On February 9, 1999 students across America will participate in the 50^{th} anniversary American High School Math Exam. In honor of this milestone, the American Mathematics Competitions has created the Fiftieth AHSME Anniversary Edition which includes one question from each of the first 49 editions of the AHSME and a special 50^{th} question. The publication, reproduction, or duplication of the problems are allowed for classroom and mathteam use. Duplication for other purposes must be obtained in writing. Send an eMail message to Professor Dick Gibbs (gibbs_d@fortlewis.edu), Chair of the CAMC.

1950-10 After rationalizing the numerator of $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$, the denominator in simplest form is

(A) $\sqrt{3}(\sqrt{3}+\sqrt{2})$ **(B)** $\sqrt{3}(\sqrt{3}-\sqrt{2})$ **(C)** $3-\sqrt{3}\sqrt{2}$ **(D)** $3+\sqrt{6}$
(E) none of these answers

1951-48 The area of a square inscribed in a semicircle is to the area of the square inscribed in the entire circle as:

(A) $1:2$ **(B)** $2:3$ **(C)** $2:5$ **(D)** $3:4$ **(E)** $3:5$

1952-44 If an integer of two digits is k times the sum of its digits, the number formed by interchanging the the digits is the sum of the digits multiplied by

(A) $9-k$ **(B)** $10-k$ **(C)** $11-k$ **(D)** $k-1$ **(E)** $k+1$

1953-50 One of the sides of a triangle is divided into segments of 6 and 8 units by the point of tangency of the inscribed circle. If the radius of the circle is 4, then the length of the shortest side is

(A) 12 units **(B)** 13 units **(C)** 14 units **(D)** 15 units **(E)** 16 units

1954-38 If $\log 2 = .3010$ and $\log 3 = .4771$, the value of x when $3^{x+3} = 135$ is approximately

(A) 5 **(B)** 1.47 **(C)** 1.67 **(D)** 1.78 **(E)** 1.63

1955-33 Henry starts a trip when the hands of the clock are together between 8 a.m. and 9 a.m. he arrives at his destination between 2 p.m. and 3 p.m. when the hands are exactly 180° apart. The trip takes

(A) 6 hr. **(B)** 6 hr. $43\frac{7}{11}$ min. **(C)** 5 hr. $16\frac{4}{11}$ min. **(D)** 6 hr. 30 min.
(E) none of these

1956-39 The hypotenuse c and one side a of a right triangle are consecutive integers. The square of the second side is

(A) ca **(B)** $\frac{c}{a}$ **(C)** $c+a$ **(D)** $c-a$ **(E)** none of these

1957-26 From a point P within a triangle, line segments are drawn to the vertices. A necessary and sufficient condition that the three triangles formed have equal areas is that the point P be

(A) the center of the inscribed circle.

(B) the center of the circumscribed circle.

(C) such that the three angles formed at P each be $120°$.

(D) the intersection of the altitudes of the triangle.

(E) the intersection of the medians of the triangle.

1958-45 A check is written for x dollars and y cents, both x and y two-digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$17.82. Then

(A) x cannot exceed 70

(B) y can equal $2x$

(C) the amount of the check cannot be a multiple of 5

(D) the incorrect amount can be twice the correct amount

(E) the sum of the digits of the correct amount is divisible by 9

1959-22 The line joining the midpoints of the daigonals of a trapezoid has length 3. If the longer base is 97, then the shorter base is

(A) 94 **(B)** 92 **(C)** 91 **(D)** 90 **(E)** 89

1960-19 Consider equation **I** : $x + y + z = 46$ where x, y, and z are positive integers, and the equation **II** : $x + y + z + w = 46$ where x, y, z, and w are positive integers. Then

(A) **I** can be solved in consecutive integers.

(B) **I** can be solved in consecutive even integers.

(C) **II** can be solved in consecutive integers.

(D) **II** can be solved in consecutive even integers.

(E) **II** can be solved in consecutive odd integers.

1961-5 Let $S = (x-1)^4 + 4(x-1)^3 + 6(x-1)^2 + 4(x-1) + 1$. Then $S =$

(A) $(x-2)^4$ **(B)** $(x-1)^4$ **(C)** x^4 **(D)** $(x+1)^4$ **(E)** $x^4 + 1$

1962-27 Let $a \textcircled{L} b$ represent the operation on two numbers, a and b, which selects the larger of the two numbers, with $a \textcircled{L} a = a$. Let $a \textcircled{S} b$ represent the operation on two numbers, a and b, which selects the smaller of the two numbers, with $a \textcircled{S} a = a$. Which of the following rules is (are) correct?

(1) $a \textcircled{L} b = b \textcircled{L} a$,

(2) $a \textcircled{L} (b \textcircled{L} c) = (a \textcircled{L} b) \textcircled{L} c$,

(3) $a \textcircled{S} (b \textcircled{L} c) = (a \textcircled{S} b) \textcircled{L} (a \textcircled{S} c)$.

(A) (1) only **(B)** (2) only **(C)** (1) and (2) only **(D)** (1) and (2) only **(E)** all three

1963-37 Given points $P_1, P_2, \ldots, P_7$ on a straight line, in the order stated (not necessarily evenly spaced). Let P be an arbitrary point selected on the line and let s be the sum of the undirected lengths

$$PP_1, PP_2, \ldots, PP_7.$$

Then s is smallest if and only if the point P is

(A) midway between P_1 and P_7 **(B)** midway between P_2 and P_6

(C) midway between P_3 and P_5 **(D)** at P_4 **(E)** at P_1

1964-15 A line through the point $(-a, 0)$ cuts from the second quadrant a triangular region with area T. The equation for the line is

(A) $2Tx + a^2y + 2aT = 0$ **(B)** $2Tx - a^2y + 2aT = 0$

(C) $2Tx + a^2y - 2aT = 0$ **(D)** $2Tx - a^2y - 2aT = 0$

(E) none of these

1965-29 Of 28 students taking at least one subject, the number taking Mathematics and English only equals the number taking Mathematics only. No student takes English only or History only, and six students take Mathematics and History, but no English. The number taking English and History only is five times the number taking all three subjects. If the number taking all three subjects is even and non-zero, the number taking English and Mathematics only is

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

1966-39 In base b the expanded fraction F_1 becomes $.3737\ldots = .\overline{37}$, and the expanded fraction F_2 becomes $.7373\ldots = .\overline{73}$. In base a the expanded fraction F_1 becomes $.2525\ldots = .\overline{25}$, and the expanded fraction F_2 becomes $.5252\ldots = .\overline{52}$. The sum of a and b, each written in base ten, is

(A) 24 **(B)** 22 **(C)** 21 **(D)** 20 **(E)** 19

1967-31 Let $D = a^2 + b^2 + c^2$, where a and b are consecutive integers and $c = ab$. Then $\sqrt{D}$ is

(A) always an even integer

(B) sometimes an odd integer, sometimes not

(C) always an odd integer

(D) sometimes rational, sometimes not

(E) always irrational

1968-32 A and B move uniformly along two straight paths intersecting at right angles in point O. When A is at O, B is 500 yards from O. In 2 minutes they are equidistant from O, and in 8 minutes more they are again equidistant from O. Then the ratio of A's speed to B's speed is

(A) $4:5$ **(B)** $5:6$ **(C)** $2:3$ **(D)** $5:8$ **(E)** $1:2$

1969-29 If $x = t^{1/(t-1)}$ and $y = t^{t/(t-1)}, t > 0, t \neq 1$, a relation between x and y is

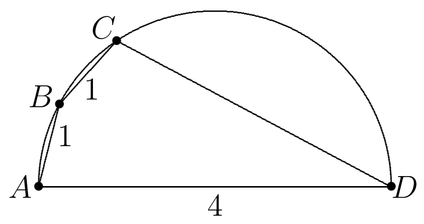
(A) $y^x = x^{1/y}$ **(B)** $y^{1/x} = x^y$ **(C)** $x^y = y^x$ **(D)** $x^x = y^y$ **(E)** none of these

1970-25 For every real number x, let $\lfloor x \rfloor$ be the greatest integer which is less than or equal to x. If the postal rate for first class mail is six cents for every ounce or portion thereof, then the cost in cents of first-class postage on a letter weighing W ounces is always

(A) $6W$ **(B)** $6\lfloor W \rfloor$ **(C)** $6(\lfloor W \rfloor - 1)$ **(D)** $6(\lfloor W \rfloor + 1)$ **(E)** $-6\lfloor -W \rfloor$

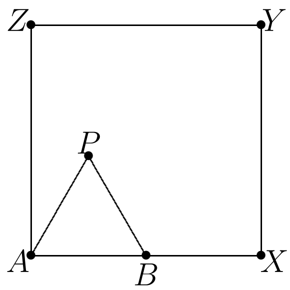
1971-31 Quadrilateral $ABCD$ is inscribed in a circle with side AD, a diameter of length 4. If sides AB and BC each have length 1, then CD has length

(A) $\dfrac{7}{2}$ **(B)** $\dfrac{5\sqrt{2}}{2}$ **(C)** $\sqrt{11}$ **(D)** $\sqrt{13}$ **(E)** $2\sqrt{3}$



1972-35 Equilateral triangle ABP with side AB of length 2 inches is placed inside a square $AXYZ$ with side of length 4 inches so that B is on side AX. The triangle is rotated clockwise about B, then P, and so on along the sides of the square until P, A, and B all return to their original positions. The length of the path in inches traversed by vertex P is equal to

(A) $20\pi/3$ **(B)** $32\pi/3$ **(C)** 12π **(D)** $40\pi/3$ **(E)** 15π



1973-31 In the following equation, each letter represents uniquely a different digit in base ten:

$$(YE)\cdot(ME) = TTT$$

The sum $E + M + T + Y$ equals

(A) 19 **(B)** 20 **(C)** 21 **(D)** 22 **(E)** 24

1974-20 Let $T = \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$; then

(A) $T < 1$ **(B)** $T = 1$ **(C)** $1 < T < 2$ **(D)** $T > 2$

(E) $T = \frac{1}{(3-\sqrt{8})(\sqrt{8}-\sqrt{7})(\sqrt{7}-\sqrt{6})(\sqrt{6}-\sqrt{5})(\sqrt{5}-2)}$

1975-25 A woman, her brother, her son, and her daughter are chess players (all relations by birth). The worst player's twin (who is one of the four players) and the best player are of opposite sex. The worst player and the best player are the same age. Who is the worst player?

(A) the woman **(B)** her son **(C)** her brother **(D)** her daughter

(E) No solution is consistent with the given information

1976-30 How many distinct ordered triples (x, y, z) satisfy the equations

$$\begin{aligned} x + 2y + 4z &= 12 \\ xy + 4yz + 2xz &= 22 \\ xyz &= 6 \end{aligned}$$

(A) none **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

1977-8 For every triple (a, b, c) of non-zero real numbers, form the number

$$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}.$$

The set of all numbers formed is

(A) $\{0\}$ **(B)** $\{-4, 0, 4\}$ **(C)** $\{-4, -2, 0, 2, 4\}$ **(D)** $\{-4, -2, 2, 4\}$

(E) none of the these

1978-22 The following four statements, and only these are found on a card:

On this card exactly one statement is false.
On this card exactly two statements are false.
On this card exactly three statements are false.
On this card exactly four statements are false.

(Assume each statement is either true or false.) Among them the number of false statements is exactly

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

1979-26 The function f satisfies the functional equation

$$f(x) + f(y) = f(x + y) - xy - 1$$

for every pair x, y of real numbers. If $f(1) = 1$, then the number of integers $n \neq 1$ for which $f(n) = n$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinite

1980-22 For each real number x, let $f(x)$ be the minimum of the numbers $4x+1, x+2$, and $-2x+4$. Then the maximum value of $f(x)$ is

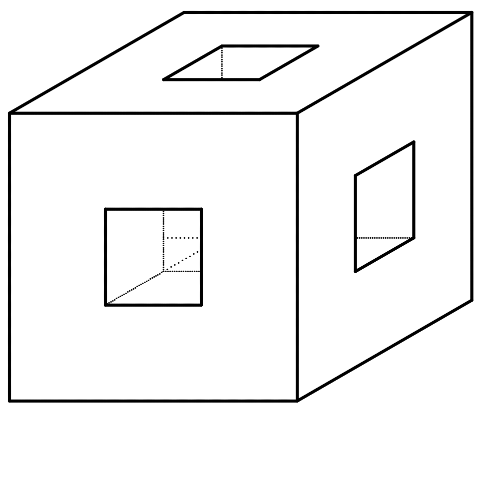
(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{5}{2}$ **(E)** $\frac{8}{3}$

1981-24 If θ is a constant such that $0 < \theta < \pi$ and $z + \frac{1}{z} = 2\cos\theta$, then for each positive integer n, $z^n + \frac{1}{z^n}$ equals

(A) $2\cos\theta$ **(B)** $2^n \cos\theta$ **(C)** $2\cos^n\theta$ **(D)** $2\cos n\theta$ **(E)** $2^n \cos^n\theta$

1982-16 In the adjoining figure, a wooden cube has edges of length 3 meters. Square holes of side one meter, centered in each face, are cut through to the opposite face. The edges of the whole are parallel to the edges of the cube. The entire surface area including the inside, in square meters, is

(A) 54 **(B)** 72 **(C)** 76 **(D)** 84 **(E)** 86



1983-26 The probability that event A occurs is $3/4$; the probability that event B occurs is $2/3$. Let p be the probability that both A and B occur. The smallest interval necessarily containing p is the interval

(A) $\left[\frac{1}{12}, \frac{1}{2}\right]$ **(B)** $\left[\frac{5}{12}, \frac{1}{2}\right]$ **(C)** $\left[\frac{1}{2}, \frac{2}{3}\right]$ **(D)** $\left[\frac{5}{12}, \frac{2}{3}\right]$ **(E)** $\left[\frac{1}{12}, \frac{2}{3}\right]$

1984-11 A calculator has a key which replaces the displayed entry with its square, and another key which replaces the displayed entry with its reciprocal. Let y be the final result if one starts with an entry $x \neq 0$ and alternately squares and reciprocates n times each. Assuming the calculator is completely accurate (e.g., no roundoff or overflow), then y equals

(A) $x^{((-2)^n)}$ **(B)** x^{2n} **(C)** x^{-2n} **(D)** $x^{-(2^n)}$ **(E)** $x^{((-1)^n 2n)}$

1985-24 A non-zero digit is chosen in such a way that the probability of choosing digit d is $\log_{10}(d+1) - \log_{10} d$. The probability that the digit 2 is chosen is exactly $1/2$ the probability that the digit chosen is in the set

(A) $\{2, 3\}$ **(B)** $\{3, 4\}$ **(C)** $\{4, 5, 6, 7, 8\}$ **(D)** $\{5, 6, 7, 8, 9\}$ **(E)** $\{4, 5, 6, 7, 8, 9\}$

1986-14 Suppose hops, skips and jumps are specific units of length. If b hops equals c skips, d jumps equals e hops, and f jumps equals g meters, then one meter equals how many skips?

(A) $\dfrac{bdg}{cef}$ **(B)** $\dfrac{cdf}{beg}$ **(C)** $\dfrac{cdg}{bef}$ **(D)** $\dfrac{cef}{bdg}$ **(E)** $\dfrac{ceg}{bdf}$

1987-12 In an office, at various times during the day the boss gives the secretary a letter to type, each time putting the letter on top of the pile in the secretary's in-box. When there is time, the secretary takes the top letter off the pile and types it. If there are five letters in all, and the boss delivers them in the order 1 2 3 4 5, which of the following could *not* be the order in which the secretary types them?

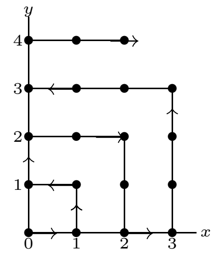
(A) 12345 **(B)** 24351 **(C)** 32415 **(D)** 45231 **(E)** 54321

1988-6 A figure is an equiangular parallelogram if and only if it is a

(A) rectangle **(B)** regular polygon **(C)** rhombus

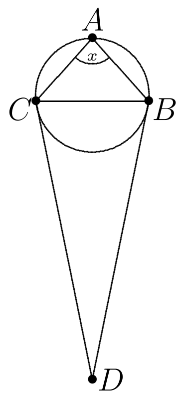
(D) square **(E)** trapezoid

1989-23 A particle moves through the first quadrant as follows. During the first minute it moves from the origin to $(1,0)$. Thereafter, it continues to follow the directions indicated in the figure, going back and forth between the positive x and y axes, moving one unit of distance parallel to an axis in each minute. At which point will the particle be after exactly 1989 minutes?



(A) $(35,44)$ **(B)** $(36,45)$ **(C)** $(37,45)$
(D) $(44,35)$ **(E)** $(45,36)$

1990-14 An acute isosceles triangle, ABC, is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$ and x is the radian measure of $\angle A$, then $x =$



(A) $\frac{3}{7}\pi$ **(B)** $\frac{4}{9}\pi$ **(C)** $\frac{5}{11}\pi$ **(D)** $\frac{6}{13}\pi$ **(E)** $\frac{7}{15}\pi$

1991-28 Initially an urn contains 100 black marbles and 100 white marbles. Repeatedly, three marbles are removed from the urn and replaced from a pile outside the urn as follows:

MARBLES REMOVED	REPLACED WITH
3 black	1 black
2 black, 1 white	1 black, 1 white
1 black, 2 white	2 white
3 white	1 black, 1 white.

Which of the following sets of marbles could be the contents of the urn after repeated applications of this procedure?

(A) 2 black marbles **(B)** 2 white marbles **(C)** 1 black marble
(D) 1 black and 1 white marble **(E)** 1 white marble

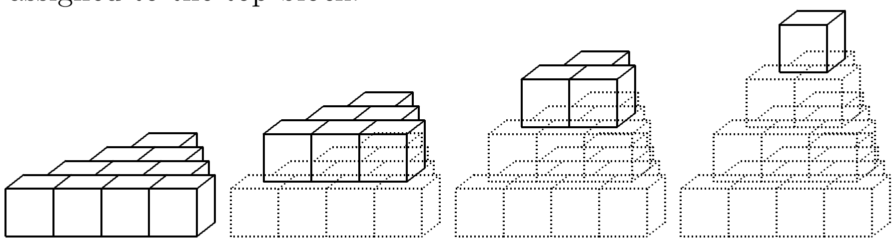
1992-14 Which of the following equations have the same graph?

I. $y = x - 2$ II. $y = \dfrac{x^2 - 4}{x + 2}$ III. $(x + 2)y = x^2 - 4$

(A) I and II only **(B)** I and III only **(C)** II and III only

(D) I, II and III **(E)** None. All the equations have different graphs

1993-22 Twenty cubical blocks are arranged as shown. First, 10 are arranged in a triangular pattern; then a layer of 6, arranged in a triangular pattern, is centered on the 10; then a layer of 3, arranged in a triangular pattern, is centered on the 6; and finally one block is centered on top of the third layer. The blocks in the bottom layer are numbered 1 through 10 in some order. Each block in layers 2, 3 and 4 is assigned the number which is the sum of the numbers assigned to the three blocks on which it rests. Find the smallest possible number which could be assigned to the top block.



(A) 55 **(B)** 83 **(C)** 114 **(D)** 137 **(E)** 144

1994-6 In the sequence

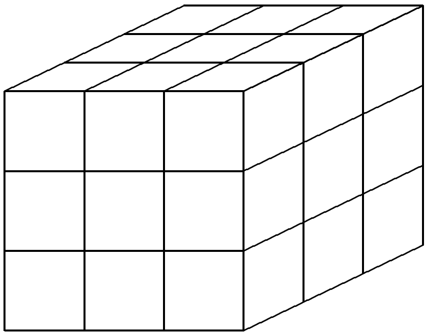
$$\ldots, a, b, c, d, 0, 1, 1, 2, 3, 5, 8, \ldots$$

each term is the sum of the two terms to its left. Find a.

(A) -3 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 3

1995-30 A large cube is formed by stacking twenty-seven congruent small cubes. The large cube is sliced by a plane that is perpendicular to one of its internal diagonals. The plane meets n of the small cubes, but does not contain any of their vertices. A possible value for n is

(A) 13 **(B)** 14 **(C)** 17 **(D)** 19 **(E)** 20



1996-27 Consider two solid spherical balls, one centered at $(0, 0, \frac{21}{2})$ with radius 6, and the other centered at $(0, 0, 1)$ with radius $\frac{9}{2}$. How many points (x, y, z) with only integer coordinates (lattice points) are there in the intersection of the balls?

(A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

1997-29 Call a positive real number *special* if it has a decimal representation that consists entirely of digits 0 and 7. For example, $\frac{700}{99} = 7.\overline{07} = 7.070707\ldots$ and 77.007 are special numbers. What is the smallest n such that 1 can be written as a sum of n special numbers?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10

(E) 1 cannot be represented as a sum of finitely many special numbers

1998-22 What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}?$$

(A) 0.01 **(B)** 0.1 **(C)** 1 **(D)** 2 **(E)** 10

1999-18 How many zeros does $f(x) = \cos(\log(x))$ have on the interval $0 < x < 1$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** infinitely many

Answers:

1950 . . . D	1951 . . . C	1952 . . . C	1953 . . . B	1954 . . . B
1955 . . . A	1956 . . . C	1957 . . . E	1958 . . . B	1959 . . . C
1960 . . . C	1961 . . . C	1962 . . . E	1963 . . . D	1964 . . . B
1965 . . . A	1966 . . . E	1967 . . . C	1968 . . . C	1969 . . . C
1970 . . . E	1971 . . . A	1972 . . . D	1973 . . . C	1974 . . . D
1975 . . . B	1976 . . . E	1977 . . . B	1978 . . . D	1979 . . . B
1980 . . . E	1981 . . . D	1982 . . . B	1983 . . . D	1984 . . . A
1985 . . . C	1986 . . . D	1987 . . . D	1988 . . . A	1989 . . . D
1990 . . . A	1991 . . . B	1992 . . . E	1993 . . . C	1994 . . . A
1995 . . . D	1996 . . . D	1997 . . . B	1998 . . . C	1999 . . . E

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