Utility-Optimal Wireless Routing in the Presence of Heavy Tails

Shuang Xia, *Member, IEEE*, Pu Wang, and Hyuck M. Kwon, *Life Senior Member, IEEE*

***Abstract*—Due to emerging mobile applications and an Internet of Things, data traffic carried by wireless networks has increased dramatically. As a result, maximizing the utilization of the limited network resource is a top priority. To date, there exists a large body of work on the stochastic network utility maximization (NUM) problem. On the one hand, the stochastic NUM problem and its associated solutions have been mainly investigated under the light-tailed (LT) data condition. On the other hand, empirical results show that data delivered by wireless networks is not only large in volume but also highly variable, which can be characterized by heavy-tailed (HT) distributions. Such HT traffic exhibits high burstiness and strong temporal correlations, which are fundamentally different from the behavior of LT traffic. In this paper, first it is shown that the classic stochastic gradient algorithms (SGAs), as effective solutions for stochastic NUM problems, fail to prevent HT traffic flows from aggressively competing with LT flows for limited resources. This leads to unbounded queueing delay for LT flows, which results in network instability. To counter such a challenge, the time-average stochastic gradient routing algorithm is proposed, which prevents competition among HT and LT traffic flows so that utility optimality and network stability can be achieved simultaneously. Moreover, because HT data have unbounded moments, such as mean and variance, it is not feasible to apply the conventional convergence analysis, which assumes the moment boundedness of traffic arrivals. To address this problem, the ordinary differential equation method is adopted to prove that the proposed algorithm still converges even with highly variable HT traffic. Comprehensive simulation results are shown to verify the derived theoretical results.**

***Index Terms*—Stochastic, network utility maximization, routing, heavy-tailed data, ordinary differential equation.**

I. Introduction

With the rapid development of Internet applications and technologies, such as wearable devices, vehicle-to-vehicle (V2V) communications, and an Internet of things, data traffic has drastically increased and needs higher-speed data communication everywhere, anytime. To address these challenges, there exists a large body of work on stochastic network utility maximization (NUM), where optimal network control problems, such as congestion control [1], routing [2]–[4], and scheduling [5], [6], have been formulated as constrained maximization problems of the utility function under stochastic dynamics in user traffic and time-varying wireless channels. The classic stochastic gradient descent (SGD) algorithm has been adopted to solve stochastic NUM problems. More specifically, these algorithms transform a constrained stochastic network utility maximization problem into a non-constrained problem by using Lagrange multipliers to define a Lagrange function. Then, Lagrange multiplier analysis is used to convert the network control decisions (e.g., scheduling, routing, and congestion control) into a Lagrange dual function, and treat the Lagrange dual variables as a function of queue length. Finally, the queue length and dual variables are updated along the stochastic gradient direction. These SGD algorithms are widely adopted in different network settings because they possess very promising features in the sense that they can guarantee network stability and maximum network utility without requiring any knowledge of the statistical information of arrival traffic flows and time-varying channel conditions. Instead, only current queue length and channel state information are required [7].

Despite their promising features, these classic SGD algorithms are generally studied and developed under the light-tailed (LT) traffic condition. However, empirical results show that the data delivered by wireless networks is not only large in volume but also high in variability, which can be characterized by heavy-tailed (HT) distribution. Therefore, heavy tailedness can be considered as a stochastic attribute of big data. HT traffic has been identified in a variety of wireless communication and computer networks, such as mobile ad-hoc networks [8], cellular networks [9], WiFi networks [10], and data center networks [11]. HT traffic is either caused by the inherent heavy-tailed distribution in the traffic source, such as file size on Internet servers, traffic volume of cellular base stations [12], message size of mobile social instant messengers, e.g., wechat and whatsapp [13], and frame length of variable bit rate (VBR) video streams [14], or by network protocols themselves, such as retransmissions and random access schemes [15]. Different from LT traffic, HT traffic exhibits high burstiness and strong temporal correlations, which can lead to a destructive impact on network performance in terms of stability [16], latency [17], [18], and connectivity [19].

The main goal of the stochastic NUM problem is to achieve maximum network utility, while ensuring queue stability for each user, e.g., to guarantee that each user has a bounded expected queue length [20]. However, because of the unique stochastic features of HT traffic, we envision that the conventional stochastic gradient algorithm (SGA) could face great dif-

Manuscript received March 19, 2018; revised July 20, 2018 and September 22, 2018; accepted October 6, 2018. Date of publication November 2, 2018; date of current version January 15, 2019. This work was supported in part by the U.S. Air Force Research Laboratory under Grants FA9453-17-1-0020 and FA2386-14-1-0026, and in part by the U.S. National Science Foundation (NSF) under Grants 1547373 and 1446557. The review of this paper was coordinated by Dr. Z. Cai. *(Corresponding author: Shuang Xia.)*

S. Xia and H. M. Kwon are with the Department of Electrical Engineering and Computer Science, Wichita State University, Wichita, KS 67260 USA (e-mail: sxxia@shcokers.wichita.edu; hyuck.kwon@wichita.edu).

P. Wang is with the Department of Computer Science, University of North Carolina at Charlotte, Charlotte, NC 28223 USA (e-mail: pu.wang@uncc.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2018.2879353

ficulties in reaching utility optimality and queue stability at the same time. In particular, our previous research [16] shows that the queue with HT traffic arrival (i.e., HT flow queues) inherently experiences heavy-tailed distributed queueing delay. Therefore, compared to LT traffic arrivals (i.e., LT flow queues), HT queues have a higher probability of accumulating with a very large queue length. Nevertheless, the classic SGA updates the dual variables according to users' queue length, which may offer HT queues more service opportunities since they have a larger queue length than LT queues. Consequently, LT queues are starved and may not be served until they have an equal or larger queue length than HT flow queues. This leads to unbounded queue length or queue instability.

In this paper, we formally prove the instability of classic SGD-based routing algorithms in the presence of HT traffic and then develop new routing algorithms to maximize network utility, while guaranteeing network stability regardless of traffic type. In particular, this paper considers a multi-hop wireless network that is formed by a collection of interconnected wireless routers. By exploiting the properties of regenerative processes [21] along with asymptotic queueing analysis [22], it is shown that under the classic SGA, the tail distribution of the LT flow queue is at least one order heavier than the tail distribution of the HT flow arrivals. According to moment theory [23], this implies that the queueing delay of the LT queue, i.e., the queue with LT arrivals, is of unbounded mean. To counter such a challenge, we propose a time-average stochastic gradient routing algorithm (TA-SGRA), which separates the queue-length update process and the dual-variables update process so that both utility optimality and queue stability can be simultaneously achieved in the presence of HT traffic. More specifically, to prove network stability, we show that under the TA-SGRA, the transmission rate for each flow converges to a constant as time proceeds. Consequently, the LT queues do not need to compete with HT queues for transmission time, and therefore, all queues behave as single-user single-server ($G1/G/1$) queues. We prove that such a feature, combined with the last-in-first-out (LIFO) intra-queue scheduling policy, can guarantee the bounded average queueing delay for both LT and HT queues. To prove that the network utility converges to the optimal one, we adopt a novel ordinary differential equation (ODE) approach because the HT traffic has an unbounded variance, which is not suitable for applying the conventional convergence analysis that is based on the Euclidean distance to the optimal set.

The contributions of this paper are summarized as follows:

- We prove that the classic SGD-based routing algorithms are not effective for HT data delivery by causing queue instability and infinite average queueing delay.
- We propose a TA-SGRA, which, to the best of our knowledge, is the first routing algorithm that can simultaneously achieve utility optimality and network stability in the presence of heavy tails.
- We adopt the ODE-based method to demonstrate the convergence property of the TA-SGRA, which cannot be proven using conventional convergence analysis due to the existence of unbounded moments of HT traffic.

The rest of this paper is organized as follows. Section III introduces the preliminaries and system model. Section IV analyzes the stability performance of classic SGD-based routing in the presence of HT traffic. Section V introduces the proposed TA-SGRA and proves that it can achieve optimal utility while guaranteeing network stability. Section VI shows simulation results that verify our theoretical consequences. Section VII concludes this paper.

II. Related Work

So far, the research on optimal control of wireless networks under heavy-tailed traffic is limited. In particular, maximum power weight scheduling (MPWS) policies were proposed in [22], [24], [25] for single-hop wireless networks (e.g., WiFi and cellular networks), where scheduling decisions are made based on queue backlog or head-of-line (HoL) queueing delay raised up to the α-th power, where α is determined by the burstiness or heavy tailness of the traffic flows. Intuitively, by properly selecting α to allocate more service opportunities to LT queues, MPWS can guarantee that all LT queues experience a bounded average queueing delay, completely shielding those LT queues from the destructive impact of HT traffic. MPWS policies are further extended to multi-hop wireless networks (e.g., sensor networks and wireless mesh networks) [26]. Although MPWS policies and their variants can lead to bounded queueing delay for LT flow queues, they can neither ensure the delay boundness of the HT flow queues, nor achieve utility optimality. To improve the delay performance of HT flow queues, we recently proposed the delay-based maximum-weight scheduling policy with the LIFO service discipline (LIFO-DMWS) [27]. LIFO-DMWS is proven to be throughput optimal in the sense that no matter whether the incoming traffic flows are HT or LT, all queues can experience bounded average queueing delay as long as the incoming traffic rates are within the network capacity region. Despite its promising performance under the destructive impact of HT traffic, LIFO-DMWS is designed for single-hop networks and is difficult to extend to multiple-hop networks. More importantly, LIFO-DMWS cannot achieve utility optimality. In this paper, we aim to propose the optimal control algorithm for multi-hop wireless networks, the first one in the literature that can achieve utility optimality, which ensures the delay boundedness of both HT and LT flow queues.

III. System Model and Preliminaries

A. Preliminaries

In this paper, we use the following notations: For any two real functions $a(t)$ and $b(t)$, $a(t) \sim b(t)$ denotes $\lim_{t\to\infty} a(t)/b(t) = 1$. Also, let $F(x) = P(X \le x)$ denote the cumulative distribution function (CDF) of a non-negative random variable (r.v.) X. Let $\overline{F}(x) = P(X > x)$ denote its tail distribution function. Table I lists these and other notations used in this paper.

Definition 1 (Heavy Tail): [28] Random variable X is HT, if for all $\theta > 0$

$$\lim_{x\to\infty} e^{\theta x}\overline{F}(x) = \infty, \tag{1}$$

and random variable X is LT if it is not heavy-tailed.

The above definition indicates that an r.v. is HT if its tail distribution decreases slower than exponentially. An r.v. is LT if its tail distribution decreases exponentially or faster. Representative HT distributions include Pareto and log-normal, and the typical LT distributions include exponential and Poisson. Based on the existence of the moments of an r.v., we define the tail coefficient of a nonnegative random variable.

TABLE I
NOTATIONS

Notation	Description
$A_i^f(t)$	Number of packets that arrive at queue i for flow f during time slot t with mean $E[A_i^f(t)] = a_i^f$
$a(t), b(t)$	Real positive function
$D(\cdot)$	Lagrange dual function
$\overline{F}(\cdot)$	Tail distribution function
$F(\cdot)$	Cumulative distribution function of nonnegative r.v.
$\mathcal{F}$	Set of traffic flows
$g_i(t)$	Stochastic gradient of user i at time slot t
L	Number of links
$\mathcal{L}$	Set of links
$\mathcal{L}(\cdot)$	Lagrange function
N	Number of network users
$\mathcal{N}$	Set of nodes
$Q_i^f(t)$	Number of packets in node i for flow f by the end of time slot t
$q_{i,j}(t)$	Number of packets in queue i for delivering to node j by end of time slot t
$r_{i,j}^f$	Transmission rate between node i and node j for flow f
$\bar{r}_{i,j}^f$	Time-average transmission rate between nodes i and j for flow f
$R_{i,j}$	Channel capacity of link (i, j)
$\mathcal{S}$	Set of collision-free link rate vectors
$U(\cdot)$	Concave, non-decreasing utility function on R_+
$\mathcal{V}(\cdot)$	Slowly varying function
$W_{i,j}^s(t)$	Indicator function for collision-free link rate vector between link i, j at time slot t
w_s	Probability of selecting collision-free link rate vector
α_1, α_2	Step size
β	Tail index
$\lambda_i, \upsilon_i, \mu$	Lagrange variables

Definition 2 (Tail Coefficient): [28] The tail coefficient $\kappa(X)$ of a nonnegative random variable X is defined by

$$\kappa(X) = -\lim_{t\to\infty} \frac{\log \Pr(X > x)}{\log x} = \sup\{k \geq 0 : E[X^k] < \infty\}. \tag{2}$$

The tail coefficient defines the maximum order above which an r.v. possesses infinite moments. In addition, in this paper, we focus on an important class of HT distributions, namely regularly varying distributions, which inherently have finite tail coefficients and infinite moments.

Definition 3 (Regularly Varying): [28] Random variable X is called regularly varying with tail index $\beta > 0$, denoted by $X \in \mathcal{RV}(\beta)$, if

$$\overline{F}(x) \sim x^{-\beta}\mathcal{V}(x), \tag{3}$$

where $\mathcal{V}(x)$ is a slowly varying function.

The tail index β indicates how heavy the tail distribution is, where the smaller values of β imply heavier tails. Moreover, for an r.v. $X \in \mathcal{RV}(\beta)$, the tail coefficient κ_X of X is equal to the tail index β, which defines the maximum order of bounded moments that X can have. Specifically, if $0 < \beta < 1$, then X has an infinite mean and an infinite variance. If $1 < \beta < 2$, then X has a finite mean and an infinite variance.

B. System Model

Consider a multi-hop network $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$, where $\mathcal{N}$ is the set of nodes, and $\mathcal{L}$ is the set of links between nodes with $|\mathcal{N}| = N$ and $|\mathcal{L}| = L$, respectively. This paper considers a single wireless channel between nodes model. Time is slotted with a unit slot size. We denote the capacity of link $(i, j) \in \mathcal{L}$ as $R_{i,j}$ and define the neighborhood of node i as the set $\mathcal{N}_i$. Let $\mathcal{F}$ denote a set of traffic flows. Each flow $f \in \mathcal{F}$ has a source node $s(f)$ and a destination node $d(f)$, where $s(f), d(f) \in \mathcal{N}$, $(s(f), d(f)) \in \mathcal{L}$, and $s(f) \neq d(f)$ for all flows f. Moreover, each node maintains two type of queues according to the corresponding layer:

At the network layer, each node maintains a flow queue for each flow. Let $Q_i^f(t)$ denote the queue length of the flow queue that temporally stores the packets for flow f at node i. Let $D_i^f(t)$ be the corresponding queueing delay. Let $A_i^f(t)$ denote the number of packets that arrive at queue i for flow f during time slot t with an average traffic rate or traffic intensity of $E[A_i^f(t)] = a_i^f$. Then, the flow queue $Q_i^f(t)$ of node i for flow f can be represented by

$$Q_i^f(t+1) = \left[Q_i^f(t) - \sum_{j\in\mathcal{N}_i} \left(r_{i,j}^f(t) - r_{j,i}^f(t)\right) + A_i^f(t)\right]^+, \tag{4}$$

where $[\cdot]^+ = \max(\cdot, 0)$, and $r_{i,j}^f(t)$ is the to-be-determined routing parameter, which is the number of packets of flow f delivered from node i to node j during time slot t with mean $E[r_{i,j}^f(t)] = r_{i,j}^f$. If $A_i^f(t) \in HT$, then we say that $Q_i^f(t)$ is an **HT queue**. Otherwise, $Q_i^f(t)$ is an **LT queue**.

Definition 4 (HT and LT Queues): A flow queue $Q_i^f(t)$ is called an HT queue if it has HT traffic arrivals, i. e., $A_i^f(t) \in HT$. Otherwise, it is called an LT queue.

At the media access control (MAC) layer, each node builds link queues for its neighbor nodes. In addition, we use a protocol model in [29]–[31] to define the collision-free link. More specifically, consider N nodes arbitrarily located on a plane, and let $d_{i,j}$ denotes the distance between nodes i and j. The communication range is denoted by Z_i, and $Z_i^{'}$ denotes the interference range for node i. Then, a successful transmission can be made by following two conditions:

$$d_{i,j} \leq Z_i \tag{5}$$

$$d_{k,j} \geq Z_k^{'}, \qquad \forall k \in N, \tag{6}$$

where the first condition means that the distance between node i and node j is within the communication range Z_i, and the second condition tells us that node j is out of the interference range of any other nodes k. Thus, we use S to denote the collection of all collision-free link vectors. A collision-free link vector $s \in \mathcal{S}$ is a subset of links that can be activated without any collisions among each other. Let $\mathcal{R} = \{R^1, R^2, ..., R^S\}$ denote the set of all collision-free link rate vectors, where $R^s = [R_{(i.j)}^s]_{(i,j)\in\mathcal{L}}$ and $R_{(i.j)}^s$ is the data rate of link $(i.j)$ when link vector s is selected. Furthermore, to guarantee network stability, the routing rate of all flows should be within the link capacity region, which

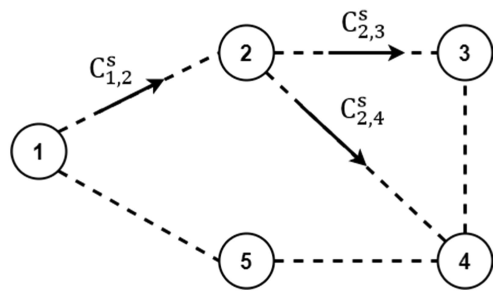

Fig. 1. Network model.

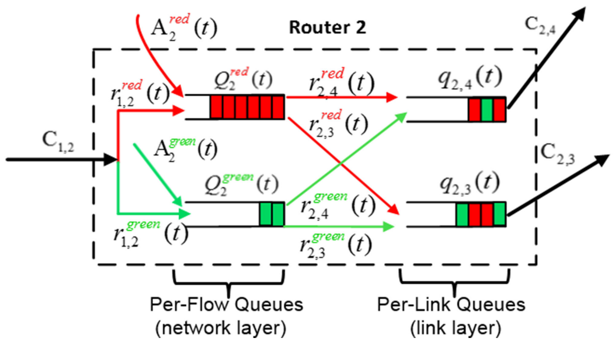

Fig. 2. Cross-layer network model details.

is a convex hull $Co(\mathcal{R})$. Let $W_s(t)$ be an indicator function at time slot t for vector R^s. When $W_s(t) = 1$, the link rate vector R^s is selected, and thus $E[W_s(t)] = w_s$ is the probability that the collision-free link capacity vector s is selected. Then, the link capacity region can be represented by

$$Co(\mathcal{R}) = \left\{ \sum_{s=1}^{S} w_s R^s | (\forall s : w_s \geq 0) \cap \sum_{s=1}^{S} w_s = 1 \right\}. \quad (7)$$

Thus, considering link (i, j), its average data rate is given by

$$C^s_{i,j} = \sum_{s=1}^{S} w_s R^s_{i,j}. \quad (8)$$

More specifically, Fig. 1 shows a simple network topology with five nodes, where node 2 receives packets from node 1 with rate $C^s_{1,2}$ under a collision-free link capacity vector $R^s_{i,j}$ and also sends packets to nodes 3 and 4 with rates $C^s_{2,3}$ and $C^s_{2,4}$, respectively. Fig. 2 shows the details of node 2 shown in Fig. 1. In node 2, there are two types of flows: red and green. Thus, there are two flow queues in node 2. Furthermore, since node 2 has two neighbor nodes, it also maintains two link queues: $q_{2,3}$ and $q_{2,4}$. In addition, let $q_{i,j}(t)$ denote the MAC-layer link queue containing packets stored at node i for delivery to neighboring node j, and let $D_{i,j}(t)$ denote the corresponding queueing delay.

The link queue $q_{i,j}(t)$ of node i receives a number $\sum_{f\in\mathcal{F}} r^f_{i,j}(t)$ of packets from its network layer, and a maximum number of $C^s_{i,j}(t)$ packets can be released from link queue $q_{i,j}(t)$ and sent over the wireless link (i, j) between nodes i and j. As a result, the link queue $q_{i,j}(t)$ evolves as

$$q_{i,j}(t+1) = \left[q_{i,j}(t) - C^s_{i,j}(t) + \sum_{f\in\mathcal{F}} r^f_{i,j}(t) \right]^+, \quad (9)$$

where $C^s_{i,j}(t) = \sum_{s=1}^{S} W_s(t) R^s_{i,j}$ denotes the instantaneous data rate of the wireless link between nodes i and j under the collision-free link rate vector s.

C. System Stability

Definition 5 (Strong Stability): [27] A network is strongly stable if all flow queues and link queues experience a bounded average queueing delay, i.e.,

$$E[D^f_i(t)] < \infty \; \forall i \in \mathcal{N}, f \in \mathcal{F}, \quad (10)$$

$$E[D_{i,j}(t)] < \infty. \; \forall (i, j) \in \mathcal{L} \quad (11)$$

IV. Network Stability Analysis for Stochastic Gradient Routing Algorithm

In this section, we first formulate the utility routing problem as a concave optimization framework and utilize the classic gradient descent algorithm [32, Section 9.3] to solve it. Then, we prove that the classic stochastic gradient routing algorithm cannot reach strong stability in the presence of HT traffic. In particular, we define a non-decreasing concave utility function $U(\cdot)$, which characterizes diminishing marginal returns. Then, we formulate the utility-optimal routing problem as follows:

$$\begin{aligned}
&Find && r^f_{i,j}(t), W_s(t) && \forall (i,j) \in \mathcal{L}, \forall f \in \mathcal{F}, \forall s \in \mathcal{S}\\
&Maximize && \sum_{f,i,j} U(r^f_{i,j}) &&\\
&\text{s.t.} && \sum_{j\in\mathcal{N}_i} (r^f_{i,j} - r^f_{j,i}) \geq a^f_i, && \forall f \in \mathcal{F}, i \in \mathcal{N}\\
& && \sum_{f\in\mathcal{F}} r^f_{i,j} \leq \sum_{s=1}^{S} w_s R^s_{i,j}, && \forall (i,j) \in \mathcal{L}, \forall s \in \mathcal{S}\\
& && r^f_{i,j} \geq 0, && \forall f \in \mathcal{F}, \forall (i,j) \in \mathcal{L}\\
& && 0 \leq w_s \leq 1, && \forall s \in \mathcal{S}\\
& && \sum_{s=1}^{S} w_s = 1 && \forall s \in \mathcal{S}\\
& && E[D^f_i(t)] < \infty, && \forall f \in \mathcal{F}, i \in \mathcal{N}\\
& && E[D_{i,j}(t)] < \infty, && \forall (i,j) \in \mathcal{L},
\end{aligned} \quad (12)$$

where the first constraint is the flow-conservation constraint, which enforces the arrival traffic rate at each router, which is smaller than the departure traffic rate; the second constraint is a link-conservation constraint, which means the average link rate $\sum_{s=1}^{S} w_s R^s_{i,j}$ is larger than the average arrival traffic rate $\sum_{f\in\mathcal{F}} r^f_{i,j}$; the third and fourth constraints are primal variable feasibility constraints; the fifth constraint implies that the total probability of selecting collision-free link vectors is 1; and the last two constraints enforce the boundedness of the queueing delay for flow queues and link queues, respectively.

To solve the above NUM problem, we define Lagrangian multipliers λ_i and the Lagrange function as

$$\mathcal{L}(r,\lambda) = \sum_{f,i,j} U(r^f_{i,j}) + \sum_{f,i} \lambda^f_i \left(\sum_{j\in\mathcal{N}_i} (r^f_{i,j} - r^f_{j,i}) - a^f_i \right). \quad (13)$$

Then, we reorder eq. (13) and obtain

$$\mathcal{L}(r,\lambda) = \sum_{f,i,j} \left(U(r^f_{i,j}) + r^f_{i,j}(\lambda^f_i - \lambda^f_j) \right) - \sum_{f,i} \lambda^f_i a^f_i. \quad (14)$$

Thus, we have the Lagrange dual function as

$$D(\lambda) = \sup_r \mathcal{L}(r,\lambda). \quad (15)$$

To find the primal Lagrangian maximizers $(r^f_{i,j})^*$, we have

$$\begin{aligned} & (r^f_{i,j})^* = \arg\max_r \mathcal{L}(r,\lambda) \\ \text{s.t.} \quad & \sum_{f\in\mathcal{F}} r^f_{i,j} \le \sum_{s=1}^{S} w_s R^s_{i,j}, \quad && \forall (i,j)\in\mathcal{L}, \forall s\in\mathcal{S} \\ & r^f_{i,j} \ge 0, && \forall f\in\mathcal{F}, \forall (i,j)\in\mathcal{L} \\ & \sum_{s=1}^{S} w_s = 1, && \forall s\in\mathcal{S} \\ & 0 \le w_s \le 1, && \forall s\in\mathcal{S}. \end{aligned} \quad (16)$$

To obtain the Lagrangian maximizers $(r^f_{i,j})^*$, we first find the flow that has the maximum dual difference between nodes i and j as

$$f^* = \arg\max_{f\in F} (\lambda^f_i - \lambda^f_j). \quad (17)$$

Then, we select a collision-free link-rate vector R^s and obtain the primal Lagrangian maximizers as

$$\begin{aligned} (r^{f^*}_{i,j})^* &= \arg\max_{r^{f^*}_{i,j} = w_s R^s_{i,j}} \mathcal{L}(r,w,\lambda) \\ &= \arg\max_{r^{f^*}_{i,j} = w_s R^s_{i,j}} \sum_{i,j} \left(U(r^{f^*}_{i,j}) + r^{f^*}_{i,j}(\lambda^{f^*}_i - \lambda^{f^*}_j) \right), \end{aligned} \quad (18)$$

The last equation holds because the last term of eq. (14) does not affect the primal Lagrangian maximizers $(r^{f^*}_{i,j})^*$. Then, we define the stochastic gradient $(g^{f^*}_i)^*(t)$ by taking the derivative of the objective function in eq. (13) with respect to λ^f_i as

$$(g^{f^*}_i)^*(t) = \sum_{j\in\mathcal{N}_i} \left((r^{f^*}_{i,j})^*(t) - (r^{f^*}_{j,i})^*(t) \right) - A^{f^*}_i(t). \quad (19)$$

Therefore, the dual variable λ can be updated in the direction of the stochastic gradient descent as

$$\begin{aligned} \lambda^{f^*}_i(t+1) &= \lambda^{f^*}_i(t) - \alpha_1(t)(g^{f^*}_i)^*(t) \\ &= \lambda^{f^*}_i(t) - \alpha_1(t) \Bigg(\sum_{j\in\mathcal{N}_i} \big((r^{f^*}_{i,j})^*(t) \\ &\qquad - (r^{f^*}_{j,i})^*(t) \big) - A^{f^*}_i(t) \Bigg)^+, \end{aligned} \quad (20)$$

Algorithm 1: Conventional Stochastic Gradient Routing Algorithm.

Observe $Q^f_i(0)$. Initialize $\lambda^f_i(0) = Q^f_i(0)$ for all nodes
for $t = 0, 1, 2, \ldots$ **do**
  for all neighbors $j \in \mathcal{N}_i$ **do**
    Find the largest dual-variable difference flow f^*
    $f^* = \arg\max_{f\in F} \left(\lambda^f_i(t) - \lambda^f_j(t) \right)$
    Find optimal primary variables $(r^{f^*}_{i,j})^*(t)$
    $(r^{f^*}_{i,j})^*(t) = \arg\max_{r^{f^*}_{i,j}(t) = W_s(t) R^s_{i,j}} \sum_{i,j} U$
    $(r^{f^*}_{i,j}(t)) + r^{f^*}_{i,j}(t)\left(\lambda^{f^*}_i(t) - \lambda^{f^*}_j(t)\right)$
    Transmit packets at $(r^{f^*}_{i,j})^*(t)$ between nodes i and j for flow f^*
  end for
  Update the dual variable $\lambda^{f^*}_i(t+1) = \lambda^{f^*}_i(t) - \alpha_1(t)$ $\left[\sum_{j\in\mathcal{N}_i} \left((r^{f^*}_{i,j})^*(t) - (r^{f^*}_{j,i})^*(t) \right) - A^{f^*}_i(t) \right]^+$
end for

where $\alpha_1(t)$ is the step size for all routers i. Now, we summarize the above classic stochastic gradient routing algorithm in Algorithm 1. The Algorithm 1 is the classic stochastic gradient descent algorithm. Therefore, it converges as time proceeds [33]. At each time round t, the computation complexity is $O(N)$, where N is the maximum number of neighboring nodes for each node, which is equal to the total number of nodes in the network. If arrival traffic $A^f_i(t)$ is light tailed, it is easy to show that Algorithm 1 can achieve utility optimality and network stability. Next, we show that Algorithm 1 is not strongly stable in the presence of HT traffic.

Theorem 1: For any router, if there is a heavy-tailed flow with a tail index smaller than two, i.e.,

$$\min_{i\in\mathcal{N}} \kappa(A^{f_H}_i(t)) < 2, \quad (21)$$

then all flow queues in this router can experience unbounded queueing delay (i.e., $E[D^f_i(t)] = \infty$).

Proof: See Appendix A. ■

Remark 1: Intuitively, the instability of classic SGD-based routing is due to the fact that HT flow can induce quick queue-length build-up and consequently create large a queue-length difference between two neighboring nodes. This leads to a large dual variable difference between two neighboring nodes because the dual variable is positively proportional to the queue length. Then, the classic SGD-based routing seeks to route data in directions that maximize the differential dual variable. Consequently, HT flows receive more service opportunities, while LT flows are starved. This leads to an unbounded average queueing delay.

V. Time-Average Stochastic Gradient Routing Algorithm

In this section, to counter the instability problems of the classic stochastic gradient routing algorithm, we propose a TA-SGRA that can achieve utility optimality and queue stability simultaneously, even under a heavy-tailed environment. In particular, we first reconsider the Lagrange dual function in eq. (15),

$$\begin{aligned} & D(\lambda) = \sup_{r} \mathcal{L}(r,\lambda) \\ \text{s.t.} \quad & \sum_{f\in\mathcal{F}} r_{i,j}^f \le \sum_{s=1}^{S} w_s R_{i,j}^s, && \forall (i,j)\in\mathcal{L}, \forall s\in\mathcal{S} \\ & r_{i,j}^f \ge 0, && \forall f\in\mathcal{F}, \forall (i,j)\in\mathcal{L} \\ & \sum_{s=1}^{S} w_s = 1, && \forall s\in\mathcal{S} \\ & 0\le w_s \le 1, && \forall s\in\mathcal{S}. \end{aligned} \tag{22}$$

To find primal Lagrangian maximizers $(r_{i,j}^f)^*$, we redefine a new Lagrange function as

$$\bar{\mathcal{L}}(r,w,\mu,\upsilon) = \mathcal{L}(r,\lambda) + \sum_{i,j}\mu_{i,j}\left(\sum_{s=1}^{S} w_s R_{i,j}^s - \sum_{f\in\mathcal{F}} r_{i,j}^f\right) + \sum_{i,j}\upsilon_{i,j} r_{i,j}^f, \tag{23}$$

where $\mu_{i,j}$ and $\upsilon_{i,j}$ are Lagrange multipliers. The expression $\sum_{s=1}^{S} w_s R_{i,j}^s - \sum_{f\in\mathcal{F}} r_{ij}^f$ comes from the first constraint in eq. (22), and $r_{i,j}^f \ge 0$ comes from the second constraint in eq. (22). Then, we have a new dual function of eq. (23) as

$$\begin{aligned} & D(\mu,\upsilon) = \sup_{r,w} \bar{\mathcal{L}}(r,w,\mu,\upsilon) \\ \text{s.t.} \quad & \sum_{s=1}^{S} w_s = 1, && \forall s\in\mathcal{S} \\ & 0\le w_s\le 1, && \forall s\in\mathcal{S}, \end{aligned} \tag{24}$$

and a dual problem as

$$\min_{\mu,\upsilon} D(\mu,\upsilon) = \min_{\mu,\upsilon}\sup_{r,w} \bar{\mathcal{L}}(r,w,\mu,\upsilon). \tag{25}$$

According to $\bar{\mathcal{L}}(r,w,\mu,\upsilon)$ defined in (23), the determination of Lagrange maximizers r and w is equivalent to the maximization of separate summands, which corresponds to the optimization of the MAC-layer parameter w and network-layer parameter r, respectively. In particular, **at the MAC layer**, to find the maximum value of w_s^*, we need to solve the following optimization problem.

$$\begin{aligned} & w_s^* = \arg\max_{w_s} \sum_{i,j}\sum_{s=1}^{S} \mu_{i,j} w_s R_{i,j}^s \\ \text{s.t.} \quad & \sum_{s=1}^{S} w_s = 1, && \forall s\in\mathcal{S} \\ & 0\le w_s\le 1, && \forall s\in\mathcal{S}. \end{aligned} \tag{26}$$

Note that, at time slot 0, we randomly select a collision-free link rate vector R^s, $s\in\mathcal{S}$. Then, considering time slot t, $t\ge 1$, $W_s(t)$ denotes an indicator function of two values, 1 and 0, depending on whether a collision-free link-rate vector $R^s = \{R_1^s, R_2^s, ..., R_L^s\}$ is active or not. If the collision-free link-rate vector R^s is active, then $W_s(t)=1$; otherwise, $W_s(t)=0$. In addition, if the link l is in the collision-free link-rate vector R^s, then $R_l^s > 0$; otherwise, $R_l^s = 0$. Therefore, if it is active (i.e., $W_s(t)=1$), then $\sum_{i,j}\mu_{i,j}(t-1)W_s(t)R_{i,j}^s$ will be maximum out of

$$\left\{\sum_{i,j}\mu_{i,j}(t-1)W_1(t)R_{i,j}^1, \sum_{i,j}\mu_{i,j}(t-1)W_2(t)R_{i,j}^2, ..., \sum_{i,j}\mu_{i,j}(t-1)W_S(t)R_{i,j}^S\right\}, \quad t\ge 1. \tag{27}$$

Thus, $\sum_{i,j}\sum_{s=1}^{S}\mu_{i,j}(t-1)W_s(t)R_{i,j}^s$ will be equal to $\sum_{i,j}\mu_{i,j}(t-1)W_s^*(t)R_{i,j}^s$. Then, we denote

$$\begin{aligned} W_s^*(t) &= \arg\max_{W_s}\left\{\sum_{i,j}\mu_{i,j}(t-1)W_1(t), \sum_{i,j}\mu_{i,j}(t-1) W_2(t), \dots, \sum_{i,j}\mu_{i,j}(t-1)W_S(t)\right\} \\ &= \arg\max_{W_s}\left\{\sum_{i,j}\mu_{i,j}(t-1)W_1(t)R_{i,j}^1, \sum_{i,j}\mu_{i,j}(t-1) W_2(t)R_{i,j}^2, ..., \sum_{i,j}\mu_{i,j}(t-1)W_S(t)R_{i,j}^S\right\} \\ &= \arg\max_{W_s}\sum_{i,j}\sum_{s=1}^{S}\mu_{i,j}(t-1)W_s(t)R_{i,j}^s, \quad t\ge 1. \end{aligned} \tag{28}$$

Furthermore, the time-average $w_s^*(t)$ can be computed by

$$w_s^*(t) = \begin{cases} W_s^*(t), & t=0 \\ \sum_{k=1}^{t}\frac{W_s^*(k)}{t} & t\ge 1. \end{cases} \tag{29}$$

At the network layer, to find the primal Lagrangian maximizers $(r_{i,j}^f)^*$, we first re-order eq. (23) and obtain

$$\bar{\mathcal{L}}(r,w,\lambda,\mu,\upsilon) = \sum_{f,i,j}\left(U(r_{i,j}^f) + r_{i,j}^f(\lambda_i^f - \lambda_j^f)\right) - \sum_{f,i}\lambda_i^f a_i^f + \sum_{i,j}\mu_{i,j}\left(\sum_{s=1}^{S} w_s R_{i,j}^s - \sum_{f\in\mathcal{F}} r_{i,j}^f\right) + \sum_{i,j}\upsilon_{i,j} r_{i,j}^f. \tag{30}$$

Then, considering Karush-Kuhn-Tucker (KKT) conditions, we take the first derivative of the Lagrange function in eq. (30) with respect to $r_{i,j}^f$, which yields

$$\frac{\partial\bar{\mathcal{L}}(r,\lambda,\mu,\upsilon)}{\partial r_{i,j}^f} = U'(r_{i,j}^f) + (\lambda_i^f - \lambda_j^f) - \mu_{i,j} + \upsilon_{i,j}. \tag{31}$$

By stationarity condition, we have

$$U'(r_{i,j}^f) + (\lambda_i^f - \lambda_j^f) - \mu_{i,j} + \upsilon_{i,j} = 0. \tag{32}$$

In addition, by applying the complementary slackness condition [32, Section 5.5],

$$\upsilon_{i,j} r_{i,j}^f = 0 \quad \text{and} \quad \upsilon_{i,j} \geq 0 \tag{33}$$

$$\mu_{i,j}\left(\sum_{s=1}^{S} w_s R_{i,j}^s - \sum_{f\in\mathcal{F}} r_{i,j}^f\right) = 0 \quad \text{and} \quad \mu_{i,j} \geq 0. \tag{34}$$

From eq. (32), we obtain $\upsilon_{i,j}$ as

$$\upsilon_{i,j} = \mu_{i,j} - (\lambda_i^f - \lambda_j^f) - U'(r_{i,j}^f). \tag{35}$$

Because $\upsilon_i \geq 0$, we obtain the following relationship:

$$U'(r_{i,j}^f) \leq \mu_{i,j} - (\lambda_i^f - \lambda_j^f). \tag{36}$$

In addition, since $U_i(\cdot)$ is a concave increasing, $U_i'(\cdot)$ is monotone, decreasing, and positive. Then, we obtain

$$r_{i,j}^f \geq U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big). \tag{37}$$

If $U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big) > 0$, then $r_{i,j}^f > 0$. Therefore, to meet the complementary slackness condition [32, Section 5.5] in eq. (33), we conclude that $\upsilon_{i,j} = 0$. Then, according to eq. (32), we have

$$r_{i,j}^f = U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big), \tag{38}$$

if $U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big) \leq 0$. This combined with (37) implies that $r_{i,j}^f \leq 0$. Thus, we can conclude that $r_{i,j}^f = 0$. Then, we get

$$r_{i,j}^f = \begin{cases} U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big), & U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big) > 0 \\ 0 & U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big) \leq 0 \end{cases}. \tag{39}$$

Therefore, to re-express eq. (39), we obtain

$$r_{i,j}^f = \Big[U'^{-1}\big(\mu_{i,j} - (\lambda_i^f - \lambda_j^f)\big)\Big]^+. \tag{40}$$

Taking the summation on both sides of eq. (40) and according to the second equality constraint in eqs. (12) and (28) yields

$$\sum_{f\in\mathcal{F}} U'^{-1}\left(\Big[\big(\mu_{i,j}(t) - (\lambda_i^f(t) - \lambda_j^f(t))\big)\Big]^+\right) = w_s^*(t) R_{i,j}^s. \tag{41}$$

Since the dual variables $\lambda_i^f(t)$, $\lambda_j^f(t)$ and link rate $w_s^*(t)R_{i,j}^s$ are known, the dual variable $\mu_{i,j}(t)$ can be calculated by using eq. (41). In addition, according to the complementary slackness condition, we know that $\mu_{i,j}(t) > 0$. Then, we can easily obtain the routing rate $r_{i,j}^f(\lambda,\mu,t)$ through

$$\begin{aligned}(r_{i,j}^f)^*(t) &= (r_{i,j}^f)^*(\lambda,\mu,t) \\ &= U'^{-1}\left(\Big[\big(\mu_{i,j}(t) - (\lambda_i^f(t) - \lambda_j^f(t))\big)\Big]^+\right).\end{aligned} \tag{42}$$

Taking the partial derivatives of the dual function (24) with respect to λ yields the stochastic gradient $g_i^f(t)$ as

$$g_i^f(t) = \sum_{j\in\mathcal{N}_i}\Big((r_{i,j}^f)^*(t) - (r_{j,i}^f)^*(t)\Big) - A_i^f(t). \tag{43}$$

Thus, we can update our stochastic gradient using Lagrangian maximizer $(r_{i,j}^f)^*(t)$ in (42). Moreover, we can calculate the time-average value $(\overline{r}_{i,j}^f)^*(t)$ as follows.

$$\begin{aligned}(\overline{r}_{i,j}^f)^*(t) &= \frac{1}{t}\sum_{k=1}^{t}(r_{i,j}^f)^*(k) \\ &= \frac{t-1}{t}(\overline{r}_{i,j}^f)^*(t-1) + \frac{1}{t}(r_{i,j}^f)^*(t), \quad t\geq 1.\end{aligned} \tag{44}$$

Now, we adopt the LIFO intra-queue scheduling policy for flow queues, which are updated as follows:

$$Q_i^f(t+1) = \left[Q_i^f(t) - \sum_{j\in\mathcal{N}_i}\big((\overline{r}_{i,j}^f)^*(t) - (\overline{r}_{j,i}^f)^*(t)\big) + A_i^f(t)\right]^+. \tag{45}$$

For link queues, we adopt the first-in-first-out (FIFO) intra-queue scheduling policy, and link queues evolve as follows:

$$q_{i,j}(t+1) = \left[q_{i,j}(t) - W_s^*(t)R_{i,j}^s + \sum_{f\in\mathcal{F}}\overline{r}_{i,j}^f(t)\right]^+. \tag{46}$$

In addition, we can also update multiplier λ along the stochastic gradients (43) using

$$\begin{aligned}\lambda_i^f(t+1) &= \lambda_i^f(t) - \alpha_2(t) g_i^f(t) \\ &= \Bigg[\lambda_i^f(t) - \alpha_2(t)\Bigg(\sum_{j\in\mathcal{N}_i}\big((r_{i,j}^f)^*(t) \\ &\qquad - (r_{j,i}^f)^*(t)\big) - A_i^f(t)\Bigg)\Bigg]^+,\end{aligned} \tag{47}$$

where $\alpha_2(t)$ is the step size for all routers i. Note that $(\overline{r}_{i,j}^f)^*(t)$ is used in the flow-queue length and link-queue length update in eqs. (45) and (46), not in the stochastic gradient descent in eq. (47).

The proposed time-average stochastic gradient routing algorithm is summarized in Algorithm 2, we will prove in Theorem 2 that Algorithm 2 converges as time proceeds. Such convergence is also verified experimentally in Figs. 12 and 13. At each time round t, the computation complexity is $O(N)$. Initially, we set $\lambda_i^f(0) = Q_i^f(0)$ for all routers and let flow queues follow the LIFO policy and link queues the FIFO policy. Then, **at the network layer**, we first calculate the Lagrange dual variable $\mu_{i,j}$ in step 6. Based on $\mu_{i,j}$, we can further compute the maximum routing rate for each router in step 9, which leads to a time-average routing rate of $(\overline{r}_{i,j}^f)^*(t)$. Then, **at the MAC layer**, we calculate the optimal value $W_s^*(t)$ in step 15, and time-average $w_s^*(t)$ in step 18. Then, each node can update its flow-queue function in step 20 and link-queue function in step 21. In addition, each node also updates the Lagrange multiples λ in steps 22. More specifically, the proposed TA-SGRA has two key features: First, we separate routing and scheduling algorithms to satisfy the layered architecture of network routers. Second, we

Algorithm 2: Time-Average Stochastic Gradient Routing Algorithm (TA-SGRA).

Observe $Q_i^f(0)$. Initialize $\lambda_i^f(0) = Q_i^f(0)$ for all routers
Let flow queues adopt the LIFO intra-queue scheduling policy and let link queues adopt the FIFO policy
for $t = 0, 1, 2, ...$ **do**
  When $t = 0$, randomly select a collision-free link-rate vector $R^s, s \in \mathcal{S}$. i.e., $W_s^*(0) = 1$, and let $w_s^*(0) = W_s^*(0)$
  for $j \in \mathcal{N}_i$ **do**
    At the network layer, compute $\mu_{i,j}(t) > 0$ such that
    $\sum_{f\in\mathcal{F}} U'^{-1}\left(\left[\left(\mu_{i,j}(t) - (\lambda_i^f(t) - \lambda_j^f(t))\right]^+\right)$
    $= w_s^*(t) R_{i,j}^s$
    Compute the Lagrange maximizer $(r_{i,j}^f)^*(t)$, where
    $(r_{i,j}^f)^*(t) = U'^{-1}\left(\left[\mu_{i,j}(t) - \left(\lambda_i^f(t) - \lambda_j^f(t)\right)\right]^+\right)$
    Compute the time-average routing rate $(\overline{r}_{i,j}^f)^*(t)$, where
    $(\overline{r}_{i,j}^f)^*(t) = \begin{cases} 0, & t = 0 \\ \frac{t-1}{t}(\overline{r}_{i,j}^f)^*(t-1) + \frac{1}{t}(r_{i,j}^f)^*(t) & t \geq 1 \end{cases}$
    Transmit packets at rate $(\overline{r}_{i,j}^f)^*(t)$
  end for
  At the MAC layer, find an optimal $W_s^*(t+1)$ using
    $W_s^*(t+1) = \arg\max_{W_s} \sum_{i,j}\sum_{s=1}^{S} \mu_{i,j}(t-1)$
    $\cdot W_s(t+1) R_{i,j}^s$
  Compute the time-average $w_s^*(t+1)$, where
    $w_s^*(t+1) = \sum_{k=1}^{t+1} \frac{W_s^*(k)}{t+1}$
  Update the flow-queue function $Q_i^f(t+1) = \left[Q_i^f(t) - \sum_{j\in\mathcal{N}_i}\left((\overline{r}_{i,j}^f)^*(t) - (\overline{r}_{j,i}^f)^*(t)\right) + A_i^f(t)\right]^+$
  Update the link-queue function $q_{i,j}(t+1) = \left[q_{i,j}(t) - W_s^*(t) R_{i,j}^s + \sum_{f\in\mathcal{F}} \overline{r}_{i,j}^f(t)\right]^+$
  Update the dual variable $\lambda_i^f(t+1) = \left[\lambda_i^f(t) - \alpha_2(t)\left(\sum_{j\in\mathcal{N}_i}\left((r_{i,j}^f)^*(t) - (r_{j,i}^f)^*(t)\right) - A_i^f(t)\right)\right]^+$
end for

also decouple the queue-length update process in eq. (45) and the dual-variable update process in eq. (47) so that we can use the time-average Lagrange maximizer to manage the transmission rates and the instantaneous Lagrange maximizer to control the dual variables.

A. Utility and Stability Analysis

In this section, we first prove that the TA-SGRA is utility optimal. More specifically, since our algorithm uses an iterative method, which means that it generates a sequence of improving approximate solutions for the problem, we can prove that the utility converges to the optimal value over time. In addition, the conventional convergence analysis is based on the Euclidean distance to the optimal set [34]. Hence, it requires a traffic arrival of bounded mean and variance. However, in our case, if the arrival traffic is heavy tailed, then the arrival will have the unbounded moments, such as variance. To overcome this issue, we adopt the ordinary differential equation method in [35, Section 2], which treats the discrete stochastic approximation scheme as a discretization version of the ODE with HT noise that is asymptotically negligible in the εth mean. Then, we have the following theorem:

Theorem 2: The time-average stochastic gradient routing algorithm is utility optimal.

Proof: See Appendix B. ■

Next, we will prove that the network is strongly stable under the TA-SGRA. The following theorem first shows that the time-average transmission rate converges to some constant as time proceeds so that LT flow queues will no longer compete with HT flow queues for transmission time.

Theorem 3: Under the proposed Algorithm 2, the time-average transmission rate $\overline{r}(t)$ of each flow, even in the presence of both LT and HT traffic, converges to a constant almost sure, i.e.,

$$\lim_{t\to\infty} \overline{r}^f(t) = U^{-1}(D^*) \; almost \; sure, \tag{48}$$

where $D^* = U(r^{*f})$.

Proof: See Appendix C. ■

Theorem 3 implies that LT and HT traffic flows eventually receive dedicated network resources without competing with each other. Under such a condition, we show that all flow queues are strongly stable by employing the LIFO intra-queue scheduling policy.

Theorem 4: Under the proposed time-average stochastic gradient routing algorithm, the LT flow queue has a bounded mean, even if an HT traffic flow is present.

Proof: According to Theorem 3, we know that when $t \to \infty$, the transmission rate of each flow converges to a constant value, which implies that every flow queue behaves as a $G1/G1/1$ queue asymptotically with a constant service rate. As a result, all LT traffic flows have a bounded average queueing delay, i.e., $E[D_i^f(t)] < \infty, f \in LT$. For HT flow, the LIFO discipline allows the waiting time or queueing delay to be "as heavy as" the service time in the case of HT arrival. Specifically, the tail distribution of the queueing delay of a $G1/G1/1$ queue with the LIFO discipline follows $\Pr(D_i^f(t) > x|\text{LIFO}) \sim \frac{1}{1-\rho}\Pr(B_i^f(t) > x(1-\rho))$ [36], where $B_i^f(t)$ denotes service time. This indicates that if traffic arrival is HT with tail index $\kappa(A_i^f(t))$, then the service time $B_i^f(t)$ for $A_i^f(t)$ follows an HT distribution, which asymptotically behaves as

$$\lim_{t\to\infty} \frac{\log \Pr(B_i^f(t) > x|\text{LIFO})}{\log x} = -\kappa(A_i^f(t)). \tag{49}$$

Consequently, the queueing delay tail distribution will behave as

$$\lim_{t\to\infty} \frac{\log \Pr(D_i^f(t) > x|\text{LIFO})}{\log x} = -\kappa(A_i^f(t)). \tag{50}$$

This means that the queueing delay is "as heavy as" the arrival process. Since the arrivals have a bounded mean, this implies that $\kappa(A_i^f(t)) > 1$. By Definition 2, this indicates that the average queueing delay of HT queues is bounded. ■

Theorem 4 proves that at the network layer, flow queues have a bounded queueing delay. Next, we evaluate the stability of the link queues by the following theorem:

Theorem 5: Under the proposed time-average stochastic gradient routing algorithm, the link queue has a bounded mean.

Proof: See Appendix D. ■

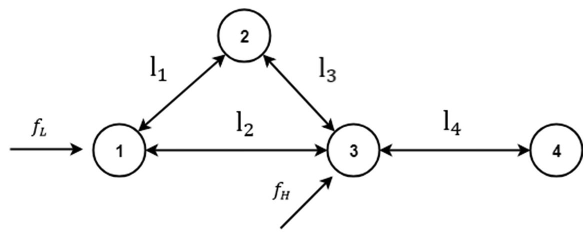

Fig. 3. Simulation model.

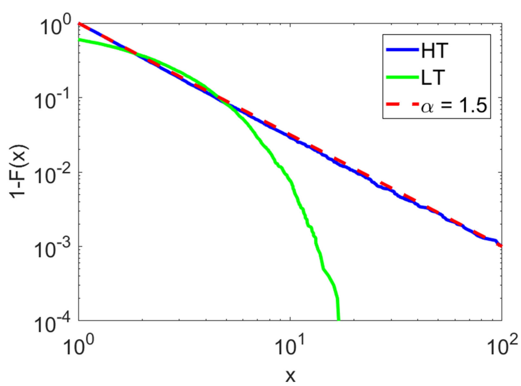

Fig. 4. Tail distribution of arrival traffic.

According to Theorems 4 and 5, we know that both flow queues and link queues have a bounded average queueing delay. Thus, the network is strongly stable.

VI. Simulation Results

In this section, we use simulations to verify our theoretical results. We first define a log utility function as the proportional fairness metric in [37]. Thus, the utility function is set to $U(r_{i,j}^f) = \log(r_{i,j}^f)$. Moreover, we select Pareto and exponential distributions to represent HT and LT distributions, respectively. We refer to a random variable $X \in \mathcal{PAR}(\alpha, x_m)$, if it follows a Pareto distribution with parameters α and x_m, i.e., $P(X > x) = (x_m/x)^\alpha$. We refer to a random variable $X \in \mathcal{EXP}(\lambda)$, if it follows an exponential distribution with parameter λ, i.e., $P(X > x) = e^{-\lambda x}$. All of the following simulation results are run in the 10^6 time slots and plotted on log-log coordinates, by which an HT distribution can manifest itself as a straight line with the slope equal to the negative value of the tail index α.

To illustrate our theoretical results, we use a small, four-node network topology, as shown in Fig. 3. At the network layer, at node 1, there is an LT flow f_L with an arrival process $A_1^{f_L}(t) \in \mathcal{EXP}(1/2)$ and $a_1^{f_L} = E[A_1^{f_L}] = 1/\lambda_1 = 2$. The destination of this LT flow f_L is node 4. At node 3, an HT flow is injected into the network with arrival processes $A_3^{f_H}(t) \in \mathcal{PAR}(1.5, 1)$ and $a_3^{f_H} = E[A_3^{f_H}(t)] = \alpha x_m/(\alpha - 1) = 3$, and the HT flow destination is also node 4. Fig. 4 shows the complimentary cumulative distribution function (CCDF) of the HT and LT flows used in our simulations. Observe in Fig. 4 that the CCDF or tail distribution of LT arrival process decays much faster than the CCDF or tail distribution of HT arrival process. This means that compared with the LT flow, the HT traffic flow has a much higher probability to generate a large volume of packets instantaneously, Therefore, the HT traffic is generally very bursty. Also, Fig. 5 shows the instantaneous traffic of HT and LT flows over time. Observe in Fig. 5 that even though the average arrival number of packets of the HT and LT traffics are close to each other, e.g., 3 and 2 packets per time slot, respectively, the instantaneous number of arrival packets for the HT traffic can be much larger than its average traffic arrivals, while the instantaneous traffic volume of LT flow do not deviate too much from its mean. For example, observe that 750 packets arrive at a slot time around 5000 for the HT traffic. However, the instantaneous number of arrived packets for the LT traffic is generally not far away from its average arrivals of 2 packets per time slot. Such highly variable and bursty traffic arrivals for the HT traffic cause challenges in the cross-layer network flow control designs. In addition, at the MAC layer, we assume that the link capacity $R_{i,j}$ between two nodes is 16, and $R^s = [R^s_{(1,2)}, R^s_{(2,3)}, R^s_{(1,3)}, R^s_{(3,4)}]$. From Fig. 3, we can conclude that there are four collision-free link-rate vectors R^1, R^2, R^3 and R^4, which are [16 0 0 0], [0 16 0 0], [0 0 16 0], and [0 0 0 16] respectively. The probabilities of selecting link-rate vectors R^1, R^2, R^3, and R^4 are w_1, w_2, w_3, and w_4, respectively.

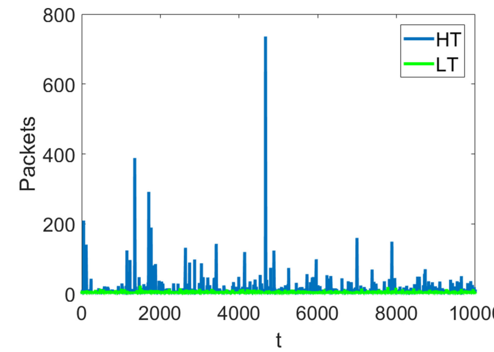

Fig. 5. Instantaneous generated packets of arrival traffic.

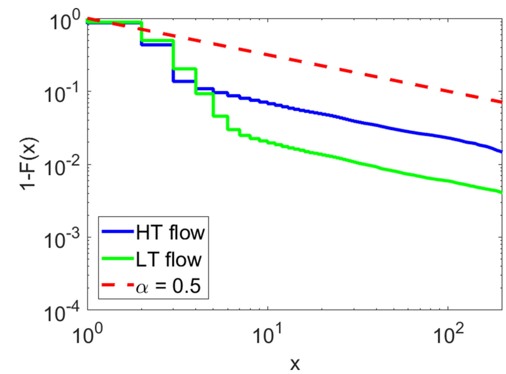

Fig. 6. Queue-delay tail distribution under conventional FIFO SGRA.

Because both HT and LT flows have the same destination, which is node 4, we observe and analyze HT and LT flows between nodes 3 and 4. In this case, according to Theorem 1, the light-tailed traffic flow f_L will have an unbounded average queue delay when the conventional FIFO SGRA (Algorithm 1) is used. As shown in Fig. 6, the queueing delay of the LT flow (i.e., f_L) has a tail distribution that exhibits itself as a straight line parallel to that of the reference Pareto distribution with tail index $\alpha = 0.5$. This indicates that the queueing delay of LT flow is heavy tailed with a tail index equal to 0.5. This, by moment theory [23], indicates that the LT flow experiences unbounded queueing delay. Such a high queueing delay is due to the fact that under classic SGD algorithms, LT flows are starved,

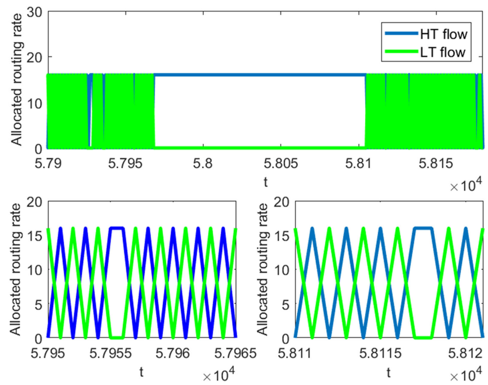

Fig. 7. Routing rate under conventional SGRA.

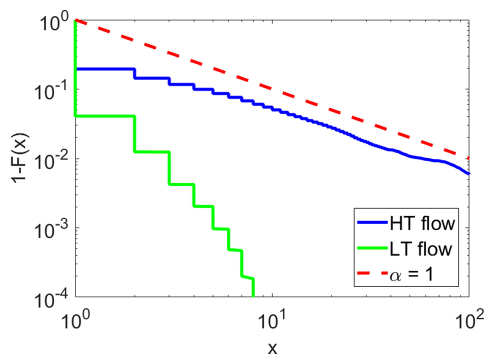

Fig. 8. Queue-delay tail distribution under proposed TA-SGRA.

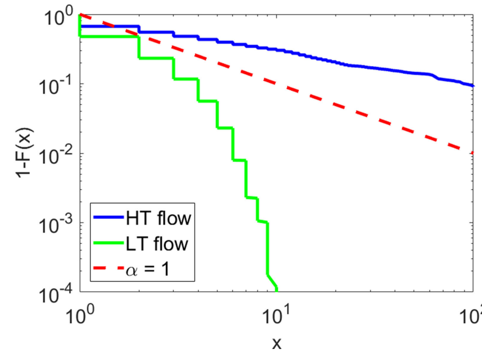

Fig. 9. Queue-delay tail distribution under proposed FIFO TA-SGRA

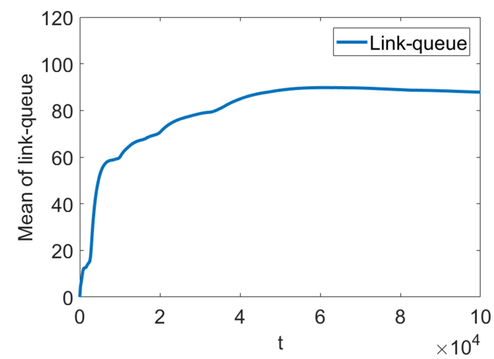

Fig. 10. Bounded mean of link-queue length.

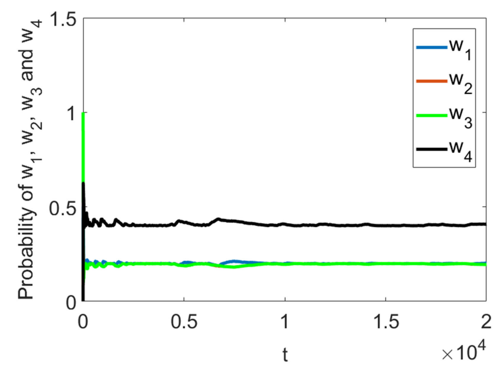

Fig. 11. Converged probability of selected link-rate vector.

while HT flows receive considerable resources. As shown in Fig. 7, before time slot 57960, since the queue-length difference of HT and LT flows between two neighboring nodes is very small, which leads to a small dual variable difference between two neighboring nodes because the dual variable is positively proportional to the queue length, they compete for the chance to transmit under the classic SGRA. Thus, we observe that the two curves oscillate before the LT traffics not being served in Fig. 7. However, due to the high variability and burstiness of HT traffic, a large number of packets can arrive abruptly during one time slot, which leads to a large queue-length difference, and the dual-variable difference between two neighboring nodes at these time slots becomes large. Then, to reduce the queue-length difference for the HT flow under the conventional SGRA, the HT flow will keep transmitting at rate 16, and the LT flow will be starved with zero rates between time slots 57960 and 58100. After time slot 58100, since the queue length difference of the HT flow becomes smaller than or equal to the queue length difference of the LT flow, competition resumes between the HT and LT flows. Hence, in Fig. 7, the two curves oscillate again.

Now, we verify the stability performance of the proposed TA-SGRA under the same network settings as in the previous case. Specifically, we define the step size in Algorithm 2 as $\alpha_2(t) = 1/(t+1)$, which satisfies $\sum_{t=0}^{\infty} \alpha_2(t) = \infty$ and $\sum_{t=0}^{\infty} \alpha_2(t)^2 < \infty$. It is shown in Fig. 8 that the queueing delay of both HT and LT flows decreases faster than the reference Pareto parameter $\alpha = 1$, which means that the average queueing delay of both LT and HT flows is bounded. To show the benefit of the LIFO policy, Fig. 9 displays the performance of the proposed TA-SGRA with the FIFO policy adopted for flow queues. It can be observed that the HT flow has a queueing delay, whose tail distribution decays much slower than the reference Pareto distribution with tail index 1, which indicates that the HT flow's queueing delay has a tail index smaller than 1 and thus has an unbounded mean. In addition, as shown in Fig. 10, the mean of the link-queue length converges to a constant value, which means the link queue is also of a bounded average queueing delay.

Furthermore, to show that the proposed TA-SGRA is utility optimal, we first set the initial probabilities of the selected collision-free link rate vectors R^1, R^2, R^3, and R^4 to zero and show that each selected vector of probabilities also converges as time proceeds. As shown in Fig. 11, the probabilities of w_1, w_2, w_3, and w_4 converge to 0.4, 0.2, 0.2, and 0.2, respectively. Thus, the mean of the data rate between nodes 3 and 4 is $16 \times 0.4 = 6.4$. Consequently, in Fig. 12, we can see that the allocated routing rate of HT and LT flows converges to 3.2, the sum of which is smaller than the link capacity 6.4. In this case, the TA-SGRA algorithm can reach the maximum utility of $2 \times ln(3.2) = 2.3263$, as shown in Fig. 13.

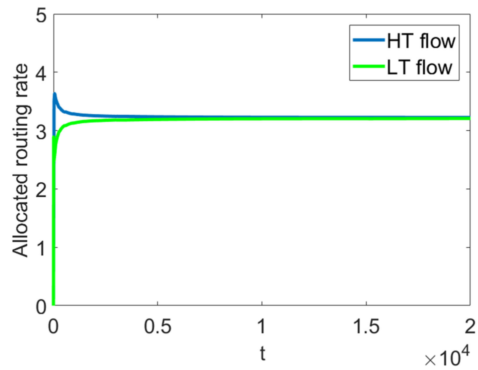

Fig. 12. Allocated routing rate under proposed TA-SGRA.

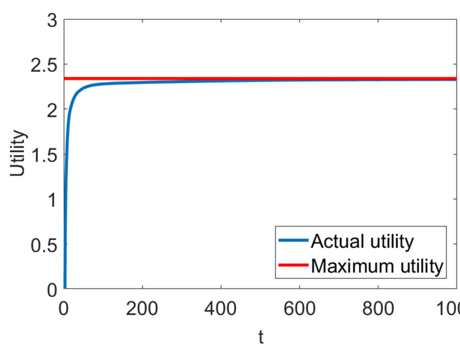

Fig. 13. Network utility under proposed TA-SGRA.

VII. Conclusion

In this paper, we investigate the impact of heavy-tail data on the performance of utility-maximum routing algorithms. We first proved that under the classic SGD-based routing algorithm, the tail distribution of the LT flow queue is at least one order heavier than the tail distribution of an HT flow queue, which implies that the LT flow queue has an unbounded queue length. To counter such a challenge, we proposed a new stochastic routing algorithm, namely the TA-SGRA, which separates the queue-length update process and Lagrange dual-variable process so that both utility optimality and queue stability can be achieved simultaneously. More specifically, it has been proven that the time-average transmission rate for each flow converges to a constant as time proceeds, which addresses the solution of the problem that the HT flow queue competes with the LT flow queue for transmission time. In addition, to address the methodology challenges brought about by the unbounded moments of HT data, we adopted an ordinary differential equation approach to prove the convergence and stability properties of the proposed TA-SGRA. To the best of our knowledge, TA-SGRA is the first routing algorithm in the literature that can simultaneously achieve network stability and utility optimality in the presence of heavy tails. Multi-channel scenarios and a network with propagation variations, such as white space and fading channels, will be good problems for future work.

Appendix A
Proof of Theorem 1

Proof: To prove that the queue length of an LT flow f_L is unbounded under the conventional SGRA, we consider a node i that has both HT and LT flow arrivals. In addition, we assume that the queueing system is in the steady state. Then, the queue-length process is in a positive recurrent regenerative process. Moreover, we let T denote the time interval between two consecutive instances when all queues are empty. Then, $E[T] < \infty$ always holds. More specifically, assume at time slot zero that the HT flow arrival ($i.e.$, f_H) with the smallest tail index, i.e., $\kappa(A_i^{f_H}(t)) = \min_{f\in\mathcal{F}}\kappa(A_i^f(t))$, receives a file of size M number of packets, and all other flows receive no traffic. Furthermore, let T_M denote the first time slot when the flow-queue length difference of flow f_H becomes less than or equal to the flow-queue length difference of the flow f_L between nodes i and j. In particular, node j, $j \in \mathcal{N}_i$, is the neighbor of node i. Then, we can denote T_M using the following equation:

$$T_M := \min\{t > 0 | Q_i^{f_L}(t) - Q_j^{f_L}(t) \geq Q_i^{f_H}(t) - Q_j^{f_H}(t)\} \quad i \in \mathcal{N}, j \in \mathcal{N}_i, (f_L, f_H) \in \mathcal{F}. \tag{51}$$

Under Algorithm 1, we not only know that the LT flow will not obtain any service since it has a smaller flow-queue difference than the HT flow, but also that the LT flow will keep receiving new arrivals $A_i^{f_L}(t)$ from the external network and packets from last-hop neighbor nodes j, $j \in \mathcal{N}_i$. At the same time, the HT flow is always served at $r_{i,j}^{f_H}$ rate until time slot T_M. Thus, for the LT flow, we have

$$Q_i^{f_L}(T_M) = \sum_{t=0}^{T_M-1}\left(A_i^{f_L}(t) + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L}(t)\right). \tag{52}$$

According to the strong law of large numbers (SLLN) [38, Chapter 1], we have $\sum_{t=0}^{T_M-1}(A_i^{f_L}(t) + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L}(t)) \geq (a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})T_M - \delta_M$ and $\sum_{t=0}^{T_M-1}\sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H}(t) \geq \sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H} T_M - \varsigma_M$, with probability 1. Thus, according to step 7 of Algorithm 1, we have the following relationship, which shows the first time slot of the queue length of LT flow larger or equal to the queue length of HT flow, as

$$\left(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L}\right) T_M - \delta_M \geq M - \left(\sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H} T_M - \varsigma_M\right), \tag{53}$$

where M is number of packets, $r_{j,i}^{f_L} = \arg\max_{r_{j,i}^{f_L} = w_s R_{j,i}^s} \sum_{j,i}(U(r_{j,i}^{f_L}) + r_{j,i}^{f_L}(Q_j^{f_L} - Q_i^{f_L}))$, and $r_{i,j}^{f_H} = \arg\max_{r_{i,j}^{f_H} = w_s R_{i,j}^s} \sum_{i,j}(U(r_{i,j}^{f_H}) + r_{i,j}^{f_H}(Q_i^{f_H} - Q_j^{f_H}))$. In addition, there exists a constant K, such that

$$\begin{aligned} T_M &\geq \frac{M + \varsigma_M + \delta_M}{a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L} + \sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H}} \\ &\geq \frac{M}{a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L} + \sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H}} = KM, \end{aligned} \tag{54}$$

where $K = \frac{1}{a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L} + \sum_{j\in\mathcal{N}_i} r_{i,j}^{f_H}}$, and $\delta_M, \varsigma_M > 0$. In addition, by the property of the regenerative process with cycle length T, we have

$$\Pr(Q_i^{f_L} > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}$$
$$= \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{I}_{\left\{Q_i^{f_L}(\tau) > \frac{KM\left(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L}\right)}{2}\right\}}$$
$$= \frac{E\left[\sum_{t=0}^{T} \mathbb{I}_{\left\{Q_i^{f_L}(t) > \frac{KM\left(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L}\right)}{2}\right\}}\right]}{E[T]}. \quad (55)$$

Then, we obtain

$$E\left[\sum_{t=0}^{T} \mathbb{I}_{\left\{Q_i^{f_L}(t) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right\}}\right]$$
$$\geq E\Bigg[\mathbb{I}(A_i^{f_H}(0) > M)$$
$$\cdot \sum_{t=0}^{T} \mathbb{I}\left(Q_i^{f_L}(t) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right)\Bigg]$$
$$\geq \Pr(A_i^{f_H}(0) > M)$$
$$\cdot \sum_{t=\frac{T_M}{2}}^{T_M} \Pr\left(Q_i^{f_L}(t) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right). \quad (56)$$

By the queueing dynamic in eq. (4) and $Q_i^{f_L}(0) = 0$, we have $Q_i^{f_L}(t) = \sum_{\tau=0}^{t-1}[A_i^{f_L}(\tau) - \sum_{j\in\mathcal{N}_i}(r_{i,j}^{f_L}(\tau) - r_{j,i}^{f_L}(\tau))\mathbb{I}_{\{Q_i^{f_L}(\tau)>0\}}]$. This implies that

$$\lim_{M\to\infty} \frac{1}{M}\sum_{t=\frac{T_M}{2}}^{T_M} \Pr\left(Q_i^{f_L}(t) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right)$$
$$\geq \lim_{M\to\infty} \frac{1}{M}\sum_{t=\frac{KM}{2}}^{KM} \Pr\Bigg(\sum_{\tau=0}^{t-1} A_i^{f_L}(\tau) >$$
$$\frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\Bigg)$$
$$= \frac{K}{2}. \quad (57)$$

The last equality in eq. (57) holds, due to the fact that flow f_H occupies the entire channel service during the time interval $0 \leq t \leq KM/2$ so that $\Pr\left(\sum_{\tau=0}^{t-1} A_i^{f_L}(\tau) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right) = 1$ always holds when $KM/2 \leq t \leq KM$. Then, we have

$$Pr\left(Q_i^{f_L} > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right)$$
$$\geq \frac{\Pr(A_i^{f_H}(0) > M)}{E[T]}$$
$$\cdot \frac{\sum_{t=\frac{T_M}{2}}^{T_M} \Pr\left(Q_i^{f_L}(t) > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right)}{E[T]}, \quad (58)$$

according to [39, Page 7], and combining eq. (55) with eqs. (56)–(57), it follows from the condition in eq. (21) that

$$\lim_{M\to\infty} \frac{\log\left[\Pr\left(Q_i^{f_L} > \frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right)\right]}{\log\left[\frac{KM(a_i^{f_L} + \sum_{j\in\mathcal{N}_i} r_{j,i}^{f_L})}{2}\right]}$$
$$\geq -\min_{f\in\mathcal{F}} \kappa(A_i^f(t)) + 1 \geq -1. \quad (59)$$

By applying the moment theorem [23], the steady-state queue length and queueing delay, $E[Q_i^{f_L}]$ and $E[D_i^{f_L}]$, respectively, are of unbounded mean. Thus, the strong stability cannot be achieved under the classic stochastic gradient algorithm. ■

APPENDIX B
PROOF OF THEOREM 2

Proof: To explore convergence and stability properties of our algorithm and prove that it is utility optimal, we adopt a similar technique in [40], which uses the ordinary differential equation approach.

From eq. (47), we have vector $\lambda^f(t+1)$:

$$\lambda^f(t+1) = \lambda^f(t) - \alpha_2(t)g^f(t)$$
$$= \lambda^f(t) + \alpha_2(t)\Bigg(a_i^f - \sum_{j\in\mathcal{N}_i}(r_{i,j}^f(t) - r_{j,i}^f(t))$$
$$+ \alpha_2(t)\big(A^f(t) - a_i^f\big)\Bigg)$$
$$= \lambda^f(t) + \alpha_2(t)(h^f(t) + \hat{A}^f(t)), \quad (60)$$

where $h^f(t) := a_i^f - \sum_{j\in\mathcal{N}_i}\left(r_{i,j}^f(t) - r_{j,i}^f(t)\right)$ and $\hat{A}^f(t) := A^f(t) - a_i^f$. Since, according to the first constraint of eq. (12), we know $h^f(t)$ is larger than some negative constant c and smaller than 0, it is in a strong form of uniform continuity in the range [c, 0], i.e., Lipschitz [41, Section 12.3]. Moreover, for stochastic gradient algorithms, the associated ODE is $\dot{\lambda}^f(t) = -g^f(\lambda^f(t))$. Thus, we have the following relationship

$$\dot{\lambda}^f(t) = h^f(\lambda^f(t)). \quad (61)$$

Then, we define $j(t) = \sum_{k=0}^{t-1}\alpha_2(k)$, and let $n = \inf\{k : j(k) > j(t) + T\}$ and $T = j(n) - j(t)$. Therefore, the continuous, piecewise linear interpolated version of $\lambda^f(t)$ can be defined by $\bar{\lambda}^f(j(t)) = \lambda^f(t)$, with linear interpolation on each interval T. In addition, to find the supreme of the difference norm

$\sup_{t\le k\le n}||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))||$ when $t\le k\le n$, we rewrite $\bar{\lambda}^f(j(k))$ and $\lambda_t^f(j(k))$, respectively, as

$$\bar{\lambda}^f(j(k))=\bar{\lambda}^f(j(t))+\sum_{i=t}^{k-1}\alpha_2(i)\big(h^f(\bar{\lambda}^f(j(i))+\hat{A}^f(i)\big) \tag{62}$$

and

$$\begin{aligned}\lambda_t^f(j(k))&=\bar{\lambda}^f(j(t))+\int_{j(t)}^{j(k)}h^f(\lambda_t^f(v))dv\\&=\bar{\lambda}^f(j(t))+\sum_{i=t}^{k-1}\Bigg\{\alpha_2(i)h^f(\lambda_t^f(j(i)))\\&\quad+\int_{j(i)}^{j(i+1)}\big(h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i))\big)dv\Bigg\},\end{aligned} \tag{63}$$

where $\int_{j(i)}^{j(i+1)}dv=\alpha_2(i)$. Then, we subtract eq. (63) from eq. (62), obtaining

$$\begin{aligned}&\sup_{t\le k\le n}||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))||\\&\le\sup_{t\le k\le n}\sum_{i=t}^{k-1}\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i)))||dv\\&\quad+\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|.\end{aligned} \tag{64}$$

Then, we let

$$I=\sup_{t\le k\le n}\sum_{i=t}^{k-1}\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i)))||dv \tag{65}$$

and

$$II=\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|. \tag{66}$$

For term I, according to eq. (63), we know that

$$\lambda_t^f(j)=\lambda^f(t)+\int_{j(t)}^{j(n)}h^f(\lambda_t^f(v))dv, \tag{67}$$

where $j\in[j(t),j(n)]$, and $\overline{\lambda}^f(j(t))=\lambda^f(t)$. Then, we take the norm on both sides of eq. (67) and obtain

$$||\lambda_t^f(j)||\le||\lambda^f(t)||+\int_{j(t)}^{j(n)}||h^f(\lambda_t^f(v))||dv. \tag{68}$$

Since h^f is Lipschitz continuous and grows linearly, we have $||h^f(x)-h^f(0)||\le L||x||$ and $||h^f(x)||\le||h^f(0)||+L||x||$, where $L>0$ denotes the Lipschitz constant. Then, we obtain

$$||h^f(\lambda_t^f(v))||\le||h^f(0)||+L||\lambda_t^f(v)||. \tag{69}$$

By defining $B_0=\lambda_t^f(j(t))=\overline{\lambda}(j(t))=\lambda^f(t)\le\sup_t||\lambda^f(t)||$, we can rewrite eq. (68) as

$$\begin{aligned}||\lambda_t^f(j)||&\le||\lambda^f(t)||+\int_{j(t)}^{j(n)}||h^f(0)+L(\lambda_t^f(v))||dv\\&=(B_0+T||h^f(0)||)+L\int_{j(t)}^{j(n)}||h^f(\lambda_t^f(v))||dv.\end{aligned} \tag{70}$$

In addition, by using Gronwall's inequality in [42], we further have

$$\begin{aligned}||\lambda_t^f(j)||&\le(B_0+T||h^f(0)||)e^{L\int_{j(t)}^{j(n)}||h^f(\lambda_t^f(v))||dv}\\&\le(B_0+T||h^f(0)||)e^{LT},\end{aligned} \tag{71}$$

where, since $h^f(\cdot)=1$, $\int_{j(t)}^{j(b)}||h^f(\lambda_t^f(v))||dv=j(b)-j(t)=T$ always holds. Then, according to Lipschitz continuity,

$$\begin{aligned}||h^f(\lambda_t^f(j))||&\le||h^f(0)||+L||\lambda_t^f(j)||\\&\le||h^f(0)||+L(B_0+T||h^f(0)||)e^{LT}\\&=B,\end{aligned} \tag{72}$$

where $B:=||h^f(0)||+L(B_0+T||h^f(0)||)e^{LT}$. Moreover, when $j\in[j(i),j(i+1)]$, we have

$$||\lambda_t^f(j)-\lambda_t^f(j(i))||\le\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))||dv\le B\alpha_2(i). \tag{73}$$

Since h^f has Lipschitz continuity, we obtain the following relationship:

$$\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i)))||dv\le BL(\alpha_2(i))^2. \tag{74}$$

Then, we take the summation on both sides of eq. (74) and obtain

$$\sum_{i=t}^{k-1}\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i)))||dv\le\sum_{i=t}^{k-1}BL(\alpha_2(i))^2, \tag{75}$$

and by taking the limitation on both sides of eq. (75), we have

$$\begin{aligned}&\lim_{t\to\infty}\sup_{t\le k\le n}\sum_{i=t}^{k-1}\int_{j(i)}^{j(i+1)}||h^f(\lambda_t^f(v))-h^f(\lambda_t^f(j(i)))||\\&\le\lim_{t\to\infty}\sum_{i=t}^{k-1}BL(\alpha_2(i))^2=0,\end{aligned} \tag{76}$$

where $\lim_{t\to\infty}\sum_{i=t}^{k-1}(\alpha_2(i))^2=0$. Thus, eq. (76) holds.

For II, according to [43], we have

$$P\left(\sup_{t\le k\le n} ||\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)|| \ge x\right) \le \frac{K\left(\sum_{i=t}^{n}(\alpha_2(i))^{\frac{\beta^2-1}{\beta}+1}\right)^{\frac{\beta}{\beta+1}}}{x^\beta} \quad (77)$$

for $x > K\left(\sum_{i=t}^{n}(\alpha_2(i))^{\frac{\beta^2-1}{\beta}+1}\right)^{\frac{1}{\beta+1}}$, where β is larger than 1. We define the following:

$$\begin{aligned}
\mu(t) &:= K\left(\sum_{i=t}^{n}(\alpha_2(i))^{\frac{\beta^2-1}{\beta}+1}\right)^{\frac{1}{\beta+1}} \\
&= K\left(\sum_{i=t}^{n}\alpha_2(i)^{\frac{\beta^2-1}{\beta}}\alpha_2(i)\right)^{\frac{1}{\beta+1}} \\
&= K\Bigg(\alpha_2(t)^{\frac{\beta^2-1}{\beta}}\alpha_2(t)+\alpha_2(t+1)^{\frac{\beta^2-1}{\beta}}\alpha_2(t+1), ..., \\
&\qquad +\alpha_2(n)^{\frac{\beta^2-1}{\beta}}\alpha_2(n)\Bigg)^{\frac{1}{\beta+1}} \\
&\le K\left((T+1)\alpha_2(t)^{\frac{\beta^2-1}{\beta}}\alpha_2(t)\right)^{\frac{1}{\beta+1}} \\
&\le K(T+1)^{\frac{1}{\beta+1}}\alpha_2(t)^{\frac{\beta-1}{\beta}} \qquad (78)
\end{aligned}$$

Eq. (78) holds since $\sup_t ||\alpha_2(t)|| \le 1$. In addition, when $t \to \infty, \mu(t) \to 0$. Furthermore, we use a similar method in [44], for $1 < \varepsilon < \beta$, and we obtain the following relationship:

$$\begin{aligned}
&E\left[\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|^\varepsilon\right] \\
&\le K\int_0^\infty x^{\varepsilon-1}P\left(\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|\ge x\right)dx \\
&= K\int_0^{\mu(t)} x^{\varepsilon-1}P\left(\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|\ge x\right)dx \\
&\quad + K\int_{\mu(t)}^\infty x^{\varepsilon-1}P\left(\sup_{t\le k\le n}\left\|\sum_{i=t}^{k-1}\alpha_2(i)\hat{A}^f(i)\right\|\ge x\right)dx \\
&\le K\mu(t)^\varepsilon + K\int_{\mu(t)}^\infty x^{\varepsilon-1}\left(\frac{\mu(t)^\beta}{x^\beta}\right)dx \\
&= K\mu(t)^\varepsilon + K\mu(t)^\beta\int_{\mu(t)}^\infty x^{\varepsilon-\beta-1}dx \\
&= K\mu(t)^\varepsilon + K\mu(t)^\beta\frac{1}{\varepsilon-\beta-1}x^{\varepsilon-\beta}|_{\mu(t)}^\infty \\
&= K\mu(t)^\varepsilon - \frac{1}{\varepsilon-\beta-1}K\mu(t)^\varepsilon \\
&= \hat{K}\mu(t)^\varepsilon. \qquad (79)
\end{aligned}$$

where $\hat{K} = (1-\frac{1}{\varepsilon-\beta-1})K$. Moreover, according to eq. (78), we know that when $t\to\infty$, $T_{II}\to 0$. Then, combining (76) and (79), we obtain

$$\lim_{t\to\infty}\sup_{t\le k\le n}||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))|| = 0. \quad (80)$$

Then, considering the linear interpolation error in [35, Section 2.1], if $j(k)\le j\le j(k+1)$, we have

$$\bar{\lambda}^f(j) = \kappa\bar{\lambda}^f(j(k)) + (1-\kappa)\bar{\lambda}^f(j(k+1)) \quad (81)$$

for some $\kappa\in[0,1]$. Thus,

$$\begin{aligned}
||\bar{\lambda}^f(j)-\lambda_t^f(j)|| &= ||\kappa(\bar{\lambda}^f(j)-\lambda_t^f(j(k))) \\
&\quad + (1-\kappa)(\bar{\lambda}^f(j)-\lambda_t^f(j(k+1)))||. \quad (82)
\end{aligned}$$

In addition, eq. (82) can be upper bounded as

$$\begin{aligned}
&||\bar{\lambda}^f(j)-\lambda_t^f(j)|| \\
&\le \kappa||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))|| \\
&\quad + (1-\kappa)||\bar{\lambda}^f(j(k+1))-\lambda_t^f(j(k+1))|| \\
&\quad + \kappa\int_{j(k)}^{j}||h^f(\lambda_t^f(v))||dv \\
&\quad + (1-\kappa)\int_{j}^{j(k+1)}||h^f(\lambda_t^f(v))||dv. \quad (83)
\end{aligned}$$

Thus, we obtain

$$\begin{aligned}
&\lim_{t\to\infty}\sup_{j\in[j(t),j(n)]}||\bar{\lambda}^f(j)-\lambda_t^f(j)|| \\
&\le \lim_{t\to\infty}\sup_{t\le k\le n}||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))|| = 0. \quad (84)
\end{aligned}$$

By taking the εth mean on both sides of eq. (84), we get

$$\begin{aligned}
&\lim_{t\to\infty}E\left[\sup_{j\in[j(t),j(n)]}||\bar{\lambda}^f(j)-\lambda_t^f(j)||^\varepsilon\right] \\
&\le \lim_{t\to\infty}E\left[\sup_{t\le k\le n}||\bar{\lambda}^f(j(k))-\lambda_t^f(j(k))||^\varepsilon\right] = 0, \quad (85)
\end{aligned}$$

where $1<\varepsilon<\beta$. Then, according to [44], we know that

$$\lim_{t\to\infty}E\left[||\lambda^f(t)-\lambda^*||^\varepsilon\right] = 0 \quad (86)$$

which implies that

$$\lim_{t\to\infty}E[r^f(t)-r^{*f}] = 0. \quad (87)$$

Furthermore, we can conclude that

$$\lim_{t\to\infty}\left(U(E[r^f(t)])-U(r^{*f})\right) = 0. \quad (88)$$

Thus, we can conclude that the TA-SGRA is utility optimal, since the utility converges to a optimal value U^* as time proceeds. ■

Appendix C
Proof of Theorem 3

Proof: To prove that the time-average transmission rate converges to a constant, we use the same technique in [5]. Then, considering strong convexity [32], we have

$$f(y) - f(x) \leq \left\langle f^{'}(x), y - x \right\rangle + \frac{L}{2}||y - x||^2 \tag{89}$$

where $< \cdot, \cdot >$ denotes the inner product, and G is a positive constant. Moreover, since $\lambda^f(t+1) = \lambda^f(t) - \alpha_2(t) g^f(t)$ and according to eq. (89), we obtain the following relationship:

$$\begin{aligned} D(\lambda^f(t+1)) &\leq D(\lambda^f(t)) + \left\langle D^{'}(\lambda^f(t)), \lambda^f(t+1) - \lambda^f(t) \right\rangle \\ &\quad + \frac{L}{2}||\lambda^f(t+1) - \lambda^f(t)||^2 \\ &= D(\lambda^f(t)) - \alpha_2(t)||D^{'}(\lambda^f(t))||^2 \\ &\quad + \frac{L}{2}\alpha_2(t)^2||D^{'}(\lambda^f(t))||^2 \\ &= D(\lambda^f(t)) - \left(1 - \frac{L}{2}\alpha_2(t)\right)\alpha_2(t)||D^{'}(\lambda^f(t))||^2. \end{aligned} \tag{90}$$

In addition, since $g^f(t) = D^{'}(\lambda^f(t))$, we have

$$\begin{aligned} &E[D(\lambda^f(t)) - D(\lambda^f(t+1))] \\ &\quad \geq \left(1 - \frac{L}{2}\alpha_2(t)\right)\alpha_2(t) E[||g^f(t)||]^2. \end{aligned} \tag{91}$$

According to Theorem 2, when $t \to \infty$, we obtain

$$\begin{aligned} \lim_{t\to\infty} E[D(\lambda^f(t)) - D(\lambda^f(t+1))] &= \lim_{t\to\infty} E[D(\lambda^f(t)) - D^*] \\ &= 0, \end{aligned} \tag{92}$$

where D^* is the optimal value of the dual function. Therefore, from the first-order optimality condition, we can conclude that

$$\lim_{t\to\infty} E[||g^f(\lambda^f(t))||] = 0. \tag{93}$$

Since strong duality exists, the minimum value of the dual function is the optimal solution of the primal function. Then, we obtain

$$U(r^{*f}) = D^*, \tag{94}$$

and thus,

$$r^{*f} = U^{-1}(D^*). \tag{95}$$

In addition, from eq. (44), we have

$$\bar{r}^f(t) = \frac{t-1}{t}\bar{r}^f(t-1) + \frac{1}{t}r^f(t), \quad t \geq 1. \tag{96}$$

and

$$t\bar{r}^f(t) - (t-1)\bar{r}^f(t-1) - r^f(t) = 0. \tag{97}$$

Furthermore, by taking the expectation on both sides of eq. (97), we have

$$E[t\bar{r}^f(t) - (t-1)\bar{r}^f(t-1)] = E[r^f(t)]. \tag{98}$$

Then, when $t \to \infty$, taking the limitation on both sides of eq. (98), we have the following relationship:

$$\lim_{t\to\infty} E[t\bar{r}^f(t) - (t-1)\bar{r}^f(t)] = \lim_{t\to\infty} E[r^f(t)]. \tag{99}$$

From eq. (87), we know that $\lim_{t\to\infty} E[r^f(t)] = r^{*f}$. Then, we have

$$\lim_{t\to\infty} E[\bar{r}^f(t)] = r^{*f}. \tag{100}$$

In addition, according to the convergence properties of time-average stochastic gradient descent algorithms [45], we obtain

$$P\left(\lim_{t\to\infty} \bar{r}^f(t) = r^{*f}\right) = 1 \tag{101}$$

Thus, we have

$$\lim_{t\to\infty} \bar{r}^f(t) = U^{-1}(D^*), \ almost\ sure, \tag{102}$$

which completes the proof. ■

Appendix D
Proof of Theorem 5

Proof: To prove that the link queue $q_{i,j}$ has a bounded mean, we adopt the Lyapunov drift theory. First, we define a quadratic Lyapunov function as

$$L(q(t)) = \sum_{i,j} q_{i,j}^2(t). \tag{103}$$

Then, we consider

$$\begin{aligned} &L(q(t+1)) - L(q(t)) \\ &= \sum_{i,j}(q_{i,j}(t+1) - q_{i,j}(t))(q_{i,j}(t+1) - q_{i,j}(t) + 2q_{i,j}(t)) \\ &= \sum_{i,j}(q_{i,j}(t+1) - q_{i,j}(t))^2 + \sum_{i,j} 2q_{i,j}(t)(q_{i,j}(t+1) \\ &\qquad - q_{i,j}(t)). \end{aligned} \tag{104}$$

Taking conditional expectations on eq. (104), we obtain

$$\begin{aligned} &E[L(q(t+1)) - L(q(t))|q(t)] \\ &= E\left[\sum_{i,j}(q_{i,j}(t+1) - q_{i,j}(t))^2 | q(t)\right] \\ &\qquad + E\left[\sum_{i,j} 2q_{i,j}(t)(q_{i,j}(t+1) - q_{i,j}(t))\right]. \end{aligned} \tag{105}$$

Then, we let

$$I = E\left[\sum_{i,j}(q_{i,j}(t+1) - q_{i,j}(t))^2 | q(t)\right] \tag{106}$$

$$II = E\left[\sum_{i,j} 2q_{i,j}(t)(q_{i,j}(t+1) - q_{i,j}(t))\right]. \tag{107}$$

For I, from eq. (9), we have

$$\begin{aligned} I &= E\left[\sum_{i,j}\left(\overline{r}_{i,j}(t) - \sum_{s\in S} W_s(t) R^s_{i,j}\right)^2 |q(t)\right] \\ &\leq E\left[\sum_{i,j}\overline{r}^2_{i,j}(t)|q(t)\right] \\ &= \sum_{i,j} E\left[\overline{r}^2_{i,j}(t)\right]. \end{aligned} \tag{108}$$

For II, using eq. (9), we obtain

$$\begin{aligned} II &= E\left[\sum_{i,j} 2q_{i,j}(t)\left(\overline{r}_{i,j}(t) - \sum_{s\in S} W_s(t) R^s_{i,j}\right)|q(t)\right] \\ &= E\left[\sum_{i,j} 2q_{i,j}(t)\overline{r}_{i,j}(t)|q(t)\right] \\ &\quad - E\left[\sum_{i,j} 2q_{i,j}(t)\sum_{s\in S} W_s(t)R^s_{i,j}\right]. \end{aligned} \tag{109}$$

Then, we define

$$III = E\left[\sum_{i,j} 2q_{i,j}(t)\overline{r}_{i,j}(t)|q(t)\right] \tag{110}$$

$$IV = E\left[\sum_{i,j} 2q_{i,j}(t)\sum_{s\in S} W_s(t)R^s_{i,j}\right]. \tag{111}$$

For III, we have

$$III = 2q_{i,j}(t)r_{i,j}. \tag{112}$$

For IV, we get

$$IV = 2\sum_{i,j} q_{i,j}(t)\sum_{s\in S} w_s R^s_{i,j}. \tag{113}$$

Therefore, we have

$$\begin{aligned} &E\left[\sum_{i,j} 2q_{i,j}(t)(q_{i,j}(t+1) - q_{i,j}(t))\right] \\ &= 2\sum_{i,j} q_{i,j}(t)\left(r_{i,j} - \sum_{s\in S} w_s R^s_{i,j}\right). \end{aligned} \tag{114}$$

According to the second constraint of eq. (12), we can bound eq. (114) as follows:

$$E\left[\sum_{i,j} 2q_{i,j}(t)(q_{i,j}(t+1) - q_{i,j}(t))\right] \leq 2\xi \sum_{i,j} q_{i,j}(t), \tag{115}$$

where ξ is a small constant. Then, from eqs. (105), (108), and (115), we have

$$\begin{aligned} &E[L(q(t+1)) - L(q(t))|q(t)] \\ &\leq \sum_{i,j} E[\overline{r}^2_{i,j}(t)] + 2\xi\sum_{i,j} q_{i,j}(t). \end{aligned} \tag{116}$$

Using Foster's criterion for the ergodic Markov chain, the link-queue-length process converges in distribution. Using the iterated mean and telescoping sums [46, Section 3.4], we have

$$\sum_{i,j} E[q_{i,j}(t)] \leq -\frac{1}{2\xi}\sum_{i,j} E[\overline{r}^2_{i,j}(t)], \tag{117}$$

which completes the proof. ■

REFERENCES

[1] J. Liu, N. B. Shroff, C. H. Xia, and H. D. Sherali, "Joint congestion control and routing optimization: An efficient second-order distributed approach," *IEEE/ACM Trans. Netw.*, vol. 24, no. 3, pp. 1404–1420, Jun. 2016.

[2] J. Liu, H. Y. Thomas, Y. Shi, and H. D. Sherali, "Cross-layer optimization on routing and power control of MIMO ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 913–926, Aug. 2008.

[3] A. Ribeiro, "Stochastic soft backpressure algorithms for routing and scheduling in wireless ad-hoc networks," in *Proc. IEEE Int. Workshop Comput. Advances Multi-Sensor Adaptive Process.*, 2009, pp. 137–140.

[4] M. Zargham, A. Ribeiro, and A. Jadbabaie, "Accelerated backpressure algorithm," in *Proc. IEEE Global Commun. Conf.*, 2013, pp. 2269–2275.

[5] S. Xia and P. Wang, "Stochastic network utility maximization in the presence of heavy-tails," in *Proc. IEEE Int. Conf. Commun.*, Paris, France, May 2017, pp. 1–7.

[6] S. Xia, "Distributed throughput optimal scheduling for wireless networks," Master's thesis, Dept. Elect. Eng. Comput. Sci., Wichita State University, Wichita, KS, USA, 2014.

[7] W. H. Xiong, A. Mukherjee, and H. M. Kwon, "MIMO cognitive radio user selection with and without primary channel state information," *IEEE Trans. Veh. Technol.*, vol. 65, no. 2, pp. 985–991, Feb. 2016.

[8] P. Wang and I. F. Akyildiz, "Spatial correlation and mobility aware traffic modeling for wireless sensor networks," *IEEE/ACM Trans. Netw.*, vol. 19, no. 6, pp. 1860–1873, Dec. 2011.

[9] V. Ramaswami, K. Jain, R. Jana, and V. Aggarwal, "Modeling heavy tails in traffic sources for network performance evaluation," *Comput. Intell., Cyber Secur. Comput. Models*, vol. 246, pp. 23–44, 2014.

[10] A. Ghosh, R. Jana, V. Ramaswami, J. Rowland, and N. K. Shankaranarayanan, "Modeling and characterization of large-scale Wi-Fi traffic in public hot-spots," in *Proc. IEEE INFOCOM 2011*, Shanghai, China, 2011, pp. 2921–2929.

[11] S. Kandula, S. Sengupta, A. Greenberg, P. Patel, and R. Chaiken, "The nature of data center traffic: Measurements & analysis," in *Proc. ACM 9th SIGCOMM Conf. Internet Meas.*, New York, NY, 2009, pp. 202–208.

[12] V. Ramaswami, K. Jain, R. Jana, and V. Aggarwal, "Modeling heavy tails in traffic sources for network performance evaluation," in *Computational Intelligence, Cyber Security and Computational Models*. Berlin, Germany: Springer, 2014, pp. 23–44.

[13] X. Zhou, Z. F. Zhao, R. P. Li, Y. F. Zhou, J. Palicot, and H. G. Zhang, "Understanding the nature of social mobile instant messaging in cellular networks," *IEEE Commun. Lett.*, vol. 18, no. 3, pp. 389–392, Mar. 2014.

[14] K. Park and W. Willinger, *Self-Similar Network Traffic and Performance Evaluation*. New York, NY, USA: Wiley-Interscience, 2000.

[15] J. Tan and N. B. Shroff, "Transition from heavy to light tails in retransmission durations," in *Proc. IEEE INFOCOM*, San Diego, CA, 2010, pp. 1–9.

[16] P. Wang and I. F. Akyildiz, "On the stability of dynamic spectrum access networks in the presence of heavy tails," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 870–881, Feb. 2015.

[17] S. Xia and P. Wang, "Distributed throughput optimal scheduling in the presence of heavy-tailed traffic," in *Proc. IEEE Int. Conf. Commun.*, London, UK, Jun. 2015, pp. 3490–3496.

[18] S. Xia, P. Wang, and Z. Sun, "Distributed timely-throughput optimal scheduling for wireless networks," in *Proc. IEEE Global Commun. Conf.*, Austin, TX, Dec. 2014, pp. 4820–4826.

[19] P. Wang and I. F. Akyildiz, "Improving network connectivity in the presence of heavy-tailed interference," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5427–5439, Oct. 2014.
[20] M. J. Neely, "Stability and capacity regions or discrete time queueing networks," unpublished paper, 2010. [Online]. Available: http://arxiv.org/abs/1003.3396v1
[21] K. Sigman and R. W. Wolff, "A review of regenerative processes," *SIAM Rev.*, vol. 35, no. 2, pp. 269–288, 1993.
[22] P. Wang and I. F. Akyildiz, "Asymptotic queueing analysis for dynamic spectrum access networks in the presence of heavy tails," *IEEE J. Sel. Areas Commun.*, vol. 3, no. 3, pp. 514–522, Mar. 2013.
[23] D. J. Daley and C. M. Goldie, "The moment index of minima (ii)," *Statist. Probability Lett.*, vol. 76, no. 8, pp. 831–837, 2006.
[24] M. G. Markakis, E. H. Modiano, and J. N. Tsitsiklis, "Max-weight scheduling in queueing networks with heavy-tailed traffic," *IEEE/ACM Trans. Netw.*, vol. 22, no. 1, pp. 257–270, Feb. 2014.
[25] S. C. Lin, P. Wang, I. F. Akyildiz, and M. Luo, "Delay-based maximum power-weight scheduling in queueing networks with heavy-tailed traffic," *IEEE/ACM Trans. Netw.*, vol. 25, no. 4, pp. 2540–2555, Aug. 2017.
[26] M. G. Markakis, E. H. Modiano, and J. N. Tsitsiklis, "Delay stability of back-pressure policies in the presence of heavy-tailed traffic," *IEEE/ACM Trans. Netw.*, vol. 24, no. 4, pp. 2046–2059, Aug. 2016.
[27] S.-C. Lin, P. Wang, I. F. Akyildiz, and M. Luo, "Throughput-optimal LIFO policy for bounded delay in the presence of heavy-tailed traffic," in *Proc. IEEE GLOBECOM*, Washington, DC, 2016, pp. 1–7.
[28] T. Rolski, H. Schmidli, V. Schmidt, and J. L. Teugels, *Stochastic Processes for Insurance and Finance*, vol. 505. Hoboken, NJ, USA: Wiley, 2009.
[29] Y. Chen, X. Wang, and L. Cai, "On achieving fair and throughput-optimal scheduling for TCP flows in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 7996–8008, Dec. 2016.
[30] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," *Wireless Netw.*, vol. 11, no. 4, pp. 471–487, 2005.
[31] Y. Shi, T. Hou, J. Liu, and S. Kompella, "How to correctly use the protocol interference model for multi-hop wireless networks," in *Proc. 10th ACM Int. Symp. Mobile ad hoc Netw. Comput.*, 2009, pp. 239–248.
[32] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge Univ. Press, 2004.
[33] L. Bottou, "Stochastic gradient descent tricks," in *Neural Networks: Tricks of the Trade*. Berlin, Germany: Springer, 2012, pp. 421–436.
[34] S. Boyd and A. Mutapcic, "Subgradient methods," Stanford Univ., Stanford, CA, USA, Lecture Notes EE364b, 2006.
[35] V. S. Borkar, *Stochastic Approximation: A Dynamical Systems Viewpoint*. Gurgaon, India: Hindustan Book Agency, 2008.
[36] A. Wierman and B. Zwart, "Is tail-optimal scheduling possible?" *Oper. Res.*, vol. 60, no. 5, pp. 1249–1257, 2012.
[37] G. Tychogiorgos, A. Gkelias, and K. Leung, "Utility-proportional fairness in wireless networks," in *IEEE 23rd Int. Symp. Pers. Indoor Mobile Radio Commun.*, Sydney, NSW, Australia, 2012, pp. 839–844.
[38] S. Keshav, *Mathematical Foundations of Computer Networking*. Reading, MA, USA: Addison-Wesley, 2012.
[39] J. Nair, K. Jagannathan, and A. Wierman, "When heavy-tailed and light-tailed flows compete: The response time tail under generalized max-weight scheduling," *IEEE/ACM Trans. Netw.*, vol. 24, no. 2, pp. 982–995, Apr. 2016.
[40] S. Xia, P. Wang, and H. M. Kwon, "Stochastic network utility maximization in the presence of heavy-tailed traffic," *IEEE Trans. Commun.*, Oct. 2018. [Online]. Available: https://drive.google.com/open?id=1cxCZT1PXUg_G5vECO06_Z6YafaaKI1uj
[41] K. Eriksson, D. Estep, and C. Johnson, *Applied Mathematics: Body and Soul: Volume 1: Derivatives and Geometry in IR3*. Berlin, Germany: Springer, 2013.
[42] H. Ye, J. Gao, and Y. Ding, "A generalized Gronwall inequality and its application to a fractional differential equation," *J. Math. Anal. Appl.*, vol. 328, no. 2, pp. 1075–1081, 2007.
[43] A. Joulin, "On maximal inequalities for stable stochastic integrals," *Potential Anal.*, vol. 26, no. 1, pp. 57–78, 2007.
[44] V. Anantharam and V. S. Borkar, "Stochastic approximation with long range dependent and heavy tailed noise," *Queueing Syst.*, vol. 71, no. 1/2, pp. 221–242, 2012.
[45] B. T. Polyak and A. B. Juditsky, "Acceleration of stochastic approximation by averaging," *SIAM J. Control Optim.*, vol. 30, no. 4, pp. 838–855, 1992.
[46] B. S. Thomson, J. B. Bruckner, and A. M. Bruckner. *Elementary Real Anal.* ClassicalRealAnalysis.com, 2008.

Shuang Xia (S'15) received the B.E. degree in automation from the Wuhan University of Science and Technology, Wuhan, China, in 2010, and the M.S. and Ph.D. degrees in electrical engineering from Wichita State University, Wichita, KS, USA, in 2014 and 2018, respectively.

His research interests include wireless sensor networks, cognitive radio networks, software defined networks, Interment of multimedia things, and machine learning.

Pu Wang received the B.E. degree in electrical engineering from the Beijing Institute of Technology, Beijing, China, in 2003, and the M.E. degree in electrical and computer engineering from the Memorial University of Newfoundland, St. John's, NL, Canada, in 2008, and the Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology, under the guidance of Prof. Ian F. Akyildiz in August 2013. Before joining the UNC Charlotte, he was an Assistant Professor with the Department of Electrical Engineering and Computer Science, Wichita State University, Wichita, KS, USA, since 2013. He received the John A. See Innovation Award from Wichita State University in 2016 and the BWN Lab Researcher of the Year Award from Georgia Tech in 2012. He received the Best Student Paper runner-up award at IEEE SCC 2016 and the TPC Top Ranked Paper award at IEEE DySPAN 2011. He was also named the Fellow of the School of Graduate Studies, Memorial University of Newfoundland in 2008. His current research focuses on modeling, analysis, and stochastic optimization of networked systems, with applications in software defined networking, cyber-physical systems, Internet of Things, wireless sensor networks, cognitive radio networks, and electromagnetic nanonetworks.

Hyuck M. Kwon (S'82–M'84–LSM'96) was born in South Korea on May 9, 1953. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, South Korea, in 1978 and 1980, respectively, and the Ph.D. degree in computer, information, and control engineering from the University of Michigan, Ann Arbor, MI, USA, in 1984. From 1985 to 1989, he was an Assistant Professor with the Department of Electrical Engineering and Computer Science, University of Wisconsin, Milwaukee, WI, USA. From 1989 to 1993, he was a Principal Engineer with Lockheed Engineering and Sciences Company, Houston, TX, USA, working on the National Aeronautics and Space Administration space shuttle and space station satellite communication systems. Since 1993, he has been with the Department of Electrical Engineering and Computer Science, Wichita State University, Wichita, KS, USA, where he is currently a Full Professor. In addition, he has held several visiting and consulting positions in communication system industries, a Visiting Associate Professor position with Texas A&M University, College Station, TX, USA, in 1997; a Visiting Professor position with the Korea Advanced Institute of Science and Technology, Daejeon, South Korea, and George Mason University, Fairfax, Virginia, in 2005, 2012, and 2017; and an Air Force Summer Faculty Fellow with the Air Force Research Laboratory, Kirtland Air Force Base, Albuquerque, NM, USA, in the summer of 2014, 2015, 2016, and 2018. His current research interests include wireless, massive multiple-input multiple-output, millimeter-wave, orthogonal frequency-division multiple-access, cooperative, code-division multiple-access, and frequency-hopping spread-spectrum satellite communication systems.