Study Guide for Test 1.

The real test will have less questions and you will have 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Below you find sample questions and indications which theorems and proofs you will have to remember from the book. Review also all past homework questions as questions similar to them might appear on the test.

1. Define prime, composite, even and odd numbers.
2. Define the relation "x divides y".
3. Using only the notion of distance define "C is between A and B". (A, B, and C are points in the plane.)
4. Transform the statement "all rabbits run fast" into an "if-then" statement.
5. Transform the statement "the square of an odd integer is odd" into an "if-then" statement.
6. Explain why a mathematician believes that "the present king of France is bald". (Hint: think of vacuous truths".)
7. Explain the similarity and the difference between the previous statement and the Boolean expression FALSE $\rightarrow x$.
8. Prove that $x|y$ and $y|z$ implies $x|z$.
9. Would you use lemmas to prove theorems, or is it the other way around?
10. Explain the main idea behind the proof of the following statement: "If x is an integer and $x > 1$ then $x^3 + 1$ is composite."
11. Prove that if a is a multiple of 2 and b is a multiple of 4 then $a + b$ is even. Is the converse also true?
12. Assume you have to prove "A iff B", and you have already shown "not A implies not B". What else do you need to show? "A implies B" or "B implies A"? Justify your answer.
13. Find a counterexample to the statement "if the sum of two integers is even, then both integers are even".
14. Use a truth table to verify that $(x \vee y) \wedge z$ is equivalent to $(x \wedge z) \vee (y \wedge z)$.
15. State DeMorgan's Laws and prove them using truth tables. (Note: I may also refer to any other part of Theorem 6.2 by name and ask you to state it and prove it. Or I may state a part as in the previous question and ask you to prove it.)

16. Give an example of a tautology, and of a contradiction.
17. Express the exclusive or relation using $\wedge$, $\vee$ and negation.
18. Explain how addition and multiplication modulo 2 may be used to evaluate Boolean expressions.
19. Express $A \rightarrow B$ using $\wedge$, $\vee$ and negation.
20. Use the identities given in Theorem 6.2 to show that $((x \rightarrow y) \wedge (y \rightarrow z)) \rightarrow (x \rightarrow z)$ is a tautology. (You are allowed to use the answer to the previous question.
21. Explain the difference between a set and an ordered list.
22. State and prove the multiplication principle.
23. Find the number of lists of length k whose elements are chosen from a pool of n possible elements, if repetitions are permitted. What if they are not?
24. How many ways are there to select a president, a secretary and a treasurer in a club of 25 people if no one is allowed to hold more than position?

Good luck. Gábor Hetyei