# Test III. (sample)

*You have 80 minutes to answer the questions. The usage of books or notes, or communicating with other students is not allowed. Give the simplest possible answer and show all your work. Ask me if you have questions.*

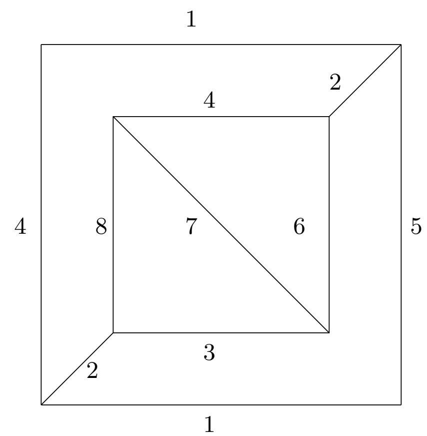
1. Draw the graph whose adjacency matrix is

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and find the number of paths of length 3 between the vertices associated to the first and the fourth line.

2. State Berge's lemma describing the connected components of $M_1 \triangle M_2$. Explain how this lemma may be used to prove Berge's theorem.

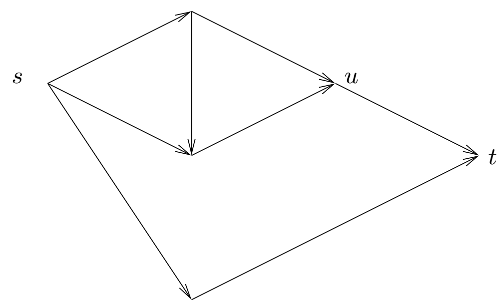
3. Find a minimum weight spanning tree in the graph shown below.




4. Draw a graph whose incidence matrix is

$$A = \begin{pmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}.$$

5. Assume that all edges of the graph below have capacity 1. Find a maximum flow and a minimum cut for source $s$ and sink $t$. How does your answer change if the capacity of the edge $(u, t)$ is changed to 2?

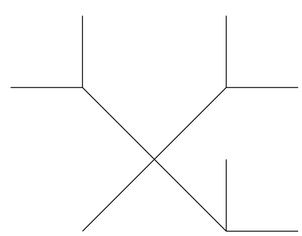



6. Does the family of sets $\{1\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 4, 5\}$ have an SDR? If yes, give one, if not give a reason why not.

7. Assume that you are given a bipartite graph, whose parts are $V_1$ and $V_2$. Assume you have found a maximum size matching $M$. Let $U_1$ be the set of unmatched vertices from $V_1$. Let $A_1$, and $A_2$ be those vertices from $V_1$, and $V_2$ respectively, which may be reached via an alternating path from $U_1$. Give a minimum size vertex cover in terms of these sets.

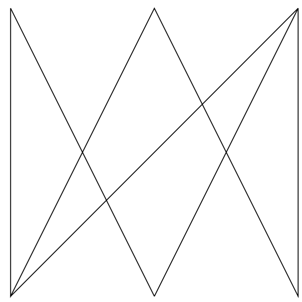
8. State Menger's theorem about vertex cuts and the maximum number of pairwise vertex-disjoint paths.

9. Find a maximum sized matching and a minimum sized cover for the graph shown below. Are the two sets of the same size? Is the graph bipartite?



10. State Hall's marriage theorem.

11. Assume you have to find a maximum sized matching on the graph shown below. Transform this question into the task of finding a maximum flow on a directed graph. **Do not solve!**



12. What is the maximum size of a matching an the minimum size of a cover for the edge-graph of an octahedron?

13. Let $E_1$ and $E_2$ be sets of independent edges, and assume $|E_1| < |E_2|$. Prove that some edge $e \in E_2$ may be added to $E_1$ such that $E_1 \cup \{e\}$ is still a set of independent edges. Which algorithm depends on this idea?

Good luck. Gábor Hetyei