Assignment 6

Oral questions

1. Prove the following converse of the inscribed angle theorem ("Star Trek Lemma"): given a circle centered at O, and three points A, B, and C such that

   (i) B and C are on the circle,
   (ii) The angle $\angle BAC$ is the half of $\angle BOC$,

   the angle A is also on the circle. (I ask you to work out only the case when $\angle BAC$ is acute and O lies in its interior, keeping in mind that there are also other cases, see the first written question. It might help if you consider, how $\angle BAC$ changes when you move the point A on a line containing O, towards O or away from it.)

Questions to be answered in writing

1. Prove that a quadrilateral is cyclic if and only if the sum of two of its opposite angles is 180^o. Explain which implication is related to the Star Trek Lemma, and which to its converse.

2. For a triangle $\triangle ABC$ let A', B', and C', respectively, be the points where the incircle is tangent to the sides BC, AC, and AB, respectively. Prove that the lines AA', BB' and CC' are concurrent. (The common intersection is the *Gergonne point*.)