Sample Final Exam Questions

The final exam will be **cumulative**. *The sample questions below are all on the material that was covered after Test 2. To prepare for questions on the earlier material, please refer to the study guides to the tests. Besides the questions listed below and on the sample tests, any question that is similar to a homework question may occur on the final.*

1. Find a parametrization of the following surfaces: a unit sphere, (I will provide the formula for spherical coordinates), a cylinder of height h whose base circle has radius R, a torus obtained by rotating the center of a unit circle around a circle of radius R.
2. Find the tangent plane of the surface given by $(u\cos(v), u\sin(v), u)$ (where $u \geq 0$) at $(1/2, \sqrt{3}/2, 1)$. Also find all points where the surface is not regular.
3. Find the surface area of the cone obtained by rotating the line segment from $(0,0)$ to $(2,4)$ around the x axis.
4. Compute $\int\int_S (x+2y)\,dS$ where S is the triangle with vertices $(1,0,1)$, $(0,1,2)$ and $(1,1,3)$.
5. Explain how the surface integral $\int\int_S \mathbf{F}\cdot\,dS$ of a vector function $\mathbf{F}: \mathbb{R}^3 \to \mathbb{R}^3$ may be expressed as the surface integral of a scalar function, using the unit normal $\mathbf{n}$ pointing to the outside of S. Also express $\mathbf{n}$ in terms of the tangent vectors $\mathbf{T}_u$ and $\mathbf{T}_v$.
6. Suppose a temperature function is given in $\mathbb{R}^3$ by the formula $T(x,y,z) = x^3 + y^3 + z^3$, and let S be the sphere $x^2 + y^2 + z^2 = 4$ oriented with the outward normal. Heat flows with the velocity vector field $F = -\nabla kT$ where k is the heat conductivity. Find the heat flux across the surface S if $k = 0.5$.
7. Give an example of a surface that is not orientable.
8. Compute the surface integral of $\mathbf{F}(x,y,z) = x\mathbf{i} - z\mathbf{j}$ over the graph of $g(x,y) = xy$, whose domain is $[0,1]\times[0,2]$.
9. Use Green's theorem to find $\int_C y\,dx - x\,dy$ where C is the boundary of the unit square $[0,1]\times[0,1]$, oriented in the counterclockwise direction.
10. Illustrate how to extend Green's theorem to non-simple regions by dividing the annular region represented in Figure 1 below into simple regions, orienting their boundaries, and indicating which contributions would cancel.
11. Verify Stokes theorem for the surface $(\cos(v), \sin(v), u)$, $u \in [0,2]$, $v \in [0, 2\pi]$ and the vector field $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j}$.
12. State all equivalent definitions of a conservative vector field.

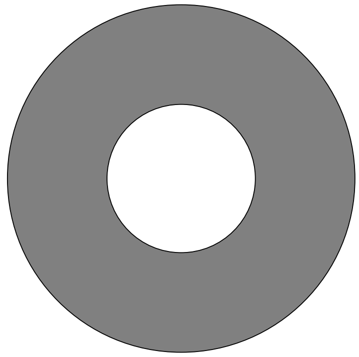

Figure 1: Non-simple region to which Green's theorem is applicable

13. Explain how Stokes theorem implies that a vector field $\mathbf{F}$ with zero curl must satisfy $\int_C \mathbf{F}\, ds = 0$ for any oriented simple closed curve.

14. Is there a vector field $\mathbf{G}$ satisfying $\nabla \times \mathbf{G} = (x, -y, 0)$? How about finding a $\mathbf{G}$ satisfying $\nabla \times \mathbf{G} = (x, -y, z)$?

15. Evaluate $\int\int_{\partial W} \mathbf{F} \cdot\, dS$ where W is the cube $[0,1] \times [0,1] \times [0,1]$ and $\mathbf{F}(x,y,z) = x\mathbf{i} - y\mathbf{j} + z\mathbf{k}$. Perform the calculation directly and then check it using Gauss theorem.

16. Let $F = (3x, 2y, z)$. Let W be the solid cylinder of radius 1, coaxial with the z-axis, and lying between $z = 0$ and $z = 1$. Let S denote the closed surface boundary of W. Use Gauss' divergence theorem to calculate $\int\int_S \mathbf{F} \cdot\, dS$. [Hint: The volume of the cylinder W is π.]

Good luck. Gábor Hetyei