UNIVERSITY OF NORTH CAROLINA CHARLOTTE
1997 HIGH SCHOOL MATHEMATICS CONTEST
March 10, 1997

1. Which of the five fractions is largest?

   (A) $\frac{25038876541}{25038876543}$ **(B)** $\frac{25038876543}{25038876545}$ **(C)** $\frac{25038876545}{25038876547}$
   (D) $\frac{25038876547}{25038876549}$ **(E)** $\frac{25038876549}{25038876551}$

2. An eight-bit binary word is a sequence of eight digits each of which is either 0 or 1. The number of different eight-bit binary words is

   (A) 32 **(B)** 64 **(C)** 128 **(D)** 256 **(E)** 512

3. If $P(x) = 2x^3 + kx^2 + x$, find k such that $x - 1$ is a factor of $P(x)$.

   (A) -3 **(B)** $-1/3$ **(C)** 0 **(D)** $1/3$ **(E)** 3

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4. If $\log_2 x + \log_2 5 = \log_2 x^2 - \log_2 14$, then $x =$

   (A) 0 **(B)** 70 **(C)** both 0 and 70 **(D)** $\log_2 70$ **(E)** 2^{70}

5. Which of the following could be the exact value of n^4, where n is a positive integer?

   (A) 1.6×10^{20} **(B)** 1.6×10^{21} **(C)** 1.6×10^{22} **(D)** 1.6×10^{23} **(E)** 1.6×10^{24}

6. The following table gives the distribution of families in the town of Colville in 1991 by the number of children. If there were 5000 families, how many families had no children.

Number of Children	0	1	2	3	4 or more
Percent of Families	n%	19%	18%	10%	9%

   (A) 2000 **(B)** 2200 **(C)** 2350 **(D)** 2500 **(E)** 2800

7. A chemist has a solution consisting of 5 ounces of propanol and 17 ounces of water. She would like to change the solution into a 40% propanol solution by adding z ounces of propanol. Which of the following equations should she solve in order to determine the value of z?

(A) $\dfrac{5}{z+17} = \dfrac{40}{100}$ **(B)** $\dfrac{z+5}{22} = \dfrac{40}{100}$ **(C)** $\dfrac{z+5}{17} = \dfrac{40}{100}$

(D) $\dfrac{z+5}{z+17} = \dfrac{40}{100}$ **(E)** $\dfrac{z+5}{z+22} = \dfrac{40}{100}$

8. How many integers n satisfy $|n^3 - 222| < 888$?

(A) 11 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

9. Seven women and five men attend a party. At this party each man shakes hands with each other person once. Each woman shakes hands only with men. How many handshakes took place at the party?

(A) 31 **(B)** 35 **(C)** 45 **(D)** 56 **(E)** 66

10. Suppose $ab < 0$. Which of the following points could **not** satisfy $y = ax + b$?

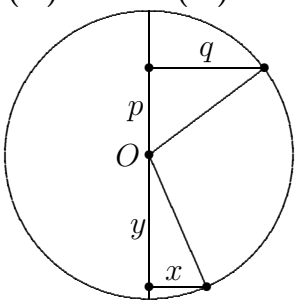
(A) $(0, 1)$ **(B)** $(1, 0)$ **(C)** $(-1, 0)$ **(D)** $(0, -1)$ **(E)** $(1, 1)$

11. Let x and y be two real numbers satisfying $x + y = 6$ and $xy = 7$. Find the value of $x^3 + y^3$.

(A) 55 **(B)** 62 **(C)** 78 **(D)** 90 **(E)** 216

12. In the figure below, the two triangles are right triangles with sides of lengths x, y, p, and q, as shown. Given that $x^2+y^2+p^2+q^2 = 72$, find the circumference of the circle.

(A) 8π **(B)** 9π **(C)** 12π **(D)** 24π **(E)** 36π



13. Three integers a, b and c have a product of 9! and satisfy $a \leq b \leq c$. What is the smallest possible value of $c - a$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 42 **(E)** 51

14. A $4 \times 4 \times 4$ cube is made from 32 white unit cubes and 32 black unit cubes. What is the largest possible percent of black surface area?

(A) 50% **(B)** 60% **(C)** 64% **(D)** $66\frac{2}{3}\%$ **(E)** 75%

15. Suppose a, b and c are real numbers for which

$$\frac{a}{b} > 1 \text{ and } \frac{a}{c} < 0.$$

Which of the following must be correct?

(A) $a + b - c > 0$ **(B)** $a > b$ **(C)** $(a - c)(b - c) > 0$

(D) $a + b + c > 0$ **(E)** $abc > 0$

16. The product of the digits of Ashley's age is the same nonzero number as it was six years ago. In how many years will it be the same again?

(A) 14 **(B)** 18 **(C)** 19 **(D)** 24 **(E)** 26

17. It takes Mathias and Anders 1188 hours to paint the Gaffney Peach. It takes Anders and Tellis 1540 hours to paint the peach; for Tellis and Hal, it takes 1890 hours; and for Hal and Mathias, it takes 1386 hours. How long would it take all four of them working together to paint the peach?

(A) 364.7 hours **(B)** 412.3 hours **(C)** 670.7 hours

(D) 729.5 hours **(E)** 824.6 hours

18. The number $N = 700,245$ can be expressed as the product of three two-digit integers, $x, y,$ and z. What is $x + y + z$?

(A) 210 **(B)** 267 **(C)** 269 **(D)** 271 **(E)** 272

19. In how many points can a line intersect the graph of the function $f(x) = x^2 \sin(x)$?

I. no points

II. one point

III. infinitely many points

(A) II only **(B)** III only **(C)** I and II only **(D)** I and III only

(E) II and III only

20. Cifarelli Builders has just completed developing a section of homes in Southeast Charlotte. The homes are numbered consecutively starting with the address 1. The contractor in charge of ordering the single-digit brass numerals that will be used on each house for its address has determined that 999 numerals need to be ordered. How many homes are in the development?

(A) 200 **(B)** 369 **(C)** 379 **(D)** 381 **(E)** 999

21. Eight people Amy, Bee, Cindy, Dennis, Eli, Fay, Gil, and Hilary attend a dinner party. They need to be seated around a circular table, but Cindy and Gil, the hosts, choose not to be seated next to one another. How many different arrangements are there which seat Gil in the seat nearest the kitchen (so he can serve the dinner)?

(A) 3600 **(B)** 4320 **(C)** 4800 **(D)** 38880 **(E)** 43200

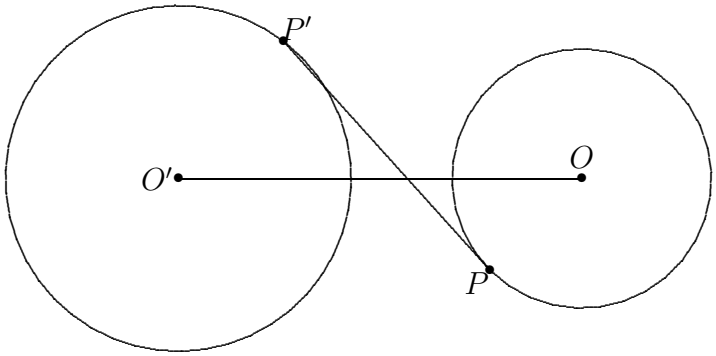
22. How many points (x, y) satisfy the equation

$$|x^2 - 1| + |y^2 - 4| = 0?$$

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** infinitely many

23. Two circles with centers O and O' have radii of 9 inches and 12 inches, respectively. The centers are 28 inches apart. How far from the center of circle O is the intersection of the line joining the centers with the common internal tangent PP'?

(A) 9 in. **(B)** 10 in. **(C)** 11 in. **(D)** 12 in. **(E)** 13 in.



24. Suppose a and b are positive integers neither of which is a multiple of 3. Then the remainder when $a^2 + b^2$ is divided by 3

(A) must be 0 **(B)** must be 1 **(C)** must be 2

(D) may be 1 or 2 but not 0 **(E)** may be $0, 1$ or 2

25. For a tetrahedron $ABCD$, a plane P is called a *middle plane* if all four distances from the vertices A, B, C, and D to the plane P are the same. How many middle planes are there for a given tetrahedron?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 7

26. You are having tea with mathematicians Ford and Fulkerson using three cups each with a 2 unit capacity. Ford has one unit in his cup and it has 50% concentration. Fulkerson also has one unit at a concentration of $33\frac{1}{3}\%$. You have only one unit of hot water in your cup. Ford and Fulkerson will share theirs with you, however, as follows: You pour some water into Ford's tea and then the rest into Fulkerson's. Ford then returns to you the same amount you gave him, and likewise, Fulkerson returns to you the amount of liquid you gave him, so that you end up with one unit of liquid. How much of your unit should you pour into Ford's to maximize the concentration of your tea?

(A) $\frac{1}{3}$ **(B)** $\frac{2}{15}$ **(C)** $\frac{12}{13}$ **(D)** $\frac{4}{5}$ **(E)** $\frac{6}{7}$

27. A *snickel* is a bug which crawls among the lattice points (points with only integer coordinates) of the plane. Each move of a snickel is eight units horizontally or vertically followed by three units in a perpendicular direction. For example, from $(0, 0)$ the snickel could move to any of the eight locations $(\pm 8, \pm 3), (\pm 3, \pm 8)$. What is the least number of moves required to get from $(0, 0)$ to $(19, 0)$?

(A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** no such sequence of moves exists