Name: **Student ID:**

Sample Test 3

The actual test will have at most 9 questions.

1. Prove that a graph has a 4 flow if and only if it is the union of two even subgraphs.
2. Prove that $\chi(G) \leq \frac{1}{2} + \sqrt{2m + \frac{1}{4}}$ where $\chi(G)$ is the chromatic number and m is the number of edges.
3. Prove that in the proof of the four color theorem we may assume that at most three countries meet at any given point. What does this imply for the dual graph?
4. Explain how the four color theorem may not apply for a map with disconnected countries. How is this issue avoided in the dual statement?
5. Using the "greedy algorithm" what upper bound can you give for $\chi(G)$? By how much does Brooks' theorem improve this bound?
6. Does the "greedy algorithm" always give a reasonable upper bound $\chi(G)$? How large can the bound obtained by the "greedy algorithm" be for bipartite graphs.
7. Explain the difference between $\chi(G)$ and $\operatorname{col}(G)$. What is the relation between them?
8. What is $\operatorname{col}(G)$ for an r-regular graph?
9. Give an example of a graph where $\operatorname{col}(G)$ is strictly more than $\chi(G)$.
10. Prove that for a k-constructible graph $\chi(G) \geq k$ holds. Prove the opposite implication when $k = 3$.
11. Prove König's theorem on the edge-chromatic number of a bipartite graph.
12. State Wizing's theorem on the edge-chromatic number of an arbitrary graph.
13. True or false: if the edge-chromatic number of a graph G is $\Delta(G)$ then G is bipartite. Justify your answer.
14. State the definition of a chordal graph, and describe its structure in terms of patching complete graphs.
15. Prove that interval graphs are chordal.
16. What is the relation between $\chi(G)$ (the chromatic number), and $\omega(G)$ (the size of a largest clique) in an arbitrary graph. Do we know that whenever $\chi(G) = \omega(G)$ the graph must be perfect?
17. Prove that every induced subgraph H of a perfect graph satisfies $|H| \leq \alpha(H) \cdot \omega(H)$.
18. Using the theorem according to which perfect graphs may be characterized by $|H| \leq \alpha(H) \cdot \omega(H)$ for every induced subgraph, prove that the complement of a perfect graph is perfect.
19. Describe the size of the intersection of two sets in terms of their incidence (or characteristic) vectors.

Good Luck. Gábor Hetyei