# UNC Charlotte 2002 Algebra
March 4, 2002

1. Suppose A and B are sets with 5 and 7 elements respectively and $A \cap B$ has 2 elements. How many elements does $A \cup B$ have?

   (A) 8   **(B)** 9   **(C)** 10   **(D)** 11   **(E)** 12

2. Find x such that
$$\frac{8+x}{8-x} = \frac{x}{x+x}.$$

   (A) $-8/3$ **(B)** $-4/3$ **(C)** $-2/3$ **(D)** 0   **(E)** 4

3. What is the length of the interval of solutions to the inequality $1 \le 3 - 4x \le 9$?

   (A) 1.75   **(B)** 2.00   **(C)** 2.25   **(D)** 2.50   **(E)** 3.25

4. Find the values of x for which $x^2 + 3x - 4 > 0$.

   (A) $x < 1$ and $x > -4$ **(B)** $x > 1$ or $x < -4$ **(C)** $x < -1$ or $x > 4$

   (D) $x > -1$ and $x < 4$ **(E)** none of **A**, **B**, **C**, or **D**

5. The equation $ax^2 - 2x\sqrt{2} + 1 = 0$ has a zero discriminant, where a is a real number. Find the root(s) of the equation.

   (A) $\pm\sqrt{2}$ **(B)** 1   **(C)** 2   **(D)** $\pm\dfrac{\sqrt{2}}{2}$ **(E)** $\dfrac{\sqrt{2}}{2}$

6. It is known that the equation $ax^2 + 5x = 3$ has a solution $x = 1$. Find the other solution.

   (A) 0.5   **(B)** 1.5   **(C)** 2   **(D)** 2.5   **(E)** 3

7. Let x_1, x_2 be the two solutions to the equation $2x^2 - x - 2 = 0$. Find the value of $\frac{1}{x_1} + \frac{1}{x_2}$.

   (A) -3 **(B)** -2 **(C)** -1 **(D)** $-1/2$ **(E)** $1/2$

8. It is known that $x = 1$ is a solution to the equation $2x^3 - 3x^2 - 4x + 5 = 0$. What are the two other solutions?

   (A) $\dfrac{-1 \pm \sqrt{41}}{4}$ **(B)** $\dfrac{1 \pm \sqrt{41}}{4}$ **(C)** $\dfrac{1 \pm \sqrt{39}}{4}$ **(D)** $\dfrac{1 \pm \sqrt{41}}{2}$ **(E)** ± 1

9. It takes 852 digits to number the pages of a book consecutively. How many pages are there in the book?

   (A) 184   **(B)** 235   **(C)** 320   **(D)** 368   **(E)** 425

10. Solve the equation $8^{\frac{1}{6}} + x^{\frac{1}{3}} = \frac{7}{3-\sqrt{2}}$.

    (A) 24   **(B)** 27   **(C)** 32   **(D)** 64   **(E)** none of **A**, **B**, **C** or **D**

11. The fraction $\frac{5x-11}{2x^2+x-6}$ was obtained by adding the two fractions $\frac{A}{x+2}$ and $\frac{B}{2x-3}$. Find the value of $A + B$.

    (A) -4 **(B)** -2 **(C)** 1   **(D)** 2   **(E)** 4

12. The slope of the line through the points that satisfy $y = 8 - x^2$ and $y = x^2$ is

    (A) 2   **(B)** 4   **(C)** 0   **(D)** -2 **(E)** -4

13. The product of the zeros of $f(x) = (2x - 24)(6x - 18) - (x - 12)$ is

    (A) -72 **(B)** 5   **(C)** 6   **(D)** 37   **(E)** 432

14. Factor $x^4 + 4y^4$ over the real numbers. Hint: Add and subtract $4x^2y^2$.

    (A) $(x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$

    (B) $(x^2 + 2xy + 2y^2)^2$

    (C) $(x^2 + 2xy - 2y^2)(x^2 + 2xy + 2y^2)$

    (D) $(x^2 - 2xy - 2y^2)(x^2 + 2xy + 2y^2)$

    (E) none of **A**, **B**, **C**, or **D**

15. What is the remainder when $x^2 + 3x - 5$ is divided by $x - 1$?

    (A) -5 **(B)** -2 **(C)** -1 **(D)** 0   **(E)** 1

16. Jeremy starts jogging at a constant rate of five miles per hour. Half an hour later, David starts running along the same route at seven miles per hour. For how many minutes must David run to catch Jeremy?

    (A) 75 minutes   **(B)** 80 minutes   **(C)** 90 minutes

    (D) 95 minutes   **(E)** 105 minutes

17. For the final exam in Professor Ahlin's class, the average(= arithmetic mean) score of the group of failing students was 62 and the average score among the passing students was 92. The overall average for the 20 students in the class was 80. How many students passed the final?

    (A) 9   **(B)** 10   **(C)** 11   **(D)** 12   **(E)** 13

18. Fifteen numbers are picked from the set $\{1, 2, 3, \ldots 20, 21\}$. Find the probability that at least three of those numbers are consecutive.

    (A) 0.1   **(B)** 0.2   **(C)** 0.4   **(D)** 0.5   **(E)** 1.0

19. Cara has 162 coins in her collection of nickels, dimes, and quarters, which has a total value of \$22.00. If Cara has twelve fewer nickels than quarters, how many dimes does she have?

    (A) 50   **(B)** 60   **(C)** 70   **(D)** 74   **(E)** 78

20. Let N denote the smallest four-digit number with all different digits that is divisible by each of its digits. What is the sum of the digits of N?

    (A) 9   **(B)** 10   **(C)** 11   **(D)** 12   **(E)** 13

21. A chain with two links is 13 cm long. A chain made from three links, as shown, of the same type is 18 cm long. How long is a chain made from 25 such links?

   **(A)** 120   **(B)** 128   **(C)** 136   **(D)** 144   **(E)** 150
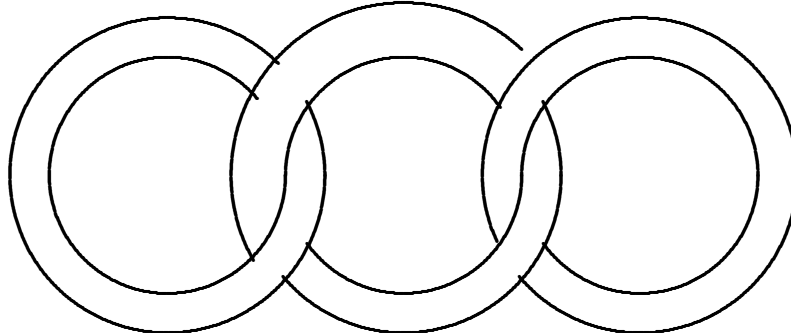
22. What is the sum of the three positive integers a, b, and c that satisfy

$$\frac{1}{a + \frac{1}{b + \frac{1}{c}}} = 3/16?$$

    (A) 6   **(B)** 7   **(C)** 8   **(D)** 9   **(E)** 11

23. A circle C contains the points $(0, 6), (0, 10)$, and $(8, 0)$. What is the x-coordinate of the center?

    (A) 6.75   **(B)** 7.25   **(C)** 7.50   **(D)** 7.75   **(E)** 8.25

24. The number 839 can be written as $19q + r$ where q and r are positive integers. What is the largest possible value of $q - r$?

    (A) 37   **(B)** 39   **(C)** 41   **(D)** 45   **(E)** 47

25. Vic can beat Harold by one tenth of a mile in a two mile race. Harold can beat Charlie by one fifth of a mile in a two mile race. If Vic races Charlie, how far ahead will Vic finish?

    (A) 0.15 miles   **(B)** 0.22 miles   **(C)** 0.25 miles

    (D) 0.29 miles   **(E)** 0.33 miles

26. The four angles of a quadrilateral form an arithmetic sequence. The largest is 15 degrees less than twice the smallest. What is the degree measure of the largest angle?

    (A) $95°$ **(B)** $100°$ **(C)** $105°$ **(D)** $115°$ **(E)** $125°$

27. What is the probability of obtaining an ace on both the first and second draws from an ordinary deck of 52 playing cards when the first card is not replaced before the second is drawn? There are four aces in such a deck.

    (A) 1/221   **(B)** 4/221   **(C)** 1/13   **(D)** 1/17   **(E)** 30/221

28. A running track has the shape shown below. The ends are semicircular with diameter 100 yards. Suppose that the lanes are each 1 yard wide and numbered from the inside to the outside. The competitor in the inside lane runs 700 yards counter clockwise. The other runners start ahead of the inside lane runner, and also run 700 yards, with all runners finishing at the same place. Approximately how much of a head start should a runner in the fifth lane receive over a runner in the first lane?
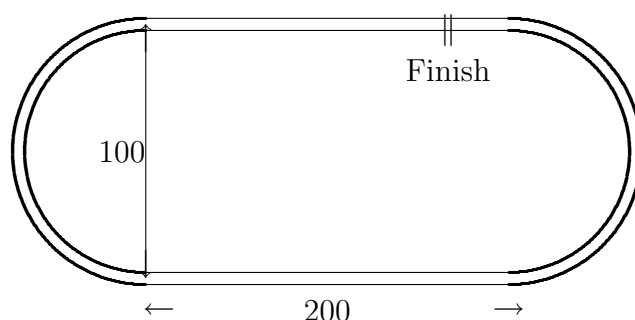
(A) 15 yards   **(B)** 20 yards   **(C)** 25 yards   **(D)** 30 yards   **(E)** 35 yards

29. Dick and Nick share their food with Albert. Dick has 5 loaves of bread, and Nick has 3 loaves. They share the bread equally. Albert gives Dick and Nick 8 dollars which they agree to share fairly. How should they divide the $8 between them?

(A) Dick should get $3 of Albert's money.   **(B)** Dick should get $4 of Albert's money.

(C) Dick should get $5 of Albert's money.   **(D)** Dick should get $6 of Albert's money.

(E) Dick should get $7 of Albert's money.

30. For what positive value of x is there a right triangle with sides $x + 1, 4x$, and $4x + 1$?

(A) 4   **(B)** 6   **(C)** 8   **(D)** 10   **(E)** 12