Assignment 9

Oral question

1. 5.5/12

Question to be answered in writing

1. Let a, b, c, d be real numbers, such that $ad - bc \neq 0$. Using that

$$\frac{az + b}{cz + d} = \begin{cases} \frac{a}{c} + \frac{b - ad/c}{cz + d} & \text{if } c \neq 0, \text{ and} \\ \frac{az + b}{d} & \text{if } c = 0, \end{cases}$$

show that every fractional linear transformation of the above form arises as a combination of horizontal translations $z \mapsto z + b$, dilations $z \mapsto az$ and "reflected inversions" $z \mapsto 1/z$. Conclude that fractional linear transformations preserve angles and the cross-ratio.