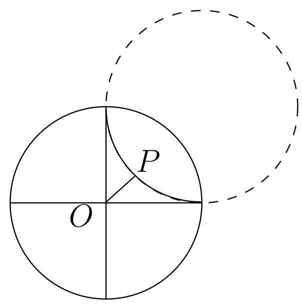
Assignment 8

Oral questions

1. Prove that the distance function $d(A,B) = |\log(AB,PQ)|$ of the Poincaré disk model is additive: if $A*C*B$ on a Poincaré line then $d(AC)+d(CB)=d(AB)$. Fix a Poincaré line with ideal points P and Q and a point A on it. Move another point B along the Poincaré line from P to Q. Show that $d(A,B)$ changes from ∞ to 0 and then back to ∞.

2. Schweikart's constant is the distance d for which the angle of parallelism is $\Pi(d) = 45^o$. Prove that for the length function of the Poincaré disk model, Schweikart's constant equals $\log(1+\sqrt{2})$. Do not use Lobachevski's Theorem (Theorem 9.16) but the formula given in Theorem 9.13, and the picture below. (Explain why d is the length of the line segment OP.)



Question to be answered in writing

1. Let a,b,c,d be real numbers, such that $ac-bd \neq 0$. Using that

$$\frac{az+b}{cz+d} = \begin{cases} \frac{a}{c} + \frac{b-ad/c}{cz+d} & \text{if } c \neq 0, \text{ and} \\ \frac{az+b}{d} & \text{if } c = 0, \end{cases}$$

show that every fractional linear transformation of the above form arises as a combination of horizontal translations $z \mapsto z+b$, dilations $z \mapsto az$ and "reflected inversions" $z \mapsto 1/z$. Conclude that fractional linear transformations preserve angles and the cross-ratio.