Sample Test I.

The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Below I am only listing questions related to the definitions, theorems, and proofs I expect you to know. There will be also application exercises, similar to the already discussed homework questions.

1. How many lists of length k can be formed using elements of the set $[n]$ if repetition of letters is allowed?
2. Explain how counting all subsets is $[n]$ is related to counting binary numbers.
3. How many lists of length k can be formed using elements of the set $[n]$ if repetition of letters is not allowed?
4. In which of the following situations would you use the product principle and in which would you use the addition principle? Explain your choice
   (a) You select 2 ladies and 3 gentlemen from a class.
   (b) You select either 2 ladies or 3 gentlemen from a class.
5. State a formula for the number of k-element subsets of an n element set. Justify your answer by using your answer to question 3 and the equivalence principle (see question 13).
6. State a formula for the number of k-element multisets taken from an n-element set. Justify your answer by reducing your formula to your answer to question 5.
7. Illustrate the phenomenon of overcounting by expressing $|A \cup B|$ in terms of $|A|$, $|B|$ and $|A \cap B|$.
8. Find the number of binary strings of length n that contain at least one 1 and at least one 0.
9. Define: a relation, a function, a one-to-one function, an onto function, domain, codomain, range, an equivalence relation.
10. State the bijection principle. Which of the preceding questions could be answered using this principle?
11. Given an equivalence relation $\mathcal{E}$ on a set A, prove that the set $P = \{\mathcal{E}(a) : a \in A\}$ of equivalence classes of $\mathcal{E}$ is a partition of A.
12. Given P a partition of a set A, prove that the relation

$$R = \{(a, b) \in A \times A \,:\, a \text{ and } b \text{ are in the same block of } P\}$$

is an equivalence relation.

13. State the equivalence principle.
14. State the basic pigeonhole principle. Use it to find the minimum size of a company in which at least two persons were born on the same day of the week.
15. What is the number of all functions from a domain A of size k to a codomain B of size n if the elements of A and B are distinct? What is the number of $1-1$ functions and what is the number of onto functions under the same conditions? Justify your answer!
16. Answer question 15 in the situation when we assume that all elements of the domain A are identical.
17. State and prove Pascal's identity.
18. Explain why Pascal's triangle is symmetric by stating and proving the corresponding formula for binomial coefficients.
19. State and prove the binomial theorem.
20. State and prove the Vandermonde formula.

Good luck. Gábor Hetyei