Assignment 9

Oral questions

1. Which of the following must always exist even in hyperbolic geometry: the incircle or the circumcircle? (Think of the crossbar theorem and of the possibility of hyperparallel lines being perpendicular bisectors.)
2. Use the additivity of defect to show that all triangles may not have the same positive defect. Is there an upper bound on the defect of a triangle? Compare this to the upper bound on the defect of a quadrilateral.
3. In neutral geometry we have the alternate interior angle theorem. Prove that in hyperbolic geometry this theorem may be strengthened to saying that two lines having a transversal with congruent alternate interior angles are hyperparallel. (Hint: find a line perpendicular to both lines.)

Question to be answered in writing

1. Assume that the lines ℓ and ℓ' have a common perpendicular line segment MM'. Prove that MM' is the shortest segment between any point of ℓ and any point of ℓ'. (Hint: Assume $A \in \ell$, $A' \in \ell'$ and compare AA' to MM'. Use the second written exercise of Assignment 8 when AA' is perpendicular to ℓ and then use the third oral exercise of Assignment 8 in the other case.)