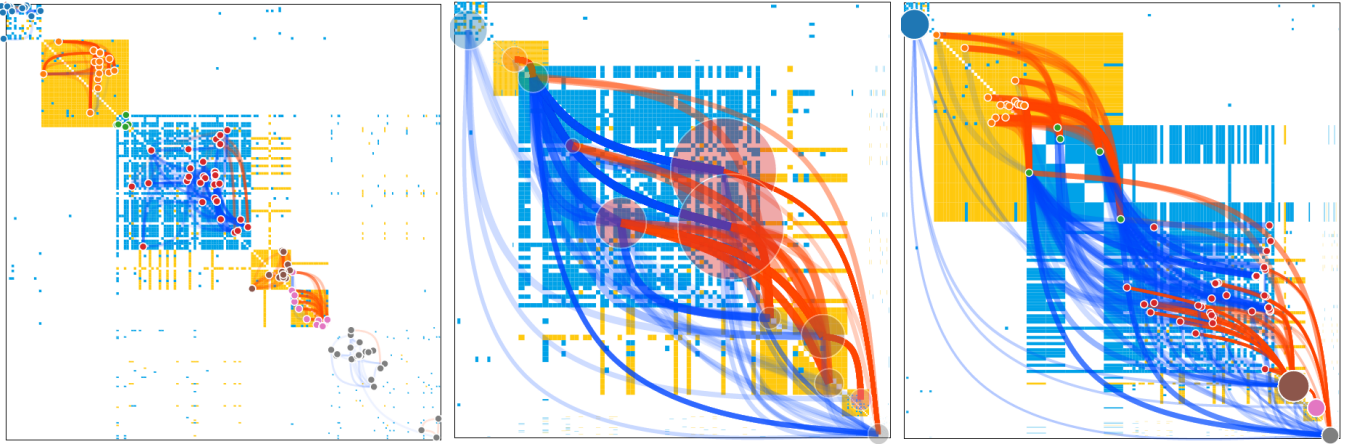
Fig. 7. A set of block-organized visualization for summarizing the Correlates of War dataset from 1993 to 2001. (Left) The network manifests seven communities with a noticeable block pattern (Western, Latin America, their anchor community, Islamic, African, Asia and the former Soviet Union, and countries with only loose connections), which matches the configuration depicted in the Clash of Civilization [17]. (Middle) Multi-level visualization enlarged two communities (Latin America and Asia) and highlighted their positive external connections. (Right) The community with nodes in green (USA, CAN, Haiti, Dominican Republic, Argentina) is the anchor of the orange (Western) and red communities (Latin America), as all the links between orange and red communities are through the green nodes.

that the layout algorithms for these approaches are very different, as matrix visualization is achieved by matrix-reordering algorithms and node-link diagram can be generated by a number of techniques, such as force-directed algorithms. Block-organized visualization arranges the nodes in the node-link diagram and rectangles in the matrix visualization consistently, which is shown to be effective on taking advantages of all representations and is efficient as operations including both user interaction and network visualization are controlled by the same block structure.

The block-organized visualization is different from adjacency matrix, as the number of blocks or communities is often significantly smaller than the number of nodes in a network. Therefore, the main drawback of adjacency matrix for not being intuitive for identifying connections is not an issue for block-organized visualization.

The co-organized node-link diagram preserves the direct connections for edges and separates the internal and external connections. The internal connections can be observed by combining the matrix and node-link diagrams, and the external connections can be observed with the curved edges in the multi-level visualization. We avoid the hair ball of node-link diagram by organizing nodes in blocks and using block structures to achieve multi-level visualization.

The block-organized visualization and spectral space exploration work together well. The block-organized visualization takes the interaction with spectral dimensions efficiently - linear performance as shown in the performance section. The network layout is based on our spectral analysis framework and the visualization provides an interaction mechanism for exploring effects of individual eigenvectors.

B. Computation Complexity

The spectral decomposition is performed with QR algorithm by a reduction to Hessenberg form [26], which is $O(N^2)$ with N as the number of nodes in the network. In our experiment, this step only takes 0.1-0.2 second for networks with up to 1222 nodes and 33428 edges. As we only use the spectral dimensions with the largest absolute eigenvalues, where $k = o(N)$, the spectral decomposition is very efficient for large sparse matrices as many social networks [1].

The complexity of the rendering pipeline is $O(N) + O(E)$ as follows. The block organization only traverses nodes once, therefore the complexity is $O(N)$. The generation of network layout for each block of size n is $O(k \times n)$, therefore the total complexity is $O(N)$. The generation of the curved edges is linear to the number of edges as $O(E)$. This is a nice feature as having a real-time interactive visualization process is crucial to visual exploration.

C. Interactive Exploration of Complex Networks

It worths to mention that spectral analysis studies the most significant data features in the spectral space. Many spectral analysis approaches are designed for balanced networks, in which the sizes of communities are comparable to each other. In such cases, small communities are often treated as sub-groups of large communities. We should be aware that the order and dimensions of communities found in the study need to be combined with prior knowledge of community sizes for generating the correct hierarchical topology structures.

Finding the number of communities k has been a challenging problem for spectral analysis. For networks similar to the two example signed networks, our spectral analysis framework has provided a clear mechanism to identify them. For complex networks, such as networks with both dense k-partite and k-block structures, the numbers of outstanding eigenvalues are not always the same as k. Users need to be aware of the variations of spectral patterns, ranging from clusters to line structures, can all suggest separate communities. For such cases, user may need to divide the nodes in the spectral space gradually with any "similar" spectral patterns during the interactive exploration process. We also need to combine all involved visualizations, spectral patterns, matrix patterns, and node-link diagram, to determine if a community needs to be

further divided and if we have found the right k for a succcssful exploration result.

VII. CONCLUSIONS AND FUTURE WORK

This paper presents a study of signed networks from both spectral analysis and visualization aspects. On the spectral analysis aspect, we have demonstrated the quasi-orthogonal relationships of spectral decomposition and topology structures of signed networks. On the visualization aspect, we have presented a block-organized approach for visualizing general signed networks through a consistent interactive exploration mechanism.

The impacts of negative edges should be further studied to visualize different types of conflict relationships. We are also interested in open-box approaches to utilize the quasi-orthogonal spectral patterns in selected sub-spaces for more intuitive visual exploration.

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