UNIVERSITY OF NORTH CAROLINA CHARLOTTE
1995 HIGH SCHOOL MATHEMATICS CONTEST
March 13, 1995

1. $\dfrac{10^{12}-10^{11}}{9}=$

(A) $\dfrac{1}{9}$ **(B)** $\dfrac{10}{9}$ **(C)** 10^3 **(D)** $\dfrac{10^{11}}{9}$ **(E)** 10^{11}

2. If $z=-x$, what are all the values of y for which

$$(x+y)^2+(y+z)^2=2x^2?$$

(A) 0 **(B)** $0,1$ **(C)** $-1,0,1$ **(D)** All positive numbers

(E) There are no values of y for which the equation is true

3. It is known that $\log_{10} 3 = .4771$, correct to four places. How many digits are there in the decimal representation of 3^{100}?

(A) 46 **(B)** 47 **(C)** 48 **(D)** 49 **(E)** 50

4. Given that $\frac{3}{2}<x<\frac{5}{2}$, find the value of

$$\sqrt{x^2-2x+1}+\sqrt{x^2-6x+9}$$

(A) 1 **(B)** 2 **(C)** $2x-4$ **(D)** $4-2x$

(E) none of **A**, **B**, **C** or **D**

5. An archer misses the target on his first shot and hits the target on the next three shots. What is the least number of consecutive hits he must achieve following the first four shots in order to hit the target on more than nine tenths of his shots?

(A) 6 **(B)** 7 **(C)** 9 **(D)** 10 **(E)** 11

6. For all integers n, $(-1)^{n^4+n+1}$ is equal to

(A) -1 **(B)** $(-1)^{n+1}$ **(C)** $(-1)^n$ **(D)** $(-1)^{n^2}$ **(E)** $+1$

7. The graph of $|x| + |y| = 4$ encloses a region in the plane. What is the area of the region?

(A) 4 **(B)** 8 **(C)** 16 **(D)** 32 **(E)** 64

8. Find the minimum value of

$$1 \circ 2 \circ 3 \circ 4 \circ 5 \circ 6 \circ 7 \circ 8 \circ 9$$

where each "$\circ$" is either a "$+$" or a "$\times$".

(A) 36 **(B)** 40 **(C)** 44 **(D)** 45 **(E)** 84

9. Given $3 = \sqrt{a} + \dfrac{1}{\sqrt{a}}$ where $a \neq 0$, find $a - \dfrac{1}{a}$.

(A) 5 **(B)** 6 **(C)** $3\sqrt{5}$ **(D)** 7 **(E)** $5\sqrt{2}$

10. The number $\log_{\frac{1}{4}} \sqrt[3]{1024}$ is equal to

(A) 5 **(B)** $\dfrac{20}{3}$ **(C)** $\dfrac{-5}{3}$ **(D)** $\dfrac{5}{3}$ **(E)** $\dfrac{-20}{3}$

11. Given that

$$x(y-a)=0, \qquad z(y-b)=0, \quad \text{and} \quad a<b,$$

which of the following must be true?

(A) $xz<0$ **(B)** $xz>0$ **(C)** $x=0$

(D) $z=0$ **(E)** $xz=0$

12. If $2^{100}=5m+k$, where k and m are integers and $0\le k\le 4$, then k is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. If $\mathtt{a}$, $\mathtt{b}$, $\mathtt{c}$ and $\mathtt{d}$ represent distinct nonzero base ten digits for which $\mathtt{aaaa}_{\mathtt{ten}}+\mathtt{bbb}_{\mathtt{ten}}+\mathtt{cc}_{\mathtt{ten}}+\mathtt{d}_{\mathtt{ten}}=\mathtt{1995}$, then $\mathtt{a}\cdot\mathtt{b}\cdot\mathtt{c}\cdot\mathtt{d}\ =$

(A) 336 **(B)** 432 **(C)** 486 **(D)** 504 **(E)** 567

14. The number of rational solutions to $x^4-3x^3-20x^2+30x+100=0$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

15. The nine numbers $N, N+3, N+6, \cdots, N+24$, where N is a positive integer, can be used to complete a three by three magic square. What is the sum of the entries of a row of such a magic square? A magic square is a square array of numbers such that the sum of the numbers in each row, each column, and the two diagonals is the same.

(A) $3N$ **(B)** $3N+6$ **(C)** $3N+12$ **(D)** $3N+24$ **(E)** $3N+36$

16. Consider the function $F : N \to N$ defined by

$$F(n) = \begin{cases} n/3 & \text{if } n \text{ is a multiple of 3} \\ 2n+1 & \text{if otherwise} \end{cases}$$

For how many positive integers k is it true that $F(F(k))) = k$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

17. Define the sequence $a_1, a_2, a_3, \ldots$ by

$$a_i = \left\lfloor 10^i \times \frac{1}{13} \right\rfloor - 10 \times \left\lfloor 10^{i-1} \times \frac{1}{13} \right\rfloor, \text{ for } i = 1, 2, \ldots.$$

The largest value of any a_i is

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

Note: For real x, $\lfloor x \rfloor$ is the largest integer that does not exceed x.

18. Triangle T has vertices $(0,30)$, $(4,0)$, and $(30,0)$. Circle C with radius r circumscribes T. Which of the following is the closest to r?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

19. If a, b and c are three distinct numbers such that

$$a^2 - bc = 7, \qquad b^2 + ac = 7, \quad \text{and} \quad c^2 + ab = 7,$$

then $a^2 + b^2 + c^2 =$

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 17

20. The midpoints of the sides of a triangle are $(1, 1)$, $(4, 3)$, and $(3, 5)$. Find the area of the triangle.

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 22

21. Let x and y be two positive real numbers satisfying

$$x + y + xy = 10 \qquad \text{and} \qquad x^2 + y^2 = 40.$$

What integer is nearest $x + y$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

22. Let $f(n)$ be the integer closest to $\sqrt[4]{n}$. Then $\sum_{i=1}^{1995} \frac{1}{f(i)} =$

(A) 375 **(B)** 400 **(C)** 425 **(D)** 450 **(E)** 500