Sample Test 1

The actual test will only have about 5 questions

1. Find the equation of the line passing through $(1, 2)$ and $(2, 5)$. Write your answer in slope-intercept form.

2. Find the equation of the line passing through $(5, 2)$, perpendicular to the line given by $y = 3/2 \cdot x - 2$.

3. Find the domain of $f(x) = \dfrac{\sqrt{5 - x}}{2x - 3}$.

4. The piecewise-defined function $f(x)$ is given by

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 1, \\ 2 - x & \text{if } x < 1. \end{cases}$$

   Find $f(0) + f(1) + f(3)$

5. Find the difference quotient of the function $f(x) = \sqrt{3x - 1}$ at $x = 2$. Simplify your answer and state the limit when $h \to 0$.

6. A pair of supply and demand functions is $p = 2(x + 1)^2$ and $p = 5 - 2x$. State which one is the supply and which one is the demand function and find the equilibrium price

7. Find $\displaystyle\lim_{x \to 3/2} \frac{4x^2 - 9}{2x - 3}$.

8. Find the horizontal asymptote of the following rational functions, if they exist and state your answer in terms of limits:
   - $f(x) = \dfrac{x^2 - 1}{3 - x^2}$
   - $g(x) = \dfrac{x^2 - x}{3 + x^3}$
   - $h(x) = \dfrac{3x^2 - x}{1 + x}$

9. Find $\displaystyle\lim_{x \to 2} \frac{\sqrt{3x - 2} - 2}{x - 2}$.

10. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 3 \\ 1 - x & \text{if } x < 3 \end{cases}$$

   What is the limit $\displaystyle\lim_{x \to 3} f(x)$? Does it exist? If not, find the one-sided limits. Is the function continuous at $x = 3$?

11. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ 2 - x & \text{if } x < 1 \end{cases}$$

What is the limit $\lim_{x\to 1} f(x)$? Does it exist? If not, find the one-sided limits. Is the function continuous at $x = 1$?

12. Let

$$f(x) = \begin{cases} 2x - 2 & \text{if } x > 2 \\ 4 - x & \text{if } x < 2 \\ 1 & \text{if } x = 2 \end{cases}$$

What is the limit $\lim_{x\to 2} f(x)$? Does it exist? If not, find the one-sided limits. Is the function continuous at $x = 2$?

Solutions:

1. The slope is $m = \frac{5-2}{2-1} = 3$. The point-slope equation is $y-2 = 3(x-1)$. After rearranging we get $y = 3x - 1$.

2. The slope of he line $y = 3/2 \cdot x - 2$ is $3/2$. The perpendicular slope is $-2/3$. The point slope equation is $y - 2 = -2/3 \cdot (x - 5)$. After rearranging we get $y = -2/3 \cdot x + 16/3$.

3. The square root of $5 - x$ is only defined if $5 - x \geq 0$, that is $5 \geq x$. The denominator can not be zero, so $2x - 3 \neq 0$, that is, $x \neq 3/2$. The domain is $(-\infty, 3/2) \cup (3/2, 5]$.

4. We have $f(0) = 2 - 0 = 2$, $f(1) = \sqrt{1} = 1$ and $f(3) = \sqrt{3}$. The sum of these numbers is $3 + \sqrt{3}$.

5. The definition of the difference quotient is $\dfrac{f(x+h) - f(x)}{h}$. Substituting $x = 2$ we get $f(2+h) = \sqrt{3(2+h) - 1} = \sqrt{5 + 3h}$ and $f(2) = \sqrt{5}$. The difference quotient is

$$\frac{\sqrt{5+3h} - \sqrt{5}}{h} = \frac{(\sqrt{5+3h} - \sqrt{5})(\sqrt{5+3h} + \sqrt{5})}{h(\sqrt{5+3h} + \sqrt{5})} = \frac{5 + 3h - 5}{h(\sqrt{5+3h} + \sqrt{5})} = \frac{3h}{h(\sqrt{5+3h} + \sqrt{5})}$$
$$= \frac{3}{\sqrt{5+3h} + \sqrt{5}}$$

As $h \to 0$, this expression goes to $\dfrac{3}{2\sqrt{5}}$.

6. The line $p = 5 - 2x$ has negative slope, this must be the demand function. The parabola $p = 2(x+1)^2$ is open up, has its vertex at $x = -1 < 0$, so it is increasing on $[0, \infty)$, this must be the supply function. Supply meets demand when $5 - 2x = 2(x+1)^2$. This can be rewritten as $2x^2 + 6x - 3 = 0$. The quadratic formula gives

$$x = \frac{-6 \pm \sqrt{60}}{4} = \frac{-3 \pm \sqrt{15}}{2}.$$

Only the positive solution is valid, so we must have

$$x = \frac{-3 + \sqrt{15}}{2}.$$

The equilibrium price is

$$p = 5 - 2 \cdot \frac{-3 + \sqrt{15}}{2} = 8 - \sqrt{15}$$

7. We have

$$\frac{4x^2 - 9}{2x - 3} = \frac{(2x-3)(2x+3)}{2x-3} = 2x + 3 \quad \text{for } x \neq 3/2$$

As $x \to 3/2$, the expression $2x + 3$ goes to $2 \cdot 3/2 + 3 = 6$.

8. The degree of the numerator is the same as the degree of the denominator in $f(x)$. The quotient of the leading terms is -1. The horizontal asymptote is $y = -1$. In terms of limits

$$\lim_{x\to\infty} f(x) = -1 \quad \text{and} \quad \lim_{x\to-\infty} f(x) = -1$$

The degree of the numerator is less than the degree of the denominator if $g(x)$. The horizontal asymptote is $y = 0$. In terms of limits

$$\lim_{x\to\infty} g(x) = 0 \quad \text{and} \quad \lim_{x\to-\infty} g(x) = 0.$$

The degree of the numerator is more than the degree of the denominator if $h(x)$. There is no horizontal asymptote. We have

$$\lim_{x\to\infty} \frac{3x^2 - x}{1 + x} = \lim_{x\to\infty} \frac{3x - 1}{1/x + 1} = \lim_{x\to\infty} 3x - 1 = \infty.$$

Similarly, $\lim_{x\to-\infty} \frac{3x^2 - x}{1 + x} = -\infty$.

9.

$$\lim_{x\to 2} \frac{\sqrt{3x-2} - 2}{x - 2} = \lim_{x\to 2} \frac{(\sqrt{3x-2} - 2)(\sqrt{3x-2} + 2)}{(x-2)(\sqrt{3x-2} + 2)} = \lim_{x\to 2} \frac{3x - 6}{(x-2)(\sqrt{3x-2} + 2)}$$
$$= \lim_{x\to 2} \frac{3}{\sqrt{3x-2} + 2} = \frac{3}{4}.$$

10. $\lim_{x\to 3^+} f(x) = 3^2 = 9$ but $\lim_{x\to 3^-} f(x) = 1 - 3 = -2$. These are different numbers. $\lim_{x\to 3} f(x)$ does not exist, the function is not continuous at $x = 3$.

11. $\lim_{x\to 1^+} f(x) = 1^2 = 1$ and $\lim_{x\to 1^-} f(x) = 2 - 1 = 1$. The right-hand limit and the left hand limit exists and they are equal. $\lim_{x\to 3} f(x) = 1$ does exist. The function is continuous at $x = 1$, because the function value $f(1) = 1$ at $x = 1$ is the same as the limit.

12. $\lim_{x\to 2^+} f(x) = 2 \cdot 2 - 2 = 2$ and $\lim_{x\to 2^-} f(x) = 4 - 2 = 2$. The right-hand limit equals the left-hand limit. $\lim_{x\to 2} f(x) = 2$ does exist. The function is not continuous at $x = 2$, because $f(2) = 1 \neq 2 = \lim_{x\to 2} f(x)$.