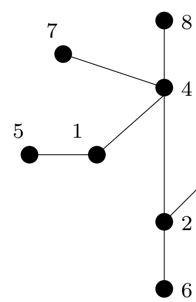
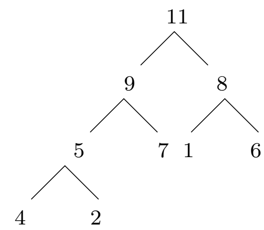
Sample Test II.

The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides looking at the sample questions below, also review all homework exercises, as questions similar to them might occur on the test.

1. State the three equivalent definitions of a tree and prove that they are equivalent.
2. Explain why each tree must have at least one leaf. (Also define leafs.)
3. State the number of edges in a tree on n vertices and prove your formula using induction and the statement in the previous question.
4. Explain how and why the formula on the number of edges in a tree follows from Euler's formula.
5. Draw the tree whose Prüfer code is 222. *Show all your work!*
6. Find the Prüfer code of the tree shown below. *Show all your work!*



7. Use bubble sort to put the following list of numbers into increasing order: $4, 1, 3, 2$. List all steps.
8. What is the number of comparisons in the bubble sort algorithm, if it is used to sort a list of n-entries? Justify your formula.
9. Use merge sort to put the following list of numbers into increasing order: $4, 1, 5, 3, 2$. Indicate all steps in your illustration.
10. What is the maximum number of comparisons used to sort a list of n items using merge sort? Prove your formula.
11. Define what is a "heap" used in the heap sort algorithm.
12. Use heap sort to sort the entries at the nodes of the heap below. Show all phases.



13. Find a breadth-first search and a depth-first search spanning tree in the edge graph of the cube.
14. Explain how depth-first search may be used to find your way out of a maze.
15. (Bonus only!) Explain how search trees may be used to solve the missionaries and cannibals puzzle, and present a solution to this puzzle.

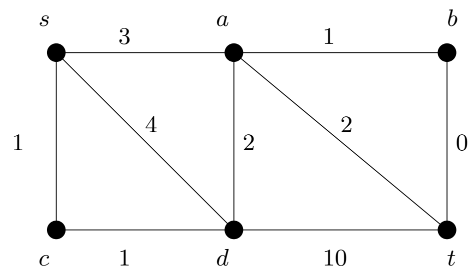


Figure 1: Graph to exercise Dijkstra's algorithm

16. Explain why Dijkstra's algorithm finds the shortest path. Illustrate the algorithm by finding the shortest path between s and t in Figure 1.
17. Prove that Kruskal's algorithm finds a minimum weight spanning tree.
18. Using Kruskal's algorithm find a minimum weight spanning tree for the graph shown in Figure 2. List the edges of the tree in the order you included them during the algorithm.

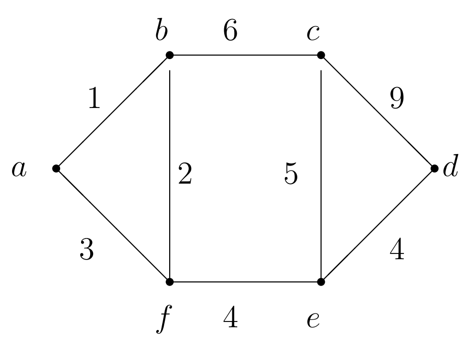


Figure 2: Graph to exercise Kruskal's algorithm

19. Find a minimum cut and a maximum flow for the network shown in Figure 3. **Show all your work!**

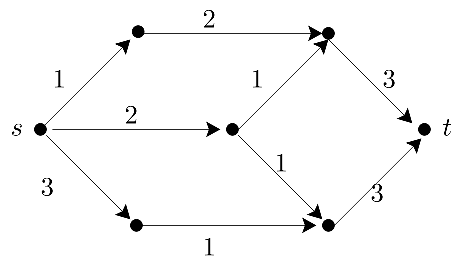


Figure 3: Network with source s and sink t

20. Give an example of a maximal flow that is not maximum. Explain how finding an augmenting path in the slack picture can help correct mistakes.

21. State the Ford-Fulkerson theorem for network flows. Explain how the network flow algorithm may be used to prove it for integer capacities.

22. State Menger's theorem (edge version) and explain how network flows may be used to prove them. (See your notes and "messenger problems" in the book). You only need to state, how would you set up the network associated to the graph.

Good luck. Gábor Hetyei