UNIVERSITY OF NORTH CAROLINA CHARLOTTE
1999 HIGH SCHOOL MATHEMATICS CONTEST
March 8, 1999

1. The sides of a triangle are in the ratio $3 : 5 : 9$. Which of the following words best describes the triangle?

   (A) obtuse **(B)** scalene **(C)** right **(D)** isosceles **(E)** impossible

2. The product of a positive number, its reciprocal, and its square is 7. Which of the following is closest to the sum of the number and its reciprocal?

   (A) 2.64 **(B)** 2.86 **(C)** 3.02 **(D)** 3.33 **(E)** 3.51

3. Given that $a = 1/x, b = 9a, c = 1/b, d = 9c, e = 1/d$, and a, b, c, and d are all distinct non-zero numbers, then x must be the same as

   (A) a **(B)** b **(C)** c **(D)** d **(E)** e

4. Let $f(x) = \sqrt{(x-2)^2}$. Compute $\sum_{x=-2}^{x=2} f(2x)$.

   (A) -7 **(B)** 0 **(C)** 7 **(D)** 14 **(E)** 16

5. What is the product of the roots of

$$(x-1)(x-3) + (x-4)(x+5) + (x-3)(x-7) = 0?$$

   (A) -1260 **(B)** -420 **(C)** $4/3$ **(D)** 10 **(E)** 36

6. Let $ABCD$ be a convex quadrilateral with the area s and let P, Q, R, and S be the midpoints of sides AB, BC, CD, and DA respectively. The sum of the areas of the triangles PBQ and RDS equals

   (A) $3s/4$ **(B)** $2s/3$ **(C)** $s/2$ **(D)** $s/4$

   (E) the ratio in question cannot be determined

7. If f is a function such that $f(3) = 2, f(4) = 2$ and $f(n+4) = f(n+3) \cdot f(n+2)$ for all the integers $n \geq 0$, what is the value of $f(6)$?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 8

   (E) it cannot be determined from the information given.

8. Which one of the following five numbers can be expressed as the sum of the squares of six odd integers (repetitions allowed).

   (A) 1996 **(B)** 1997 **(C)** 1998 **(D)** 1999 **(E)** 2000

9. There exist positive integers x, y, and z satisfying

$$28x + 30y + 31z = 365.$$

   Compute the value of $z - 2x$ for some such triplet.

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

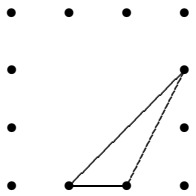
10. If x and y are positive integers for which

$$2(x - y)^2 + 4y^2 = 54,$$

    then x could be

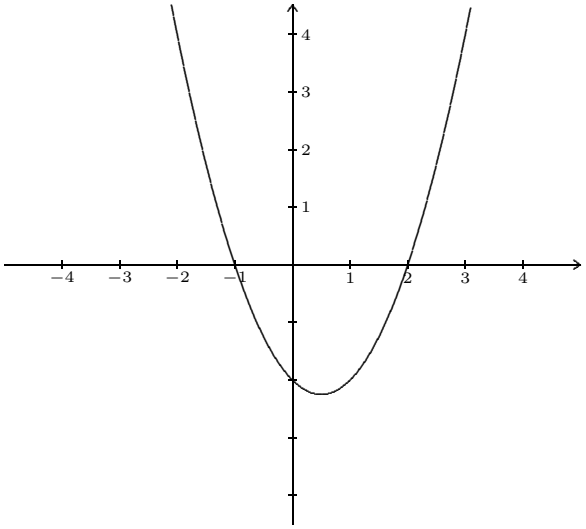
    (A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

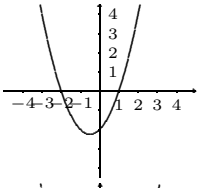
11. Twelve lattice points are arranged along the edges of a 3×3 square as shown. How many triangles have all three of their vertices among these points? One such triangle is shown.

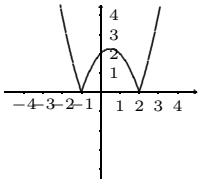


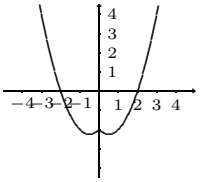
(A) 48 **(B)** 64 **(C)** 204 **(D)** 220 **(E)** 256

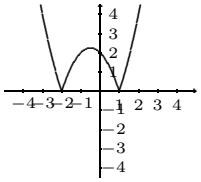
12. Let f be the function whose graph is shown. Which of the following represents the graph of $f(|x|)$?

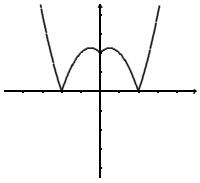


(A) 

(B) 

(C) 

(D) 

(E) 

13. Statistics have shown that in a certain college course, 65% of the students pass the first time they take it. Among those who have to repeat it, 70% pass on the second attempt, and among those who have to take it three times, 50% pass on the third attempt. What percentage of students have to take the course more than three times?

(A) 50% **(B)** 35% **(C)** 22.75% **(D)** 5.25% **(E)** 1%

14. Three integers a, b, and c have a product of 27,846 and the property that the same number N results from each of the following operations:

- a is divided by 6.
- 4 is added to b.
- 4 is subtracted from c.

What is $a + b + c$?

(A) 102 **(B)** 136 **(C)** 152 **(D)** 160 **(E)** 177

15. A non-constant polynomial function $f(x)$ satisfies

$$f(-4) = f(-2) = f(1) = f(3) = 2.$$

What is the smallest possible degree of f?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

16. Let

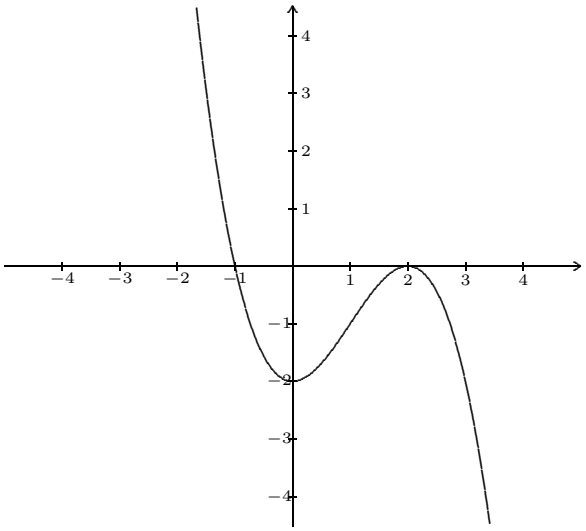
$$g(x) = \begin{cases} |x| - 2 & \text{if } x \leq 0 \\ x - 3 & \text{if } 0 < x < 4 \\ 3 - x & \text{if } 4 \leq x \end{cases}$$

Find a number x such that $g(x) = -4$.

(A) -2 **(B)** -1 **(C)** 3 **(D)** 4 **(E)** 7

17. A cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ has a graph which is tangent to the x-axis at 2, has another x-intercept at -1, and has y-intercept at -2 as shown. Find the constants a, b, c, and d. Then, $a + b + c + d =$

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2



18. Find the sum of all values of x that satisfy

$$|x + 1| + 3|x - 2| + 5|x - 4| = 20.$$

(A) 2 **(B)** 5 **(C)** 6 **(D)** 9 **(E)** 11

19. You have 10 coins, all of different weights and you can weigh them only in pairs in a two-pan balance. What is the minimal numbers of weighings needed to find the heaviest coin?

(A) 5 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 45

20. The area of a circle circumscribed about a regular hexagon is 200π. What is the area of the hexagon?

(A) $60\sqrt{3}$ **(B)** 600 **(C)** 1200 **(D)** $300\sqrt{3}$ **(E)** $600\sqrt{3}$

21. From a group of three female students and two male students, a three student committee is selected. If the selection is random, what is the probability that exactly 2 females and 1 male are selected?

(A) 0.3 **(B)** 0.4 **(C)** 0.5 **(D)** 0.6 **(E)** 0.7

22. What is the units digit of integer 3^{1999}?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 7 **(E)** 9

23. The set of all x such that

$$(|x| - 2)(1 + x) > 0$$

is exactly

(A) $x > 2$ **(B)** $|x| > 2$ **(C)** $-2 < x < -1$ or $x > 2$ **(D)** $-1 < x < 2$ **(E)** $x < -2$ or $x > 2$

24. The product of four distinct positive integers, a, b, c, and d is 8!. The numbers also satisfy

$$ab + a + b + 1 = 323 \quad (1)$$
$$bc + b + c + 1 = 399. \quad (2)$$

What is d?

(A) 7 **(B)** 14 **(C)** 21 **(D)** 28 **(E)** 35

25. Which of the equations below has roots that are the reciprocals of the roots of the equation

$$x^2 - 3x - 2 = 0?$$

(A) $2x^2 + 3x - 1 = 0$ **(B)** $2x^2 - 3x - 1 = 0$ **(C)** $2x^2 + 3x + 1 = 0$

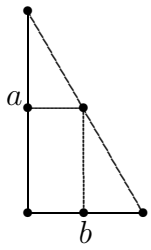
(D) $2x^2 - 3x + 1 = 0$ **(E)** none of **A**, **B**, **C** or **D**

26. How many two-digit integers are there where the tens digit is greater than the units digit?

(A) 35 **(B)** 36 **(C)** 45 **(D)** 55 **(E)** 85

27. What is the area of the largest rectangular region that can be inscribed in a right triangle with legs of length 3 and 4?

(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5 **(E)** 4



28. (tiebreaker) How many digits are there in the (decimal representation of the) integer 19^{9^9}? Recall that $2^{3^4} = 2^{(3^4)}$.