Study guide for test 1

The test may have less questions and you will have about 75 minutes to answer them. You will have to give the simplest possible answer and show all your work. The questions below are sample questions. Besides trying to answer these questions, make sure you also review all homework exercises. The test may also have questions similar to those exercises. During the test, the usage of books or notes, or communicating with other students will not be allowed.

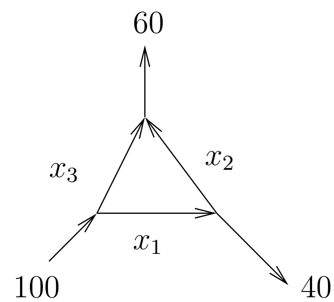
1. Suppose you are given a system of two linear equations, describing a pair of lines in the plane. What can you say about the solution set when these lines are parallel? When is the solution set infinite?
2. Write the coefficient matrix and the augmented matrix for the linear system in Exercise 1.1/11.
3. List the row operations you may use to reduce an augmented matrix into echelon form.
4. For what value(s) of h is the following matrix the augmented matrix of a consistent linear system? If the system is consistent, is the solution unique?
$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 6 & 0 & h & 5 \end{pmatrix}$$
5. In Exercise 1.2/1 describe which matrices are only in echelon form and which are in reduced echelon form.
6. Suppose you have an augmented matrix of a linear system in reduced echelon form. Describe in terms of the pivot columns which are the free variables, when is the system consistent, and when is the solution unique.
7. Find the general solution of the linear system, whose augmented matrix is given in Exercise 1.2/7 by converting your augmented matrix to reduced echelon form and interpreting the result.
8. Write the vector $\begin{bmatrix} 8 \\ 10 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
9. After setting up the appropriate linear system and transforming it it to echelon form, decide whether the set of vectors
$$\left\{ \begin{bmatrix} -1 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$$
is linearly dependent or independent.
10. After setting up the appropriate linear system and transforming it it to echelon form, decide whether the set of vectors
$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$$
is linearly dependent or independent.

11. Restate the question in Exercise 1.4/13 in the language of the previous two questions.

12. Write the solution of the homogeneous system in Exercise 1.5/5 in parametric vector form. (The augmented matrix has to be in reduced echelon form.)

13. Write the solution of the system in Exercise 1.5/11 in parametric vector form and explain how it is related to the solution of the corresponding homogeneous system.

14. Write down the system of linear equations describing the general flow pattern for the network shown in the figure. **Do not solve!**



15. Write down the matrix of the following transformations, mapping $\mathbb{R}^2$ into $\mathbb{R}^2$: reflection about the x_1 axis, reflection about the line $x_1 = x_2$, vertical expansion by a factor of 3, horizontal contraction by a factor of 1/2, reflection through the origin.

16. Write down the matrix of the vertical projection of each point of $\mathbb{R}^3$ onto the plane $x_3 = 0$.

17. Write down the matrix of the rotation of $\mathbb{R}^2$ by $\pi/4$ radians counterclockwise around the origin.

18. Given

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & -1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix},$$

compute $AB + C$

Concluding remarks: For the moment, we have skipped the economic and chemical applications in Section 1.6. From Section 2.1, I expect you to be able to answer questions similar to the last sample question.

Good luck. Gábor Hetyei