UNC Charlotte 2002 Comprehensive

March 4, 2002

1. It takes 852 digits to number the pages of a book consecutively. How many pages are there in the book?

   (A) 184 **(B)** 235 **(C)** 320 **(D)** 368 **(E)** 425

2. Solve the equation $8^{\frac{1}{6}} + x^{\frac{1}{3}} = \frac{7}{3-\sqrt{2}}$.

   (A) 24 **(B)** 27 **(C)** 32 **(D)** 64 **(E)** none of **A**, **B**, **C** or **D**

3. The fraction $\frac{5x-11}{2x^2+x-6}$ was obtained by adding the two fractions $\frac{A}{x+2}$ and $\frac{B}{2x-3}$. Find the value of $A + B$.

   (A) -4 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** 4

4. The slope of the line through the points that satisfy $y = 8 - x^2$ and $y = x^2$ is

   (A) 2 **(B)** 4 **(C)** 0 **(D)** -2 **(E)** -4

5. The product of the zeros of $f(x) = (2x - 24)(6x - 18) - (x - 12)$ is

   (A) -72 **(B)** 5 **(C)** 6 **(D)** 37 **(E)** 432

6. Factor $x^4 + 4y^4$ over the real numbers. Hint: Add and subtract $4x^2y^2$.

   (A) $(x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$

   (B) $(x^2 + 2xy + 2y^2)^2$

   (C) $(x^2 + 2xy - 2y^2)(x^2 + 2xy + 2y^2)$

   (D) $(x^2 - 2xy - 2y^2)(x^2 + 2xy + 2y^2)$

   (E) none of **A**, **B**, **C**, or **D**

7. What is the remainder when $x^2 + 3x - 5$ is divided by $x - 1$?

   (A) -5 **(B)** -2 **(C)** -1 **(D)** 0 **(E)** 1

8. Jeremy starts jogging at a constant rate of five miles per hour. Half an hour later, David starts running along the same route at seven miles per hour. For how many minutes must David run to catch Jeremy?

   (A) 75 minutes **(B)** 80 minutes **(C)** 90 minutes

   (D) 95 minutes **(E)** 105 minutes

9. For the final exam in Professor Ahlin's class, the average (= arithmetic mean) score of the group of failing students was 62 and the average score among the passing students was 92. The overall average for the 20 students in the class was 80. How many students passed the final?

   (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

10. Fifteen numbers are picked from the set $\{1, 2, 3, \ldots 20, 21\}$. Find the probability that at least three of those numbers are consecutive.

    (A) 0.1 **(B)** 0.2 **(C)** 0.4 **(D)** 0.5 **(E)** 1.0

11. Cara has 162 coins in her collection of nickels, dimes, and quarters, which has a total value of \$22.00. If Cara has twelve fewer nickels than quarters, how many dimes does she have?

    (A) 50 **(B)** 60 **(C)** 70 **(D)** 74 **(E)** 78

12. Let (h, k) denote the center and let r denote the radius of the circle given by $x^2 + 2x + y^2 - 4y = 4$. What is the sum $h + k + r$?

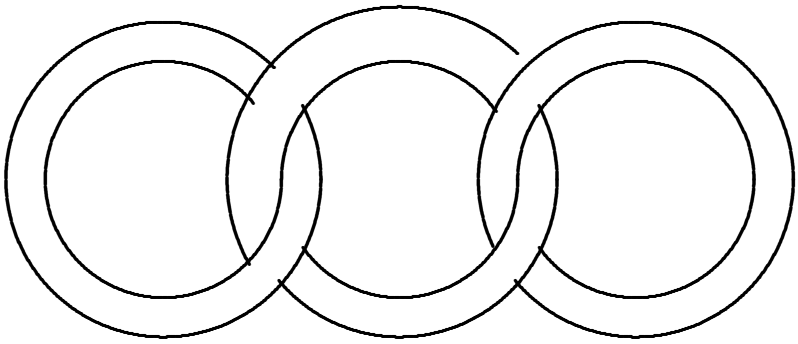
    (A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

13. Let N denote the smallest four-digit number with all different digits that is divisible by each of its digits. What is the sum of the digits of N?

    (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

14. A chain with two links is 13 cm long. A chain made from three links of the same type is 18 cm long. How long is a chain made from 25 such links?

(A) 120 **(B)** 128 **(C)** 136 **(D)** 144 **(E)** 150



15. What is the sum of the three positive integers a, b, and c that satisfy

$$a + \frac{1}{b + \frac{1}{c}} = 7.5?$$

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 11

16. A circle C contains the points $(0, 6)$, $(0, 10)$, and $(8, 0)$. What is the x-coordinate of the center?

(A) 6.75 **(B)** 7.25 **(C)** 7.50 **(D)** 7.75 **(E)** 8.25

17. The number 839 can be written as $19q + r$ where q and r are positive integers. What is the largest possible value of $q - r$?

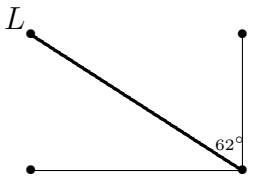
(A) 37 **(B)** 39 **(C)** 41 **(D)** 45 **(E)** 47

18. The four angles of a quadrilateral form an arithmetic sequence. The largest is 15 degrees less than twice the smallest. What is the degree measure of the largest angle?

(A) 95° **(B)** 100° **(C)** 105° **(D)** 115° **(E)** 125°

19. A family is traveling due west on a straight road that passes a famous landmark, L in the figure below. At a given time, the bearing on the landmark is $62°$ west of north. After the family has travelled 5 miles farther, the landmark is $38°$ west of north. What is the closest the family can come to the landmark if they remain on the road?

    (A) 4.25 **(B)** 4.34 **(C)** 4.55 **(D)** 4.76 **(E)** 4.85



20. Vic can beat Harold by one tenth of a mile in a two mile race. Harold can beat Charlie by one fifth of a mile in a two mile race. If Vic races Charlie, how far ahead will Vic finish?

    (A) 0.15 miles **(B)** 0.22 miles **(C)** 0.25 miles

    (D) 0.29 miles **(E)** 0.33 miles

21. For what positive value of x is there a right triangle with sides $x+1, 4x$, and $4x+1$?

    (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

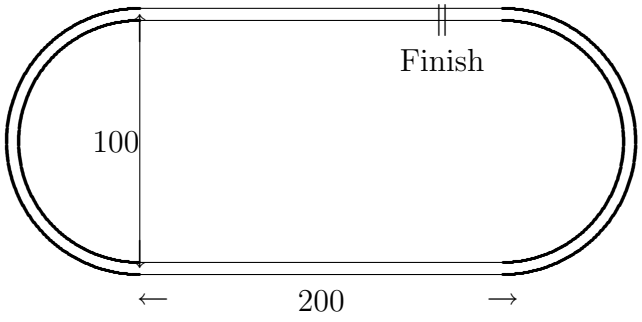
22. What is the probability of obtaining an ace on both the first and second draws from an ordinary deck of 52 playing cards when the first card is not replaced before the second is drawn? There are four aces in the deck.

    (A) 1/221 **(B)** 4/221 **(C)** 1/13 **(D)** 1/17 **(E)** 30/221

23. Let (m, n) be an ordered pair of integers such that to $5m^2 + 2n^2 = 2002$. Which of the following digits could be the units digit of n?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 7 **(E)** 9

24. A running track has the shape shown below. The ends are semicircular with diameter 100 yards. Suppose that the lanes are each 1 yard wide and numbered from the inside to the outside. The competitor in the inside lane runs 700 yards counter clockwise. The other runners start ahead of the inside lane runner, and also run 700 yards, with all runners finishing at the same place. Approximately how much of a head start should a runner in the fifth lane receive over a runner in the first lane?



(A) 15 yards **(B)** 20 yards **(C)** 25 yards **(D)** 30 yards **(E)** 35 yards

25. Dick and Nick share their food with Albert. Dick has 5 loaves of bread, and Nick has 3 loaves. They share the bread equally. Albert gives Dick and Nick 8 dollars which they agree to share fairly. How should they divide the \$8 between them?

(A) Dick should get \$3 of Albert's money. **(B)** Dick should get \$4 of Albert's money.

(C) Dick should get \$5 of Albert's money. **(D)** Dick should get \$6 of Albert's money.

(E) Dick should get \$7 of Albert's money.

26. A 12×12 square is divided into n^2 congruent squares by equally spaced lines parallel to its sides. Circles are inscribed in each of the squares. Find the sum of the areas of the circles.

(A) 6π **(B)** 12π **(C)** 24π **(D)** 36π **(E)** the answer depends on n

27. Consider a square with side length s. Let a denote the area of the inscribed circle (which touches all four edges) and let A denote the area of the circumscribed circle (which goes through all four corners). Which of the following holds?

(A) $a = \frac{1}{2}A$ **(B)** $a = \frac{1}{3}A$ **(C)** $a = \frac{2}{3}A$ **(D)** $a = \frac{2}{\pi}A$ **(E)** $a = \frac{3}{4}A$

28. What is the positive zero of the function

$$f(x) = \log\sqrt{5x+5} + \frac{1}{2}\log(2x+1) - \log 15?$$

(A) 2 **(B)** $\log 15$ **(C)** 3 **(D)** 4 **(E)** f has no positive zero

29. Suppose that the equation $x^2 - px + q = 0$ has roots $x = a$ and $x = b$. Which of the following equations has roots $x = a + \frac{1}{b}$ and $x = b + \frac{1}{a}$?

(A) $x^2 - \left(p + \frac{p}{q}\right)x + \left(q + \frac{1}{q} + 2\right) = 0.$ **(B)** $x^2 - \left(q + \frac{p}{q}\right)x + \left(p + \frac{1}{q} + 2\right) = 0.$
(C) $x^2 - \left(p + \frac{q}{p}\right)x + \left(q + \frac{1}{p} + 2\right) = 0.$ **(D)** $x^2 - \left(p + \frac{q}{p}\right)x + \left(p + \frac{1}{p} + 2\right) = 0.$
(E) $x^2 - \left(q + \frac{q}{p}\right)x + \left(q + \frac{1}{p} + 2\right) = 0$

30. Determine the sum of all the natural numbers less than 45 that are not divisible by 3.

(A) 600 **(B)** 625 **(C)** 650 **(D)** 675 **(E)** 700

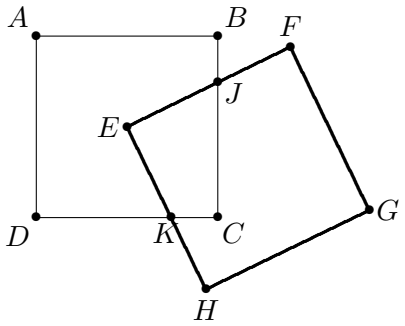
31. Some hikers start on a walk at 10 a.m. and return at 4 p.m. One third of the distance walked is uphill, one third is level, and one third is downhill. If their speed is 4 miles per hour on level land, 2 miles per hour uphill, and 6 miles per hour downhill, approximately how far did they walk?

(A) 18.4 miles **(B)** 19.6 miles **(C)** 22.1 miles

(D) 24.0 miles **(E)** 26.2 miles

32. Two squares, each with side length 12 inches, are placed so that the corner of one lies at the center of the other (see the diagram below). Suppose the length of BJ is 3. What is the area of the quadrilateral EJCK?

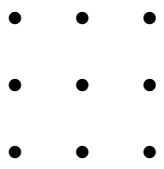
(A) 25 in^2 **(B)** 30 in^2 **(C)** 36 in^2 **(D)** 40 in^2 **(E)** 49 in^2



33. The natural numbers are arranged in groups as follows: {1}, {2,3}, {4,5,6}, {7,8,9,10}, etc. Note that there are always k numbers in the k^{th} group. What is the sum of the numbers in the 10^{th} group?

(A) 260 **(B)** 369 **(C)** 452 **(D)** 505 **(E)** 638

34. Three points are selected simultaneously and randomly from the 3 by 3 grid of lattice points shown. What is the probability that they are collinear? Express your answer as a common fraction.



(A) 1/42 **(B)** 1/21 **(C)** 2/21 **(D)** 1/7 **(E)** 1/6

35. Use each of the digits $2, 3, 4, 6, 7, 8$ exactly once to construct two three-digit numbers M and N so that $M - N$ is positive and is as small as possible. Compute $M - N$.

(A) 33 **(B)** 35 **(C)** 39 **(D)** 41 **(E)** 47

36. What is the largest factor of 11! that is one bigger than a multiple of 6?

(A) 55 **(B)** 77 **(C)** 385 **(D)** 463 **(E)** 1925