Study Guide for the Midterm Exam

Definitions and axioms to remember

1. Axioms: Euclid's postulates (I may ask you also about Birkhoff's and Hilbert's postulates, but if I do so, I will provide a copy of those), five axioms from the section "Neutral geometry".

2. Definitions: triangular and square numbers, rays, line segments, congruence, similarity, power of a point, sensed ratio, cross-ratio, parallelism, Fermat point, defect of a triangle, and the following triangle centers: centroid, orthocenter, incenter, circumcenter. You should also be able to use inner products and complex numbers.

Statements you should remember with their proof

1. From our textbook: Theorems about triangle congruences, the Star Trek Lemma (+converse, from hw), special cases such as Thales' theorem, application to cyclic quadrilaterals. Law of cosines, extended law of sines. Existence of: centroid, incenter, circumcenter, orthocenter, Euler line; alternate interior angle theorem, weak exterior angle theorem, additivity of defect.

2. From lecture and handouts: existence of a power of a point, Ceva's theorem, Menelaus theorem, Fermat point, central projection preserves cross-ratio. I expect you to be able to prove the existence of such points as the Nagel point or the Gergonne point using Ceva's theorem, but I will provide the definition of such points. (I also may define a triangle center for you that you never heard of before and ask you to use Ceva's theorem to prove its existence.)

3. From homework: Gregory's formula for π, sum of the interior angles of a triangle from Euclid's fifth postulate, distance formula, midpoint formula, existence of the Euler line (or from textbook), formula for the radius of the excircle, Star Trek lemma (+converse),

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

Statements you should know (without proof)

1. From our textbook: Pythagoras' theorem (+converse), equivalence of Playfair's postulate to Euclid's fifth postulate, Heron's formula, Pasch theorem and crossbar theorem, existence of a triangle with positive defect in the hyperbolic plane.

2. From lecture: description of Pythagorean triplets.

Of course you have to remember the standard trigonometric identities, and all facts listed in Section 4.2 (Review of Results from Euclidean Geometry).

What to expect

The exam will be *closed book*. You will have 80 minutes. Some questions may ask you to state and prove a theorem from the list I gave, others may be exercises similar to your homework assignments. There may be questions about examples, whether they have certain properties.