1. How many points (x, y) in the plane satisfy both $x^2 + y^2 = 25$ and $x^2 - 10x + y^2 - 24y = -105$?

   (A) none (B) 1 (C) 2 (D) 3 (E) more than 3

2. Let $f(x) = (2x+3)^3$ and $g(x) = x^3+x^2-x-1$. Denote the sum of the coefficients of the polynomial $h(x) = f(g(x))$ by s. Which of the following statements is true?

   (A) $s \le 0$ (B) $1 \le s \le 6$ (C) $7 \le s \le 20$ (D) $21 \le s \le 36$ (E) $s > 36$

3. The graph of the function $f(x) = ||2x| - 10|$ on the interval $[-10, 10]$ looks like

   (A) M (B) W (C) V (D) Λ (E) none of these

4. There is a unique positive number r such that the two equations $y+2x = 0$ and $(x-3)^2+(y-6)^2 = r^2$ have exactly one simultaneous solution. Which of the following statements is true?

   (A) $0 < r < 1$ (B) $1 \le r < 3$ (C) $3 \le r < 5$ (D) $5 \le r < 6$ (E) $r \ge 6$

5. The vertices of a triangle are the centers of the circles $C_1 = \{(x, y) \mid x^2 + y^2 = 1\}$, $C_2 = \{(x, y) \mid (x - 4)^2 + y^2 = 1\}$ and $C_3 = \{(x, y) \mid x^2 - 14x + y^2 - 16y = 0\}$. Let S be the area of the triangle. Which of the following statements is true?

   (A) $S \le 6$ (B) $6 < S \le 9$ (C) $9 < S \le 12$ (D) $12 < S \le 15$ (E) $S > 15$

6. How many real solutions does the following system have?

$$\begin{cases} x + y & = 2, \\ xy - z^2 & = 1. \end{cases}$$

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. Let $a > 1$. How many positive solutions has the equation

$$\sqrt{a - \sqrt{a + x}} = x\ ?$$

   (A) 1 (B) 2 (C) 0 (D) 3 (E) 4

8. The top of a rectangular box has area 40 square inches, the front has area 48 square inches, and the side has area 30 square inches. How high is the box?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 8

9. The lower two vertices of a square lie on the x-axis, while the upper two vertices of the square lie on the parabola $y = 15 - x^2$. What is the area of the square?

   (A) 9 (B) $10\sqrt{2}$ (C) 16 (D) 25 (E) 36

10. Pansies have 5 petals while lilacs have 4 petals. A bouquet has 20 flowers with a total of 92 petals. Let P be the number of pansies in the bouquet. Which of the following statements does P satisfy?

   (A) $3 \le P \le 7$ (B) $8 \le P \le 10$ (C) $11 \le P \le 14$ (D) $15 \le P \le 17$ (E) $P \ge 18$

11. A three-digit number *abc* is *palindromic* if $a = c$. What is the number of distinct three-digit palindromic numbers?

    (A) 72 (B) 84 (C) 88 (D) 90 (E) 100

12. The double of a positive number is the triple of its cube. The number is:

    (A) $\sqrt{2/3}$ (B) 1 (C) $\sqrt{3/2}$ (D) $\sqrt[3]{2}/\sqrt{3}$ (E) $\sqrt[3]{3}/\sqrt{2}$

13. Suppose a, b and c are positive integers with $a < b < c$ such that $1/a + 1/b + 1/c = 1$. What is $a + b + c$?

    (A) 6 (B) 8 (C) 9 (D) 11 (E) no such integers exist

14. A quadratic equation $x^2 - 9x + a = 0$ has two distinct roots, one of them being twice the other. Which of the following statements is true?

    (A) $a \leq 5$ (B) $5 < a \leq 10$ (C) $10 < a \leq 15$ (D) $15 < a \leq 20$ (E) $a > 20$

15. In the quadratic equation $x^2 - 7x + a = 0$ the sum of the squares of the roots equals 39. Find a.

    (A) 8 (B) 7 (C) 6 (D) 5 (E) 4

16. Evaluate $S = \cot 1^\circ \cot 2^\circ \cot 3^\circ \ldots \cot 89^\circ$.

    (A) $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) 2

17. The sides of a right triangle form an arithmetic sequence, while their sum equals 48. Find the area of the triangle.

    (A) 24 (B) 96 (C) 48 (D) 54 (E) 84

18. The ratio of the legs in a right triangle equals $3/2$, while the length of the hypotenuse is $\sqrt{52}$. Find the area of the triangle.

    (A) 12 (B) 13 (C) 26 (D) 30 (E) 169

19. The Chebyshev polynomial of the first kind of order n is defined by $T_n(\cos\alpha) = \cos n\alpha$, so that $T_0(\cos\,\alpha) = 1$ and hence $T_0(x) = 1$; $T_1(\cos\,\alpha) = \cos\,\alpha$, hence $T_1(x) = x$; $T_2(\cos\,\alpha) = \cos\,2\alpha = 2\cos^2\alpha - 1$ so that $T_2(x) = 2x^2 - 1$, etc. What is the value of $T_{10}(\sin\,\alpha)$?

    (A) $\cos 10\alpha$ (B) $\sin 10\alpha$ (C) $-\sin 10\alpha$ (D) $-\cos 10\alpha$ (E) $\frac{\sin 10\alpha}{\cos\alpha}$

20. Find the maximum value of the expression $f(x, y) = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}$ over the square Q : $-1 \leq x \leq 1,\ -1 \leq y \leq 1$.

    (A) 1 (B) $\sqrt{\frac{3}{2}}$ (C) $\frac{3}{2}$ (D) $\frac{2}{\sqrt{3}}$ (E) $\frac{4}{3}$