UNIVERSITY OF NORTH CAROLINA CHARLOTTE
1996 HIGH SCHOOL MATHEMATICS CONTEST
March 4, 1996

1. If
$$f(x,y) = (\max(x,y))^{\min(x,y)}$$
and
$$g(x,y) = \max(x,y) - \min(x,y),$$
then
$$f(g(-1,-\frac{3}{2}), g(-4,-1.75)) =$$

(A) -0.5 **(B)** 0 **(C)** 0.5 **(D)** 1 **(E)** 1.5

2. Which of the following statements below could be used to disprove "If p is a prime number, then p is three less than a multiple of four."

(A) Some even numbers are not prime.

(B) Not all odd numbers are prime.

(C) Seven is prime.

(D) Nine is not prime.

(E) Five is prime.

3. If $\log x + \log 5 = \log x^2 - \log 14$, then $x =$

(A) 2^{70} **(B)** 0 **(C)** 70 **(D)** either 0 or 70 **(E)** 70^2

4. A line l_1 has a slope of -2 and passes through the point $(r,-3)$. A second line, l_2, is perpendicular to l_1, intersects l_1 at (a,b), and passes through the point $(6,r)$. The value of a is

(A) r **(B)** $\frac{2}{5}r$ **(C)** 1 **(D)** $2r-3$ **(E)** $\frac{5}{2}r$

5. What is the probability of obtaining an ace on both the first and second draws from a deck of cards when the first card is not replaced before the second is drawn?

(A) 1/13 **(B)** 1/17 **(C)** 1/221 **(D)** 30/221 **(E)** 4/221

6. If $a, b, c,$ and d are nonzero real numbers, $\frac{a}{b} = \frac{c}{d}$, and $\frac{a}{d} = \frac{b}{c}$, then which one of the following must be true?

(A) $a = \pm b$ **(B)** $a = \pm c$ **(C)** $a = \pm d$ **(D)** $b = \pm c$

(E) none of **A**, **B**, **C** or **D**

7. If $\sqrt{2 + \sqrt{x}} = 3$, then $x =$

(A) 1 **(B)** 7 **(C)** 11 **(D)** 49 **(E)** 121

8. A cyclist rides his bicycle over a route which is $\frac{1}{3}$ uphill, $\frac{1}{3}$ level, and $\frac{1}{3}$ downhill. If he covers the uphill part of the route at the rate of 16 miles per hour and the level part at the rate of 24 miles per hour, what rate in miles per hour would he have to travel the downhill part of the route in order to average 24 miles per hour for the entire route?

(A) 32 **(B)** 36 **(C)** 40 **(D)** 44 **(E)** 48

9. The square of $2^{\sqrt{2}}$ equals

(A) 2^2 **(B)** $4^{\sqrt{2}}$ **(C)** 4^2 **(D)** $4^{2\sqrt{2}}$ **(E)** ${4^{\sqrt{2}}}^2$

10. Let A be the ratio of the volume of a sphere to the volume of a cube each of whose faces is tangent to the sphere, and let B be the ratio of the surface area of this sphere to the surface area of the cube. Then

(A) $\frac{A}{B} > 1$ **(B)** $\frac{A}{B} = 1$ **(C)** $1 > \frac{A}{B} > \frac{1}{2}$ **(D)** $\frac{A}{B} = \frac{1}{2}$

(E) $\frac{A}{B} < \frac{1}{2}$

11. The sum of the odd positive integers from 1 to n is 9,409. What is n?

(A) 93 **(B)** 97 **(C)** 103 **(D)** 167 **(E)** 193

12. What is the area of the region of the plane determined by the inequality $3 \leq |x| + |y| \leq 4$?

(A) 7 **(B)** 9 **(C)** 14 **(D)** 16 **(E)** 32

13. For how many positive integers n do there exist n consecutive integers that sum to -1? (The sum of 1 consecutive integer is just the number itself.)

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

14. A circle with radius $\sqrt{2}$ is centered at $(0, 0)$. The area of the smaller region cut from the circle by the chord from $(-1, 1)$ to $(1, 1)$ is

(A) π **(B)** $\sqrt{2} - 1$ **(C)** $\frac{\pi}{2} - 1$ **(D)** $\sqrt{2}(1 - \frac{\pi}{4})$

(E) $\frac{\sqrt{2}}{\pi}$

15. In a math class of size 50, the average score on the final exam is 68. The best ten exams are all 100. The average of the other 40 is

(A) 50 **(B)** 55 **(C)** 60 **(D)** 65 **(E)** 70

16. What is the base x for which $\log_x 729 = 4$?

(A) $3\sqrt{3}$ **(B)** $7/\sqrt{2}$ **(C)** 5 **(D)** $2\sqrt{5}$ **(E)** $\pi\sqrt{3}$

17. For what positive value of c does the line $y = -x + c$ intersect the circle $x^2 + y^2 = 1$ in *exactly* one point?

(A) $\ln 4$ **(B)** $4^{1/3}$ **(C)** $\frac{3}{2}$ **(D)** $\sqrt{2}$ **(E)** $\sin^{-1}(1)$

18. If a, b and c satisfy $a^2 + b^2 = 208$, $b^2 + c^2 = 164$, and $c^2 + a^2 = 244$, then $a^2 + b^2 - c^2 =$

(A) -36 **(B)** 20 **(C)** 108 **(D)** 120 **(E)** 180

19. A grocer has c pounds of coffee that are divided equally among k sacks. She finds n empty sacks and decides to redistribute the coffee equally among the $k+n$ sacks. When this is done, how many fewer pounds of coffee does each of the original sacks hold?

(A) $\dfrac{c}{k+n}$ **(B)** $\dfrac{c}{k+cn}$ **(C)** $\dfrac{c}{k^2+kn}$ **(D)** $\dfrac{cn}{k+n}$

(E) $\dfrac{cn}{k^2+kn}$

20. The quantities x, y, and z are positive and $xy = \frac{z}{4}$. If x is increased by 50% and y is decreased by 25%, how must z be changed so that the relation $xy = \frac{z}{4}$ remains true?

(A) z must be decreased by 12.5%

(B) z must be increased by 12.5%

(C) z must be decreased by 25%

(D) z must be increased by 25%

(E) z must be increased by 50%

21. If N is the cube of a certain positive integer, which of the following is the square of the next positive integer?

(A) $\sqrt{(N+1)}$ **(B)** $\sqrt[3]{(N+1)}$ **(C)** N^2+1

(D) $N^{2/3}+2N^{1/3}+1$ **(E)** $N^{2/3}-2N^{1/3}+1$

22. In a certain class, two-thirds of the female students and half of the male students speak Spanish. If there are three-fourths as many girls as boys in the class, what fraction of the entire class speaks Spanish?

(A) $\dfrac{5}{6}$ **(B)** $\dfrac{4}{7}$ **(C)** $\dfrac{4}{5}$ **(D)** $\dfrac{1}{3}$

(E) none of **A**, **B**, **C** or **D**

23. If $x \neq y$ and $\frac{x^3-y^3}{x-y} = 8$, then $x^2 + xy + y^2 =$

(A) 2 **(B)** 5 **(C)** 8 **(D)** 64

(E) It cannot be determined from the information given.

24. The number of integers from 1 to 10000 (inclusive) which are divisible neither by 13 nor by 51 is

(A) 9030 **(B)** 9050 **(C)** 9070 **(D)** 9090 **(E)** 9110

25. In order that 9986860883748524N5070273447265625 equal 1995^{10}, the letter N should be replaced by the digit

(A) 1 **(B)** 2 **(C)** 4 **(D)** 7 **(E)** 8

26. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 11i$ where $i = \sqrt{-1}$, then $a + b =$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

27. Three chords in a circle have lengths a, b, and c, where $c = a + b$. If the chord of length a subtends an arc of 30° and the chord of length b subtends an arc of 90°, then the number of degrees in the smaller arc subtended by the chord of length c is

(A) 120 **(B)** 130 **(C)** 140 **(D)** 150 **(E)** 160