UNC Charlotte 2003 Algebra

March 10, 2003

1. Fred has a pocket full of change, but cannot make change for a dollar. What is the greatest value of coins he could have?

   (A) \$.99 **(B)** \$1.09 **(C)** \$1.19 **(D)** \$1.29 **(E)** \$1.39

2. For what value(s) of b does the equation $2x^2 + bx + 2 = 0$ have no real roots?

   (A) $b > 2$ **(B)** $-4 < b < 4$ **(C)** $b > 2$ or $b < -2$

   (D) $b < -4$ **(E)** $b = 4$

3. A boy is 4 feet 8 inches tall. If he could walk around the earth one time on the equator, the top of his head would travel farther than his feet. Assuming that the equator is a perfect circle, how much farther would the top of his head travel than his feet (in inches)?

   (A) 56 **(B)** 56π **(C)** 96π **(D)** 112π **(E)** 3136π

4. Which of the following describes the range of the function $f(x) = x^2 - 2x$ if its domain is the three element set $\{0, 2, 4\}$?

   (A) $\{0\}$ **(B)** $\{0, 2\}$ **(C)** $\{0, 8\}$

   (D) $\{2, 8\}$ **(E)** all real numbers equal to or greater than -1

5. The slope of a line perpendicular to the line passing through the points $(1, 3)$ and $(-2, 1)$ is

   (A) $-3/2$ **(B)** $-2/3$ **(C)** 0 **(D)** $2/3$ **(E)** $3/2$

6. The numerator of a certain reduced fraction N/D is 5 less than the denominator. If 3 is subtracted from the numerator and 2 is added to the denominator, the resulting fraction is equal to one half. What is $D + N$?

   (A) 22 **(B)** 25 **(C)** 27 **(D)** 31 **(E)** 35

7. Cameron runs at a rate of 800 ft. per minute. Maggie runs at the rate of 900 ft. per minute. If Cameron starts 500 feet ahead of Maggie, how many minutes does it take Maggie to catch up with her.

   (A) 1.5 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 7

8. A recent poll showed that nearly 30% of European school children think that

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}.$$

   This is wrong, of course. Is it possible that $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$ for some real numbers a and b?

   (A) Yes, but only if $a + b = 1$ **(B)** Yes, but only if $a + b = 2$

   (C) Yes, but only if $a^2 + b^2 = 1$ **(D)** Yes, but only if $a^2 + b^2 = 0$

   (E) No, it is not possible

9. What is the number of real solutions of the equation

$$\frac{x^6 - 8}{x^2 - 2} = 12?$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

10. Which of the following is an equation of the line tangent to the circle $x^2+y^2 = 2$ at the point $(1, 1)$?

    (A) $x = 1$ **(B)** $y = 1$ **(C)** $x + y = 2$ **(D)** $x - y = 0$ **(E)** $x + y = 0$

11. Suppose N is a positive integer that is a perfect cube. Which of the following represents the next positive integer that is a perfect cube?

    (A) $N^3 + 3\sqrt[3]{N} + 1$ **(B)** $N + 3\sqrt[3]{N^2} + 3\sqrt[3]{N} + 1$ **(C)** $N^3 + 3N^2 + 3N + 1$
    (D) $N^3 + N^2 + N + 1$ **(E)** N^3

12. For what positive number x are $5, 6 + x$, and $7 + x$ the lengths of the sides a right triangle?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

13. Conard High School has 50 students who play on the baseball, football, and tennis teams. Some students play more than one sport. If 15 play tennis, 25 play baseball, 30 play football, 8 play tennis and baseball, 5 play tennis and football, and 10 play baseball and football, determine how many students play all three sports.

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Let r and s be the two solutions to the equation $x^2 - 3x + 1 = 0$. Find $r^3 + s^3$.

    (A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 24

15. How many ordered pairs of integers with a sum 23 have a product that is maximal?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

16. The sum of the zeros of $f(x) = x(2x + 3)(4x + 5) + (6x + 7)(8x + 9)$ is

    (A) $\frac{-35}{4}$ **(B)** $\frac{35}{4}$ **(C)** -70 **(D)** 70 **(E)** 0

17. Three points $A = (0, 1)$, $B = (2, a)$, and $C = (3, 7)$ are on a straight line. What is the value of a ?

    (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

18. One leg of a right triangle is two meters longer than twice the length of the other leg. The hypotenuse is four meters less than the sum of the two legs. What is the perimeter of the triangle, in meters?

    (A) 20 **(B)** 25 **(C)** 30 **(D)** 35 **(E)** 50

19. Margaret and Cyprian both have some nickels, dimes and quarters, at least one of each type and a different number of each type. Margaret has the same number of quarters as Cyprian has dimes, and she has the same number of dimes as Cyprian has nickels. She also has the same number of nickels as Cyprian has quarters. The value of their coins is the same. What is the least value they could have?

    (A) 85 **(B)** 95 **(C)** 105 **(D)** 125 **(E)** 135

20. For what value of k are the lines $2x + 3y = 4k$ and $x - 2ky = 7$ perpendicular?

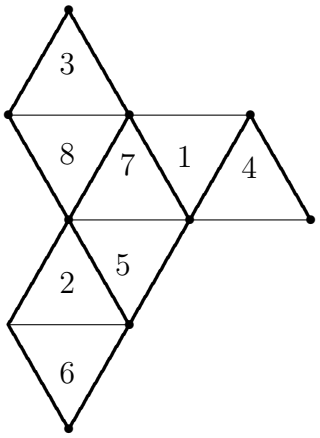
    (A) $-3/4$ **(B)** $1/6$ **(C)** $1/3$ **(D)** $1/2$ **(E)** $2/3$

21. Suppose $\{a, b, c, d, e, f\} = \{2, 3, 4, 5, 6, 7\}$. What is the least possible value of $ab + cd + ef$? (Note that a need not be 2, b need not be 3, etc.)

(A) 50 **(B)** 52 **(C)** 53 **(D)** 60 **(E)** 68

22. An octahedral net is a collection of adjoining triangles that can be folded into a regular octahedron. When the net below is folded to form an octahedron, what is the sum of the numbers on the faces adjacent to one marked with a 3?

(A) 13 **(B)** 15 **(C)** 17 **(D)** 18 **(E)** 19



23. Let M and N denote the two integers that are respectively twice and three times the sum of their digits. What is $M + N$?

(A) 27 **(B)** 36 **(C)** 45 **(D)** 54 **(E)** 60

24. Let $f(a, b, c) = \frac{a+b}{c} + \frac{b+c}{a} + \frac{a+c}{b}$. An integer u exists such that $f(u-1, u, 2u) = 8$. What is the value of $f(u^2 - 3, 2u - 3, 2u - 4)$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

25. How many positive integer triples (x, y, z) satisfy $\frac{1}{x} + \frac{y}{z} = \frac{13}{21}$, where $x < 7$ and where y and z have no common divisors?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6