Assignment 13

Oral questions

Given a great circle of a sphere, the *poles* of the circle are the endpoints of the diameter that is perpendicular to the circle. (So the poles associated to the Equator are the North Pole and the South Pole.) Conversely, given a pair of antipodal points, the polar of the pair is great circle whose poles are the given points. To a spherical triangle $\triangle ABC$ we assign its polar triangle $\triangle A'B'C'$ as follows: A' is the pole of the great circle through BC containing A, the vertices B' and C' are defined similarly.

1. Prove that the polar of the polar triangle $\triangle A'B'C'$ is the original triangle $\triangle ABC$.
2. If α, β and γ are the angles of $\triangle ABC$ then $(\pi - \alpha)R$, $(\pi - \beta)R$ and $(\pi - \gamma)R$ are the side lengths of $\triangle A'B'C'$. Here R is the radius of the sphere and angles are measured in radians.

Questions to be answered in writing

1. Find the angles of the triangle whose sides are 3, 4, and 5. (Use the hyperbolic law of cosines.)
2. Find the sides of the triangle, whose angles are $A = 10^o$, $B = 20^o$ and $C = 40^o$. (Use the dual law $\cos(C) = -\cos(A)\cos(B) + \sin(A)\sin(B)\cosh(c)$.)