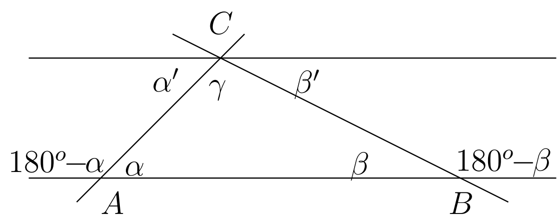
Assignment 2

Oral questions

1. List all primitive Pythagorean triples (a, b, c) that satisfy $b = 20$. (Here b is one of the legs.) How many primitive Pythagorean triples satisfy $b = 30$?

2. Complete the following proof of the theorem stating that the sum of the angles of a triangle ABC is 180^o. We draw parallel line to AB through C and use the notation introduced in the picture.



Applying Euclid's fifth postulate to the line AC and the angles $180^o - \alpha$ and α' yields $180^o - \alpha + \alpha' \geq 180^o$. As a consequence we must have $\alpha' \geq \alpha$. Similarly, applying Euclid's fifth postulate to the line BC and the angles $180^o - \beta$ and β' yields $180^o - \beta + \beta' \geq 180^o$, and so $\beta' \geq \beta$. Hence we obtain

$$\alpha + \beta + \gamma \leq \alpha' + \beta' + \gamma \leq 180^o.$$

Use Euclid's fifth postulate directly in two more situations to show that $\alpha + \beta + \gamma$ is also greater than equal to 180^o.

3. Given a line ℓ in the plane define the relation $A \sim B$ as follows: $A \sim B$ holds if either both are on ℓ or neither of them is on ℓ but they are both on the same side of ℓ. Using the plane separation axiom, prove that the relation $\sim$ is an equivalence relation. Which properties of an equivalence relation are spelled out and which are tacitly assumed? What is the number of equivalence classes? Why?

Questions to be answered in writing

1. Explain how Thales' theorem is a special case of the Star Trek Lemma. Prove Thales' theorem. Prove the Star Trek Lemma in the case when the angle $\angle BOC$ is acute and O is on the line segment AB.

2. Assume that the distance of the points O_1 and O_2 is d. Draw a circle of radius r_1 around O_1 and a circle of radius r_2 around O_2. Express, in terms of equations and inequalities for r_1, r_2 and d, necessary and sufficient conditions for the two circles to have 0, 1 or 2 points in common. (You do not have to prove your claims, but you have to consider all possibilities, including one circle containing the other one.)