Study guide for Test 2

The test may have less questions and you will have about 75 minutes to answer them. You will have to give the simplest possible answer and show all your work. The questions below are sample questions. Besides trying to answer these questions, make sure you also review all homework exercises. The test may also have questions similar to those exercises. During the test, the usage of books or notes, or communicating with other students will not be allowed.

1. Using elementary row transformations, find the inverse of the matrix given in 2.2/41 if it exists.
2. Find the inverses of the following elementary matrices: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
3. How many pivot positions does and invertible $n \times n$ matrix have? What can you say about the solution set of $A\mathbf{x} = 0$ if A is an invertible matrix? (Review also the rest of Theorem 8 on page 121.)
4. Find the determinant of the matrix
$$\begin{bmatrix} x & 1 & 0 \\ 1 & 1 & 0 \\ 5 & 0 & 2 \end{bmatrix}$$
using cofactor expansion across the first row. For which values of x is this matrix invertible?
5. Find the determinant and the matrix of cofactors of the matrix
$$\begin{bmatrix} a & 1 & 0 \\ 0 & b & 5 \\ 0 & 0 & c \end{bmatrix}.$$
Use this information to find the inverse of the matrix.
6. Find the area of the parallelogram determined by the columns of $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$.
7. Use Cramer's rule to find the values of the parameter s for which the solution set of
$$sx_1 - x_2 = 1$$
$$x_1 + 2x_2 = 0$$
is unique and find the value of x_1 in the unique solution.
8. The rank of a 6×7 matrix is 5. What is the dimension of its nullspace?
9. The matrix
$$\begin{bmatrix} 1 & 1 & 1 & 10 & 20 \\ 1 & 1 & 2 & 2 & 4 \\ -1 & -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 4 \end{bmatrix} \quad \text{is row equivalent to} \quad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
Find a basis for its column space and for its nullspace.

10. The $\mathcal{B}$-coordinates of a vector relative to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 5 \end{bmatrix} \right\}$ are

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Write the vector $\mathbf{x}$ relative to the standard basis.

11. Show that $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^2$ and write the $\mathcal{B}$-coordinates of the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

12. Find the eigenvalues of the matrix
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{bmatrix}.$$

13. Find all eigenvectors and eigenvalues of the matrix
$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

14. Suppose $\mathbf{u}$ and $\mathbf{v}$ are eigenvectors of the same matrix A, and let λ and μ be the corresponding eigenvalues. Complete the following sentences with $\lambda = \mu$ or $\lambda \neq \mu$.
    (a) If ... then $\mathbf{u}$ and $\mathbf{v}$ must be linearly independent.
    (b) If ... then any linear combination of $\mathbf{u}$ and $\mathbf{v}$ is also an eigenvector.

Concluding remarks: In the upcoming test I will only ask for eigenvectors of triangular matrices. I may ask to find the eigenvalues of any square matrix.

Good luck. Gábor Hetyei