Assignment 6

Due to the Midterm Exam, there will be no oral presentation on October 18.

Mandatory questions to be answered in writing

1. Given any cut $\xi \subset \mathbb{Q}^+$, define $\sqrt{\xi}$ by $\sqrt{\xi} := \{x \in \mathbb{Q}^+ \,|\, x^2 \in \xi\}$. Prove that $\sqrt{\xi}$ is a cut and that $\sqrt{\xi} \cdot \sqrt{\xi} = \xi$.

2. Given any cut $\xi \subset \mathbb{Q}^+$ and $n \in \mathbb{P}$, define $\xi \cdot n$ by $\xi \cdot n := \{x \in \mathbb{Q}^+ \mid \exists y \in \xi (x < y \cdot n)\}$. Without using Theorem 141, prove that $\xi \cdot n$ is a cut. Prove also that whenever ξ is a rational cut $\{x \in \mathbb{Q}^+ \,|\, x < \frac{p}{q}\}$, the set $\xi \cdot n$ is the rational cut $\{x \in \mathbb{Q}^+ \,|\, x < \frac{p \cdot n}{q}\}$.

3. Show that the order on cuts is dense, that is, given $\xi < \eta$ cuts, there is a cut ζ satisfying $\xi < \zeta < \eta$. (Hint: you may choose ζ to be a rational cut.)