Study Guide for the Final

Definitions and axioms to remember

1. Axioms: Euclid's postulates, Playfair's postulate, hyperbolic axiom.

2. Definitions: defect of a triangle, horoparallel and hyperparallel lines, fractional linear transformations (also conjugate ones!), hypercycles, horocycles, hyperbolic trigonometric functions, inversion, fractional linear transformations.

3. Poincaré disk model and upper half plane model: points, lines, angle, and distance in these models.

4. Spherical geometry: great circles, lunes, excess, antipodal points, polar triangle.

Statements you should remember with their proof

1. From our textbook: (AAA) congruence in hyperbolic geometry (Theorem 8.4), existence uniqueness of a limiting parallel ray (Theorem 8.10), hyperbolic and spherical Pythagorean Theorem (equation (16.5) and Theorem 18.2), sine and cosine of an angle in a hyperbolic or spherical right triangle (equation (16.4) and Theorem 18.3), hyperbolic and spherical laws of sines (Equation (16.8) and Theorem 18.5) and cosines (Equation (16.7), Theorems 18.6 and 18.7)

2. From lecture and handouts: inversion and all of its properties, exact formulas, with proof. Formulas for a hyperbolic and for a spherical right triangle, proof of Theorem 16.3 (Hyperbolic Law of Cosines for Angles), hyperbolic Pythagorean theorem. Description of hypercycles and horocycles in the Poincaré upper half plane model. Fractional linear transformation connecting the two Poincaré models.

3. From homework: Distance function in the Poincaré disk model is additive, hyperbolic circles are also Euclidean circles in the Poincaré disk model. Fractional linear transformations preserve angles and cross-ratio, description of fractional linear transformations preserving the upper half plane, description of rotations around i in the Poincaré upper half plane model. Complex cross ratio is real for points on a circle or line.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

Statements you should know (without proof)

1. From our textbook: classification of parallel lines (section 9.1), hyperparallel lines have a common perpendicular (Theorem 9.2), angle of parallelism depends only on the distance between the point and the line (Theorem 8.11), statements on singly and doubly asymptotic triangles in sections 9.3 and 9.4, Lobachevskii's formula for the angle of parallelism (Theorems 11.14 and 12.5), defect and excess are proportional to the area (for defect, know also the exact area). I might ask you to apply the spherical or hyperbolic law of sines or cosines. Circumference of the hyperbolic circle in the Poincaré disk model (Theorem 16.1) and circumference of a spherical circle (Lemma 18.4).

What to expect

The exam will be *closed book*. The above guide is meant to help with the mandatory part. For the optional part prepare as if it was another midterm. The mandatory part will be as long as the midterm, the optional part will have only about 5 questions.