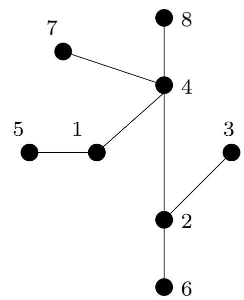
Sample Final Exam Questions (mandatory part)

The actual final exam will have a mandatory and an optional section. The optional questions will be similar to the ones on the previous (sample) tests, and need to be answered only if you do not want me to re-use your average test score. The questions below are supposed to help you prepare for the mandatory part of the final.

1. Draw the tree whose Prüfer code is 222. *Show all your work!*

2. Find the Prüfer code of the tree shown below, *Show all your work!*



3. Use bubble sort to put the following list of numbers into increasing order: $4, 1, 3, 2$. List all steps.

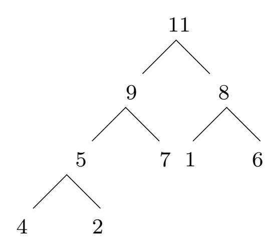
4. What is the number of comparisons in the bubble sort algorithm, if it is used to sort a list of n-entries? Justify your formula.

5. Use merge sort to put the following list of numbers into increasing order: $4, 1, 5, 3, 2$. Indicate all steps in your illustration.

6. What is the maximum number of comparisons used to sort a list of n items using merge sort? Prove your formula.

7. Define what is a "heap" used in the heap sort algorithm.

8. Use heap sort to sort the entries at the nodes of the heap below. Show all phases.



9. Find a minimum cut and a maximum flow for the network shown in Figure 1. **Show all your work!**

10. Suppose the vertices in Figure 1 are cities, and the capacities indicate how many wagons can travel along that edge each day. Suppose wagons can leave each city once a day and each edge takes 2 days to travel. Explain how would you find the number of wagons that can be sent from s to t in 12 days. (Only describe how would you set up the problem.)

11. Give an example of a maximal flow that is not maximum. Explain how finding an augmenting path in the slack picture can help correct mistakes.

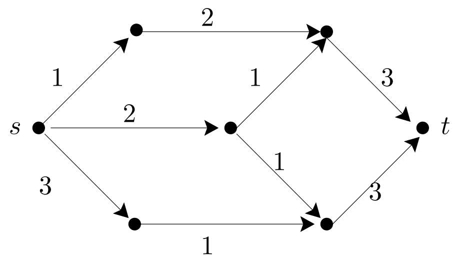


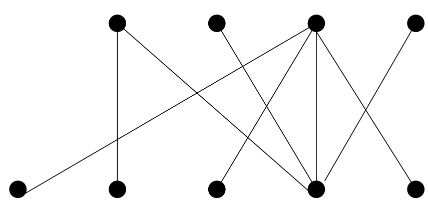
Figure 1: Network with source s and sink t

12. State the Ford-Fulkerson theorem for network flows. Explain how the network flow algorithm may be used to prove it for integer capacities. Indicate what problem you may encounter if you allowed non-integer flows and capacities, and what theorem in analysis makes the Ford-Fulkerson theorem still valid.

13. State Menger's theorem (edge version) and explain how network flows may be used to prove them. (See your notes and "messenger problems" in the book).

14. Explain how the question of finding a maximum size matching and a minimum size edge cover in a bipartite graph may be translated into the question of finding a maximum flow and a minimum cut in a network flow. Prove that the maximum size of a matching is the same as the minimum size of a cover in a bipartite graph.

15. Find a maximum size matching and a minimum size cover in the bipartite graph below by stating and solving a related network flow problem. *Show all your work!*



16. State and prove Hall's Theorem.

17. State and prove Birkhoff's Theorem.

18. Write the following doubly stochastic matrix as a convex combination of permutation matrices:

$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

19. Solve a transportation problem, just like in the following exercises in our textbook: 4.5/1,3,5,7. *Show all your work!*

20. Prove that a transportation problem has always an optimal solution S, for which the associated set of edges $E(S)$ contains no circuit.

21. Use the branch and bound method to solve the traveling salesperson problem indicated in 3.3/1.

Good Luck. Gábor Hetyei