

Assignment 1

Mandatory questions

1. What is the sixth set in the following sequence?
 1. \emptyset
 2. $\{\emptyset\}$
 3. $\{\emptyset, \{\emptyset\}\}$
 4. $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$
2. There is an infinite train in which the passenger seats are numbered with the positive integers. There is already a passenger sitting on every seat. An unknown but finite number of new passengers arrive who would like to sit next to each other. Describe a way to move the existing passengers in such a way that the new arrivals may be seated next to each other.
3. Which of the Zermelo-Fraenkel axioms implies that the intersection of two sets is a set?
4. Prove that there is only one empty set.
5. Call the sets A and B equivalent if there is a bijection between them. Prove that this is an equivalence relation.
6. Write up $P(\{1, 2, 3\})$ (=the power set of $\{1, 2, 3\}$).
7. Which of the Zermelo-Fraenkel axioms implies that no set can contain itself as an element?

Bonus questions

1. Prove that the powerset $P(A)$ of any nonempty set A has more elements than A .
2. Prove that the “Axiom of Induction” in Landau’s book is equivalent to the “second principle of induction”. (The second principle of induction states the following: Assume that $P(1)$ is true and $P(y)$ is true whenever $P(x)$ is true for all $x < y$. Then $P(x)$ is true for all positive integers x .)