

UNC Charlotte 2010 Algebra

March 8, 2010

- Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the x -axis. The value of $m + b$ is:
(A) -6 (B) -5 (C) -4 (D) -3 (E) -2
- Which of the following equations have the same graphs?
I. $y = x - 2$ II. $y = (x^2 - 4)/(x + 2)$ III. $(x + 2)y = x^2 - 4$
(A) I. and II. only (B) I. and III. only (C) II. and III. only
(D) I., II., and III. (E) None. All the equations have different graphs.
- When 97^3 is expressed in base 100 notation, what is the sum of the 'digits'? (The digits are the coefficients of 100^2 , 100 , and 1 .)
(A) 88 (B) 143 (C) 162 (D) 187 (E) 190
- Find the maximum value of $f(x) = \sqrt{x^2 + 22x + 121} + \sqrt{x^2 - 26x + 169}$ over the interval $-12 \leq x \leq 12$.
(A) 24 (B) 26 (C) 28 (D) 30 (E) 32
- How many triples of positive integers (x, y, z) satisfy both
 - $x \leq y \leq z$ and
 - $x^2 + y^2 + z^2 = 4(x + y + z) + 38$.(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3
- The number of positive integers k for which the equation $kx + 12 = 3k$ has an integer solution for x is
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

7. For all non-zero real numbers x and y such that $x - y = xy$, the value of $1/x - 1/y$ is
(A) $1/2$ (B) 0 (C) 1 (D) 2 (E) -1
8. The sum of 67 consecutive integers is 2010. What is the least of these numbers?
(A) -3 (B) -1 (C) 1 (D) 3 (E) 4
9. Suppose $f(0) = 3$ and $f(n) = f(n - 1) + 2$. Let $T = f(f(f(f(5))))$. What is the sum of the digits of T ?
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10
10. Find the volume of a rectangular box whose left side, front side, and bottom have areas of 12 square inches, 30 square inches and 90 square inches, respectively.
(A) 240 cubic inches (B) 300 cubic inches (C) 120 cubic inches
(D) 180 cubic inches (E) 360 cubic inches
11. Solve simultaneously:
- $$\begin{aligned}x + 2y + z &= 14 \\2x + y + z &= 12 \\x + y + 2z &= 18\end{aligned}$$
- (A) $x = 1, y = 3, z = 7$ (B) $x = -2, y = 4, z = 8$
(C) $x = -3, y = 12, z = 0$ (D) $x = 6, y = -4, z = 4$
(E) $x = 4, y = 4, z = 5$
12. How many positive integer divisors does $N = 6^3 \cdot 150$ have?
(A) 32 (B) 75 (C) 18 (D) 24 (E) 27

13. Let x_1 and x_2 be the real solutions of the equation $x^2 + bx + c = 0$ with $b \neq 0$. If $x_1 - x_2 = 4$ and $x_1^2 + x_2^2 = 40$, then b must be equal to
(A) 8 (B) 4 (C) 12 (D) 8 or 4 (E) 8 or -8
14. Suppose A and B are sets with 5 and 7 elements respectively and $A \cap B$ has 2 elements. How many elements does $A \cup B$ have?
(A) 14 (B) 12 (C) 8 (D) 10 (E) 18
15. For what positive value of x is there a right triangle with sides $2x + 2$, $6x$, and $6x + 2$?
(A) 8 (B) 6 (C) 4 (D) 5 (E) 9
16. Find an equation of the line tangent to the circle $x^2 + y^2 = 2$ at the point $(1, 1)$.
(A) $3x - y = 4$ (B) $y = 2x - 1$ (C) $y = -2x + 3$
(D) $3x - 2y = 1$ (E) $x + y = 2$
17. Three points $A = (0, 1)$, $B = (2, a)$ and $C = (3, 7)$ are on a straight line. What is the value of a ?
(A) 5 (B) 3 (C) 1 (D) 4 (E) 2
18. A rope maker cut a cord into three pieces. Let's name the pieces X , Y , and Z . X is 3 feet long, Y is 3 feet longer than one-fourth of Z . Z is as long as X and Y together. How long is the cord?
(A) 13 feet (B) 14 feet (C) 15 feet (D) 16 feet (E) 17 feet
19. Find all points (x, y) that have an x -coordinate twice the y -coordinate and that lie on the circle of radius of 5 with center at $(2, 6)$.
(A) $(6, 3)$ only (B) $(2, 1)$ only (C) $(4, 2)$ and $(6, 3)$
(D) $(2, 1)$ and $(0, 0)$ (E) $(6, 3)$ and $(2, 1)$

20. Four numbers are written in a row. The average of the first two numbers is 7, the average of the middle two numbers is 2.3, and the average of the last two numbers is 8.4. What is the average of the first number and the last number?
- (A) 13.1 (B) 7.7 (C) 8.85 (D) 2.3 (E) none of the above
21. What non-zero real value for x satisfies $(7x)^{14} = (14x)^7$?
- (A) $1/7$ (B) $2/7$ (C) 1 (D) 7 (E) 14
22. The sum of three numbers is 20. The first is four times the sum of the other two. The second is seven times the third. What is the product of all three?
- (A) 28 (B) 32 (C) 60 (D) 84 (E) 140
23. Jimmy and Katie run a circular track of circumference 400 meters. Jimmy runs 275 meters per minute, and Katie runs 300 meters per minute. If they started the race at the same start line and run in the same direction, how many minutes would it take for them to meet at the same spot?
- (A) 10 min (B) 12 min (C) 14 min (D) 16 min (E) 20 min
24. How many integers from 101 to 999 contain the digit 8?
- (A) 270 (B) 252 (C) 243 (D) 219 (E) 180
25. If $\log \left(3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}} \right) = A \log 3$, what is A ?
- (A) $\frac{31}{16}$ (B) $\frac{31}{32}$ (C) $\frac{16}{15}$ (D) $\frac{32}{17}$ (E) $\frac{13}{32}$