# Sample Test 1

*The actual test will have only 5 questions.*

1. Consider the following relation on the set $\{-2,-1,0,1,2\}$: $x \sim y$ means that $|x-y|=1$. Is the relation reflexive? Antireflexive? Symmetric? Draw the graph of the relation.

2. Describe the effect of joining a vertex $xy$ to a graph $G$ in terms of its adjacency matrix.

3. Draw the direct product $C_4 \times P_3$. Here $C_4$ is a cycle of length $4$, $P_3$ is a path of length $3$.

4. Draw the graph whose incidence matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

   (Here the columns are associated to edges.)

5. Show that every $5$-regular graph has an even number of vertices.

6. Assume that $x$ and $y$ are different vertices, and that there is a walk from $x$ to $y$ in the graph. Prove that there is also a path between the same two vertices.

7. Prove that a bipartite graph may not contain an odd cycle. Is the converse also true?

8. Using Dijkstra's algorithm, find a minimum weight path from $s$ to $t$ for the graph shown in Fig. 1. Show all your work.

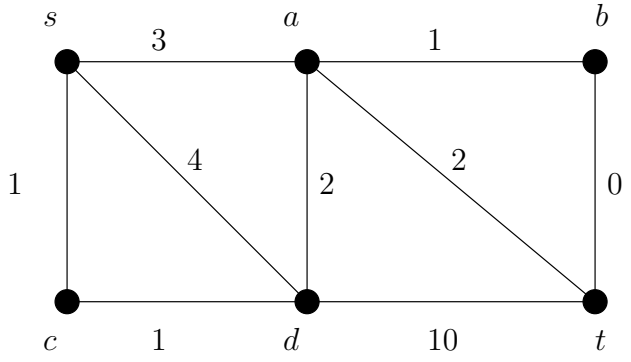


Figure 1: A weighted graph

9. For which values of $n$ does the complete graph $K_n$ have an Euler walk?

10. For which values of $m$ and $n$ does the complete bipartite graph $K_{m,n}$ have a Hamilton path?

11. Prove that an edge $xy$ is a bridge if and only of it is not contained in any cycle.

12. A graph contains $9$ connected components. If there is a bridge in the graph, how many connected components will there be after removing one edge?

13. State the equivalent definitions of a block.
14. State the inequalities between connectivity, edge-connectivity, and minimum degree in a graph. Outline the proof of the inequality.
15. Prove that a graph is a tree if and only if it is minimally connected.
16. Find the number of spanning trees of $P_2 \times P_1$.
17. Using Kruskal's algorithm, find a minimum weight spanning tree of the graph shown in Fig. 1.
18. Outline the proof of the fact that Prim's algorithm yields a minimum weight spanning tree.

Good Luck. Gábor Hetyei