Sample Final Exam Questions (mandatory part)

The actual final exam will have a mandatory and an optional section. The optional questions will be similar to the ones on the previous (sample) tests, and need to be answered only if you do not want me to re-use your average (unadjusted) test score. The list of questions below is supposed to help you prepare for the mandatory part of the final.

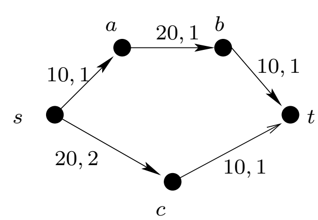
This document is subject to updates and changes until Wednesday December 5.

Last update: Wednesday, December 05, 2007

1. State the König-Egerváry theorem, and explain its relation to network flows.
2. State and prove Hall's Theorem
3. Find a maximum matching in a bipartite graph, just like in the following exercises in our textbook: 4.4/1,3.
4. Solve a transportation problem, just like in the following exercises in our textbook: 4.5/1,3,5,7.
5. Prove that a transportation problem has always an optimal solution S, for which the associated set of edges $E(S)$ contains no circuit.
6. Explain how a system of prices can be associated to a spanning tree solution and express the transportation cost of the solution in terms of the supply and demand prices. Prove your formula.
7. State and prove Birkhoff's Theorem.
8. Write the following doubly stochastic matrix as a convex combination of permutation matrices:

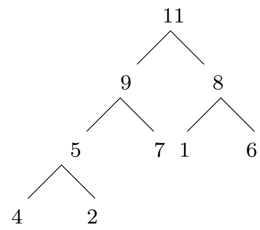
$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

9. In the directed graph below the first coordinate is the capacity, the second is transportation time, measured in hours. Trains leave from each station at the top of each hour. What is the maximum amount of material that can be transported from s to t within 4 hours? Do not solve the problem, only draw the graph of the network flow problem into which this dynamic flow problem may be converted.



10. Use bubble sort to put the following list of numbers into increasing order: $4, 1, 3, 2$. List all steps.
11. What is the number of comparisons in the bubble sort algorithm, if it is used to sort a list of n-entries? Justify your formula.
12. Use merge sort to put the following list of numbers into increasing order: $4, 1, 5, 3, 2$. Indicate all steps in your illustration.

13. What is the maximum number of comparisons used to sort a list of n items using merge sort? Prove your formula.
14. Define what is a "heap" used in the heap sort algorithm.
15. Use heap sort to sort the entries at the nodes of the heap below. Show all phases.



16. Use the branch and bound method to solve the traveling salesperson problem indicated in 3.3/1.
17. Use the approximate algorithm discussed in class to find approximate salesperson tour for the cost matrix

$$\begin{pmatrix} \infty & 4 & 10 & 5 \\ 4 & \infty & 7 & 7 \\ 10 & 7 & \infty & 11 \\ 5 & 7 & 11 & \infty \end{pmatrix}$$

18. What properties must a travel cost matrix have to make the approximate algorithm (discussed in class) applicable? How does the algorithm work? What can you guarantee: how good is the approximate solution compared to the optimal solution?

Remark: the approximate algorithm for the traveling salesperson problem discussed in class is not the same as the one in the book. It is the algorithm that derives from the proof of the Theorem in section 3.3. It involves finding a minimum weight spanning tree and transforming that into a Hamiltonian circuit.

Good Luck. Gábor Hetyei