

Assignment 4

Oral questions

1. Assume that f and g are increasing functions on the interval $I \subseteq \mathbb{R}$. Show that $f + g$ is also an increasing function on I . Give an example of two increasing functions f and g on $[0, 1]$, such that $f \cdot g$ is not increasing.
2. Exercise 19.4a
3. Exercise 19.6a
4. Exercises 19.4b and 19.6b
5. Exercise 19.8
6. Exercise 19.10
7. Show that $f(x) = \frac{1}{1+x^2}$ is uniformly continuous on \mathbb{R} .

Question to be answered in writing

1. Consider the function $f : [0, 1] \rightarrow [0, 1]$ given by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 1-x & \text{if } x \notin \mathbb{Q} \end{cases}$$

Show that f is 1-1 and that $f(f(x)) = x$. (So f is its own inverse.) Show that f is continuous only at the point $x = \frac{1}{2}$.