UNC Charlotte Algebra Competition

March 9, 2009

1. If the operation $\oplus$ is defined by $x \oplus y = 3y + y^x$, then $2 \oplus 5 =$

   (A) 10 **(B)** -10 **(C)** 40 **(D)** -26 **(E)** None of these

2. The average of two numbers is xy. If one number is equal to x, then the other number is equal to

   (A) y **(B)** $2y$ **(C)** $xy - x$ **(D)** $2xy - x$ **(E)** None of these

3. The product of three consecutive positive integers is 33 times the sum of the three integers. What is the product?

   (A) 330 **(B)** 660 **(C)** 990 **(D)** 1120 **(E)** None of these

4. Which of the following statements is false.

   A The sum of 3 consecutive integers is always divisible by 3.

   B The sum of 4 consecutive integers is always divisible by 4.

   C The sum of 5 consecutive integers is always divisible by 5.

   D The sum of 2005 consecutive integers is always divisible by 2005.

   E None of the above.

5. The line through the points $(m, -9)$ and $(7, m)$ has slope m. What is the value of m?

   (A) 3 **(B)** $-\frac{7}{9}$ **(C)** 16 **(D)** 5 **(E)** None of these

6. Let x denote the smallest positive integer satisfying $12x = 25y^2$ for some positive integer y. What is $x + y$?

   (A) 75 **(B)** 79 **(C)** 81 **(D)** 83 **(E)** 88

7. College freshman Peter has an abundance of socks. He has 15 pairs, 5 each of brown, blue and black. Sadly, he does not store them in pairs, and when he reaches for socks, he grabs them sight unseen. What is the fewest number of socks Peter must choose in order to be sure to get at least two different color matching pairs?

   (A) 7 **(B)** 10 **(C)** 12 **(D)** 13 **(E)** 15

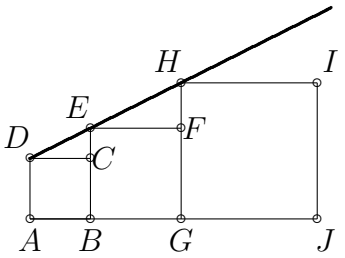
8. The ratio of $2x + y$ to $2y + x$ is 5 to 4. What is the ratio of $x + 3y$ to $3x + y$?

   (A) 3 : 5 **(B)** 5 : 7 **(C)** 7 : 9 **(D)** 9 : 11 **(E)** 11 : 13

9. Using all nine digits $1, 2, 3, 4, 5, 6, 7, 8$, and 9, build three even integers M, N, and P so that the sum of the three is as small as possible. What is that sum?

   (A) 774 **(B)** 811 **(C)** 828 **(D)** 848 **(E)** 922

10. Three adjacent squares rest on a line. Line L passes through a corner of each square as shown. The lengths of the sides of the two smaller squares are 4 cm and 6 cm. Find the length of one side of the largest square.



(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 14

11. What is the area of the triangular region in the first quadrant bounded on the left by the y-axis, bounded above by the line $7x + 4y = 168$ and bounded below by the line $5x + 3y = 121$?

(A) 16 **(B)** 50/3 **(C)** 17 **(D)** 52/3 **(E)** 53/3

12. Given that (x, y) satisfies $x^2+y^2 = 9$, what is the largest value of x^2+3y^2+4x?

(A) 22 **(B)** 24 **(C)** 36 **(D)** 27 **(E)** 29

13. Find the sum $12 + 17 + 22 + 27 + 32 + \ldots + 97$ of all the two-digit numbers whose units digit is either 2 or 7.

(A) 972 **(B)** 981 **(C)** 990 **(D)** 999 **(E)** 1008

14. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?

    (A) 116 **(B)** 188 **(C)** 52 **(D)** 45 **(E)** None of these

15. Find the sum of the x-intercepts of the function

$$g(x) = 3(2x + 7)^2(x - 1)^2 - (2x + 7)(x - 1)^3.$$

    (A) $-69/10$ **(B)** $-67/10$ **(C)** $-33/5$ **(D)** $-5/2$ **(E)** $5/2$

16. An urn contains marbles of four colors, red, yellow, blue and green. All but 25 are red, all but 25 are yellow, and all but 25 are blue. All but 36 are green. How many of the marbles are green?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

17. Each of four cubes is a different size, such that the side of the largest is twice the side of the smallest, and each adjacent size has an equal increment of length. Which of the following statements are true?

    I. The sum of the volumes of the three smallest cubes equals the volume of the largest cube.

    II. The total length of the largest cube's edges is half the sum of the other three cubes' edges.

    III. The sum of the areas of one face on the two middle cubes equals the sum of the areas of one face on the other two cubes.

    (A) I only **(B)** II only **(C)** both I and II

    (D) III only **(E)** none of these

18. Define a "prime time" to be when a digital display indicates both hours and minutes as a prime number on a twelve hour clock. Which of the following is closest to the percent of the time is the time displayed "prime time"?

    (A) 11.8 **(B)** 17.0 **(C)** 28.3 **(D)** 41.7 **(E)** none of these

19. If $(mx + 7)(5x + n) = px^2 + 15x + 14$, what is $m(n + p)$

    (A) -10 **(B)** -520 **(C)** 480 **(D)** 2 **(E)** 520

20. Solve for x: $\sqrt{1+\sqrt{3-\sqrt{1+\sqrt{2+\sqrt{x}}}}} = 1$.

(A) 3844 **(B)** 62 **(C)** $\sqrt{62}$ **(D)** 64 **(E)** 4096

21. Solve for x: $8^{3x+1} - 8^{3x} = 448$.

(A) 2/3 **(B)** 3/2 **(C)** 2 **(D)** 2.93578 **(E)** none of the above

22. What is the product of all the even divisors of 1000?

(A) 32×10^{12} **(B)** 64×10^{14} **(C)** 128×10^{16}

(D) 64×10^{18} **(E)** 10^{24}

23. Let N denote the largest number satisfying all three of the properties

(a) N is a product of three consecutive integers,

(b) N is a sum of three consecutive integers, and

(c) $N < 1000$.

What is the sum of the digits of N?

(A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

24. How many two-element subsets $\{a, b\}$ of $\{1, 2, 3, \ldots, 16\}$ satisfy ab is a perfect square?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

25. A rectangular box with integral sides has a volume of 72 cubic units. What is the least possible surface area.

(A) 100 **(B)** 108 **(C)** 114 **(D)** 120 **(E)** 290