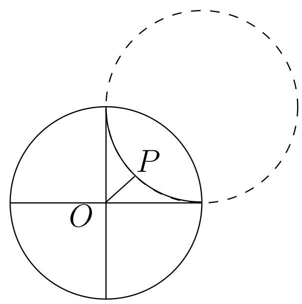
Assignment 9

Oral questions

1. Prove that the distance function $d(A,B) = |\log(AB,PQ)|$ of the Poincaré disk model is additive: if $A * C * B$ on a Poincaré line then $d(AC) + d(CB) = d(AB)$. Fix a Poincaré line with ideal points P and Q and a point A on it. Move another point B along the Poincaré line from P to Q. Show that $d(A,B)$ changes from ∞ to 0 and then back to ∞.

2. Schweikart's constant is the distance d for which the angle of parallelism is $\Pi(d) = 45^o$. Prove that for the length function of the Poincaré disk model, Schweikart's constant equals $\log(1+\sqrt{2})$. Do not use Lobachevski's Theorem (Theorem 11.4) but the formula given in Theorem 11.1, and the picture below. (Explain why d is the length of the line segment OP.)



Question to be answered in writing

1. Prove that inversion preserves the angle of two circles, using the statements on our handout, in the special case when the center of the base circle and the centers of the two other circles are collinear. Assume the center of the base circle is 0 and its radius is 1. Assume the two circles to be inverted have their centers O_1 and O_2 on the real line, at c_1 and c_2 respectively, and that they have radius r_1 and r_2 respectively. Assume the point P is an intersection of these circles. Using the law of cosines, express the cosine of $\angle O_1PO_2$ in terms of c_1, c_2, r_1, r_2. Let O_1', O_2' and P' the image of O_1, O_2 and P under the inversion. Using the formulas on our handout, show that $\angle O_1'P'O_2'$ has the same cosine.