Delay-Based Maximum Power-Weight Scheduling With Heavy-Tailed Traffic

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***Abstract*— Heavy-tailed (HT) traffic (e.g., the Internet and multimedia traffic) fundamentally challenges the validity of classic scheduling algorithms, designed under conventional light-tailed (LT) assumptions. To address such a challenge, this paper investigates the impact of HT traffic on delay-based maximum weight scheduling (DMWS) algorithms, which have been proven to be throughput-optimal with enhanced delay performance under the LT traffic assumption. First, it is proven that the DMWS policy is not throughput-optimal anymore in the presence of hybrid LT and HT traffic by inducing unbounded queuing delay for LT traffic. Then, to solve the unbounded delay problem, a delay-based maximum power-weight scheduling (DMPWS) policy is proposed that makes scheduling decisions based on queuing delay raised to a certain power. It is shown by the fluid model analysis that DMPWS is throughput-optimal with respect to moment stability by admitting the largest set of traffic rates supportable by the network, while guaranteeing bounded queuing delay for LT traffic. Moreover, a variant of the DMPWS algorithm, namely the IU-DMPWS policy, is proposed, which operates with infrequent queue state updates. It is also shown that compared with DMPWS, the IU-DMPWS policy preserves the throughput optimality with much less signaling overhead, thus expediting its practical implementation.**

***Index Terms*— Heavy tails, delay-based maximum weight (MaxWeight) policy, power-weight scheduling, fluid-limit approximations, throughput-optimal, switched networks.**

I. INTRODUCTION

LINK scheduling is a critical and even most challenging resource allocation functionality in general queueing networks, such as wireless downlinks and uplinks, input-queued switches, wireless sensor networks, ad-hoc networks, and cloud computing facilities among many others. In all these systems, not all queues can be served simultaneously, due to the constraints from wireless interference or switch matching. To fully utilize the limited network resources, throughput-optimal scheduling policies [1], often referred to as maximum weight (MaxWeight) policies, have been extensively exploited. The throughput-optimal policy can stabilize the network by guaranteeing bounded queueing delay under any feasible loads, without requiring any explicit statistical information of the arriving traffic flows and serving rates. Throughput-optimal scheduling was first introduced in the seminal work [1], which proposes queue length-based MaxWeight scheduling (QMWS), where the flow with the largest queue length is served first. Since then, numerous work has been focused on the variations or extensions of this policy in different settings. For example, the recent work in [2] investigates the response time performance (i.e., queueing delay) under generalized queue length-based MaxWeight policies. However, while these queue length-based policies have been shown to achieve excellent throughput performance, they suffer a substantial queueing delay because the long waiting time of building up large queue lengths [3] is required for a flow to be served eventually. Moreover, the queue length-based MaxWeight scheduling policies even lose the throughput optimality under flow dynamics, where certain flows only have a finite number of packets to transmit and thus cannot bring a sufficiently large queue length to establish desired queueing dynamics [4].

To address those challenges, delay-based MaxWeight scheduling (DMWS) policies [5]–[8] have been investigated recently, which utilize the head-of-line (HoL) packet waiting time as the weight instead of queue lengths. More specifically, DMWS gives a higher serving priority to the flows with a large weight as before, but the weight of a flow now increases with its HoL time until the flow gets served, even if there is not sufficient built-up queue lengths. This intuitively eases or even solve the substantial delay problem. By applying the similar concept, the DMWS policy is proven to be throughput-optimal with flow dynamics, where the flows with finite number of packets come or go as time proceeds [7].

Despite its superior throughput and delay performance, all existing DMWS policies are developed under the assumption of the conventional light-tailed (LT) traffic (i.e., Markovian or Poisson traffic), and the throughput performance of the DMWS policy in the presence of heavy-tailed (HT) traffic has not yet been fully understood. HT traffic has been widely identified in a variety of data-oriented communication and computer networks, such as WiFi networks [9], the Internet [10], [11], mobile ad-hoc networks [12], cellular networks [13], [14], and data center networks [15], [16]. Heavy-tailed traffic can be either caused by the inherent heavy-tailed distribution in the traffic source such as the file size on the Internet servers [16], [17], the message size of cellular

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base stations [13], [14], and the frame length of variable bit rate (VBR) video streams [18], or caused by the network protocols themselves such as retransmissions and random access schemes [19]. In particular, HT traffic exhibits high burstiness and strong correlations as well as statistical similarity over different timescales. The highly bursty nature can induce significant performance degradation, such as unbounded network latency [20], [21], greatly degraded network stability [22], [23], and considerably reduced connectivity [24].

In this paper, we aim to analyze the impact of hybrid HT and LT traffic on the throughput optimality of DMWS policies. Previous researches [22], [23], [25], [26] have shown the destructive impact of HT traffic on the system stability under queue length-based policies (e.g., QMWS). However, such analysis cannot be applied for the delay-based scheduling policies (e.g., DMWS), since there exists no direct relation between the instantaneous queue length and queueing delay. To counter such challenge, fluid model-based stability analysis can be adopted [27], [28], which establishes the deterministic fluid model for the original stochastic queueing model and exploit the Little's law in fluid domain to connect the fluid limit of the HoL packet delay with that of the queue length. By such a way, DMWS can be considered to be equivalent to the queue length-based counterpart (e.g., QMWS). Then, it is easy to prove that DMWS is throughput optimal with respect to steady-state stability, i.e., the convergence of queue length process in distribution for every traffic vector within the stability region. However, although under the conventional LT assumption, the steady-state stability generally implies bounded average queue lengths, under HT environments, such implication may not hold at all [25]. Hence, it is of significant importance to study the potential destructive impact of HT traffic on the boundness of queue lengths under DMWS and develop effective solution to mitigate such impact.

In this paper, we first show the network instability of the DMWS policy with hybrid HT and LT traffic. Specifically, by applying sample-path analysis (e.g., [27], [28] in the literature for queue length-based scheduling), we derive the sufficient condition under which DMWS leads to unbounded average queueing delay for LT traffic. Next, we propose the delay-based maximum power-weight scheduling (DMPWS). In particular, by jointly exploiting fluid model-based stability and moment analysis, we prove that DMPWS is throughput-optimal with respect to moment stability by admitting the largest set of traffic rates that is supportable by the network, while guaranteeing bounded queueing delay for LT traffic. Intuitively, this feature is achieved by giving higher priorities (i.e., larger power-weight) to LT traffic flows, which provide them sufficient serving opportunities when competing with HT traffic flows. Such a feature is of great importance in the sense that it prevents bursty HT traffic from significantly degrading the queueing performance of LT traffic (e.g., email deliveries, audio/voice traffic, and scalar sensing readings).

Note that the design insights of DMPWS are inspired by the previous work on QMPWS [23], [25], [26]. However, the stability and throughput optimality analysis of DMPWS are different and much more involved. We adopt fluid-limit approximations that transform the original stochastic systems into a deterministic system. While the work of queue length-based policies in [29] also adopts fluid-limit approximations to prove the unbounded delay with HT traffic, due to the merits of queue length-based designs, it simply proves the stability with stochastic queueing systems by using the conventional approaches of Foster-Lyapunnov criterion and moment bound. On the other hand, aiming at delay-based designs, we establish linear relation between queue lengths and queueing delay with deterministic systems in the fluid domain, and prove the steady-state stability of DMPWS by using fluid-domain Lyapunov drift technique. Finally, we exploit the fluid model-based moment analysis to prove the throughout optimality of DMPWS by investigating the bounds on mean return time of queue-length process.

To further enhance the practicability of DMPWS, we propose a variant DMPWS policy, called infrequent updating-DMPWS (IU-DMPWS), which only needs the infrequent queue-state (i.e., HoL packet delay) measurements. While such infrequent updates of queue information have been exploited for conventional QMWS [30], [31], the impact on the proposed DMPWS is unknown and quite involved to analyze. To this end, we prove that the IU-DMPWS policy still preserves the throughput optimality as its original DMPWS scheme, but is more favored in system implementation, such as uplink scheduling in cellular networks, due to less signaling overhead. More specifically, there is a clear tradeoff with IU-DMPWS that infrequent updates and lower overhead (or computational cost) come at the expense of larger delay, as verified by simulation results.

To the best of our knowledge, this work is the first rigorous analysis for the throughput performance of delay-based scheduling policies with HT traffic. We summarize our main contributions as follows:

- We show that the existing DMWS policy fails to achieve throughput optimality in the presence of HT traffic.
- We prove that the proposed DMPWS policy can achieve the throughput optimality with respect to moment stability, under hybrid HT and LT traffic.
- We further demonstrate that the proposed IU-DMPWS policy preserves the good merits from its original scheme with less signaling cost.

The rest of the paper is organized as follows. Section II introduces the system model and preliminaries. Section III analyzes the network instability of the DMWS policy with hybrid HT and LT traffic. To solve the instability problem, Section IV proposes the DMPWS policy and its variant IU-DMPWS. Section V provides performance evaluation and Section VI concludes the paper.

II. System Model and Preliminaries

A. Preliminaries

Definition 1 (HT Distribution): A random variable X is heavy-tailed (HT) if for all $\theta > 0$

$$\limsup_{x\to\infty} e^{\theta x}\Pr(X > x) = \infty, \tag{1}$$

or equivalently, $E[e^{zX}] = \infty$, $\forall z > 0$. On the other hand, a random variable is light-tailed (LT) if it is not HT, or equivalently, if there exists $z > 0$ so that $E[e^{zX}] < \infty$.

Informally speaking, a HT random variable has tail distribution decreasing slower than exponentially (e.g., Pareto and log-normal); a LT random variable has tail distribution decreasing exponentially or even faster (e.g., exponential and Gamma).

An important class of HT distributions is the regularly varying distribution.

Definition 2 (Regularly Varying Distribution): A random variable X is called regularly varying with tail index $\beta > 0$, denoted by $X \in \mathcal{RV}(\beta)$, if

$$\Pr(X > x) \sim x^{-\beta}\mathcal{L}(x), \tag{2}$$

where for any two real functions $a(t)$ and $b(t)$, $a(t) \sim b(t)$ denote $\lim_{t\to\infty} a(t)/b(t) = 1$ and $L(x)$ is a slowly varying function.

Regularly varying distributions are a generalization of Pareto/Zipf/power-law distributions. The tail index β indicates how heavy the tail distribution is, where smaller values of β imply heavier tail. Moreover, for a random variable $X \in \mathcal{RV}(\beta)$, tail index β defines the maximum order of bounded moments X can have. Specifically, if $0 < \beta < 1$, X has infinite mean and variance. If $1 < \beta < 2$, X has finite mean and infinite variance. Regular varying distributions can effectively and stochastically characterize lots of network attributes, such as the frame length of VBR traffic, the session duration of licensed users in WLANs, and files sizes on the Internet severs.

B. System Model

Consider a queueing network topology described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of nodes and $\mathcal{E}$ denotes the set of links. We assume that time is slotted with a unit slot length and that arrivals occur at the end of each time slot. Our model involves single-hop traffic flows (i.e., data arrives at the source node of an edge to be transmitted to the node at the other end of the edge, where it exits the network). Let F be the number of traffic flows in the network. A *traffic flow* $f \in \{1, \ldots, F\}$ consists of a discrete-time stochastic arrival process $\{A_f(t); t \in \mathbb{Z}_+\}$, a source node $s(f)$, and a destination node $d(f)$, where $s(f), d(f) \in \mathcal{V}$ and $(s(f), d(f)) \in \mathcal{E}$. Each arrival process takes values in the set of nonnegative integers, and is independent and identically distributed (i.i.d.) over time. Furthermore, the arrival processes associated with different traffic flows are mutually independent. Let $\lambda_f = E[A_f(0)] > 0$ denote the rate of traffic flow f and $\lambda = (\lambda_1, \ldots, \lambda_F)$ denote its vector. Each flow f is buffered in a dedicated queue at $s(f)$ and the service discipline within each queue is assumed to be first-come, first-served (FCFS). Moreover, we define the tail coefficient of an arrival flow $A_f(t)$ as

$$\kappa(A_h(t)) := \sup\{k \geq 0 : E[A_h^k(t)] \leq \infty\}, \tag{3}$$

which gives the maximum order of finite moments that arrival process $A_f(t)$ can have. In particular, if $A_f(t)$ is HT with tail index β, i.e., $A_f(t) \in \mathcal{RV}(\beta)$, then $\kappa(A_f(t)) = \beta$.

Let the stochastic process $\{Q_f(t); t \in \mathbb{Z}_+\}$ and $\{W_f(t); t \in \mathbb{Z}_+\}$ denote the number of packets and the HoL packet waiting time, respectively, in queue f at the beginning of time slot t. Moreover, not all traffic flows can be served simultaneously due to the interference in wireless networks or matching constraints in a switch. A set of flows that can be served simultaneously is a *feasible* schedule π. Let S denote the set of all feasible schedules that is assumed to be an arbitrary subset of the powerset of $\{1, \ldots, F\}$. For simplicity, we assume the maximum transmission rate along any links is one packet per time slot. Let $\pi_f(t)$ denote the number of packets transmitted from queue f at time t, $E[\pi_f(t)] = \pi_f \leq 1$ the average service rate of queue f where $\pi = (\pi_1, \ldots, \pi_F) \in S$, and $S(t) = (\pi_1(t), \ldots, \pi_F(t))$ the time-varying scheduling vector. Let $Y(t) = (Y_1(t), \ldots, Y_F(t))$ denote the schedule idling process at time t, where $Y_f(t) = \max\{\pi_f(t) - Q_f(t), 0\}$. $Q(t) = (Q_1(t), \ldots, Q_F(t))$ captures the queue lengths at time slot t, and its initial state $Q(0)$ can be an arbitrary element of $\mathbb{Z}_+^F$. As a result, we adopt quadruple processes $\{Q(t), W(t), Y(t), S(t)\}$ and an initial condition $Q(0)$ to completely characterize a stochastic queueing system and its time evolution.

We introduce notations used in this paper as follows. $\mathbb{I}_M$ denotes the indicator function of event M and $\|X\|_1$ indicates L^1-norm of vector (or set) X.

Definition 3 (Steady-State Stability [6]): A queueing system described in Section II-B is considered. Let Q^x denote the queue length process with initial backlog $\|Q^x(0)\|_1 = x$. If there exists a scheduling policy under which the process $\{Q(t); t \in \mathbb{Z}_+\}$ converges in distribution, i.e., there exist an $\epsilon > 0$ and a finite integer $T > 0$ such that for any sequence of processes $\{\frac{1}{x}Q^x(xT), x = 1, 2, \ldots\}$, we have

$$\limsup_{x\to\infty} E[\frac{1}{x}\|Q^x(xT)\|_1] \leq 1 - \epsilon, \tag{4}$$

then the queue length process is ergodic and the queueing system is steady-state stable.

The stability of the queueing network depends on the link transmission rates and the scheduling constraints. This relation is captured by the network capacity region as follows.

Definition 4 (Network Capacity Region [1]): The network stability region Φ of the single-hop queueing system is depicted by the set of traffic rate vectors as

$$\Phi := \{\lambda \in \mathbb{R}_+^F | \lambda \leq \sigma \text{ componentwise}, \quad \text{for some } \sigma \in IntCo(S)\}$$

where $IntCo(S) = \{\sum_{\pi\in S}\gamma_\pi\pi_f | \sum_{\pi\in S}\gamma_\pi < 1, \gamma_\pi \geq 0, \forall\pi\}$ denotes the interior of a convex hull.

If a rate vector is in the capacity region (i.e., λ can be covered by a convex combination of feasible schedules), then the traffic flows with respect to this vector is called *admissible*, and there exists a scheduling policy so that the network is steady-state stable. In the following, we further define the moment stability, which is a new stability criterion to characterize the QoS performance under HT traffic [22].

Definition 5 (Moment Stability): Given a single-hop system described above under a specific scheduling policy. If all LT flows achieve bounded moments of first-order (i.e., finite average queue length and HoL packet delay as $E[Q_f] < \infty$ and $E[W_f] < \infty$, $\forall f \in LT$), and all HT flows achieve their bounded queue-length moments of maximum-order (i.e., $E[Q_f^{\kappa(A_f(t))-1}] < \infty, \forall f \in HT$), then the queueing system is moment stable.

It has been established in [22] that under any scheduling algorithms, HT traffic flows necessarily experience unbounded average HoL packet delay (i.e., $E[W_f] = \infty, \forall f \in HT$), if the HT traffic arrivals have the tail index less than two (i.e., $A_f(t) \in \mathcal{RV}(\beta)$ with $\beta < 2$). Therefore, the network stability under hybrid LT and HT traffic flows is simply defined with respect to the delay boundness of LT traffic flows.

Definition 6 (Throughput Optimality): A scheduling algorithm is throughput-optimal under hybrid HT and LT traffic, if it can achieve moment stability for any admissible rate vectors (i.e., any rates within the network capacity region).

III. Network Instability of Delay-Based MaxWeight Scheduling (DMWS)

In this section, we first provide the stochastic model and fluid model (FM) for the DMWS policy, and then study the network instability of DMWS under hybrid HT and LT traffic, which proves that the classical DMWS policy is not throughput-optimal anymore in the presence of heavy tails.

A. Delay-Based MaxWeight Scheduling (DMWS) Policy

The queueing model (i.e., queueing network equations) of flow queue f is described by

$$Q_f(t+1) = Q_f(t) + A_f(t) - \pi_f(t) + Y_f(t), \quad \forall t \in \mathbb{Z}_+. \tag{5}$$

Under DMWS, the *weight* of a feasible schedule is the sum of the HoL time $W_f(t)$ of all queues of such schedule. Moreover, the MaxWeight policy activates a feasible schedule with the maximum weight at any given time slot. Specifically, under the DMWS policy, the scheduling vector $S(t)$ follows

$$S(t) = \operatorname*{argmax}_{\pi \in S}\{\sum_{f \in F} W_f^{\alpha_f}(t)\pi_f(t)\} \tag{6}$$

with $\alpha_f = 1$ for all $f \in F$. If there are multiple feasible schedules, $S(t)$ will choose one of them uniformly at random.

The fluid model (FM), i.e., Eqs. (7a)-(7f), is the deterministic equivalence of the original stochastic queueing model, i.e., Eqs. (5)-(6), by substituting mean arrival rates λ_f and mean service rates $\pi_f, \forall f \in F$ for the corresponding stochastic processes $A(t)$ and $S(t)$, respectively, at every regular time $t \geq 0$ (i.e., the time where derivative exists). That is, we have

$$q_f(t) \;=\; q_f(0) + \lambda_f t - \sum_{\pi \in S} s_\pi(t)\pi_f + y_f(t), \quad \forall f \in F; \tag{7a}$$

$$\sum_{\pi \in S} s_\pi(t)\pi_f \geq y_f(t), \quad \forall f \in F; \tag{7b}$$

$$\text{each } s_\pi(\cdot) \text{ and } y_f(\cdot) \text{ is non-decreasing;} \tag{7c}$$

$$\sum_{\pi \in S} s_\pi(t) = t; \tag{7d}$$

$$w_f(t) > 0 \Rightarrow \frac{dy_f(t)}{dt} = 0, \quad \forall f \in F; \tag{7e}$$

$$\sum_{f \in F} w_f^{\alpha_f}(t)\pi_f < \max_{\sigma \in S} \sum_{f \in F} w_f^{\alpha_f}(t)\sigma_f \Rightarrow \frac{ds_\pi(t)}{dt} = 0, \quad \forall \pi \in S \; \forall f \in F. \tag{7f}$$

where $s_\pi(t)$ denotes the total amount of time until t that schedule π has been activated, and $y_f(t)$ denotes the cumulative idling service for flow f until time t.

Eq. (7a) characterizes the queueing dynamic of flows; Eq. (7b) states that for each flow, the wasted service should be no more than the activated schedule π provides; Eq. (7c) gives that for each schedule, the total amount of activated time should be increasing. Eqs. (7d)-(7e) indicates the *working-conserving* discipline. Specifically, Eq. (7d) states that the service is fully utilized in every time; Eq. (7e) gives that no wasted service exists when HoL packet delay(s) is positive. Eq. (7f) is the fluid model equation for delay-based MaxWeight policy that yields a conditional decision with several deterministic variables. Specifically, Eq. (7f) implies that schedules without the maximum weight receive no service.

B. Fluid Model and Queueing Model Association

In order to use the stability of the FM to show the stability of the corresponding stochastic system later in Section IV-B, we employ fluid-limit association to specify the relation between the queueing model and FM.

Definition 7 (Fluid & Queueing Model Association [32]): A fluid model is associated with a queueing network, if

- each sequence b_j of positive integers with $b_j \to \infty$ possesses a subsequence b_{j_n} as initial backlog, i.e., $\|Q^{b_{j_n}}(0)\|_1 = b_{j_n}$, on which the components of scaled sequences $X^{b_{j_n}}(b_{j_n}t)/b_{j_n}$ from queueing network process $X = (Q(\cdot), W(\cdot), Y(\cdot), S(\cdot))$ converge uniformly over compact (u.o.c.);
- this limit, i.e., $\lim_{n\to\infty} X^{b_{j_n}}(b_{j_n}t)/b_{j_n}$, satisfies the fluid model.

In the sense of Definition 7, we respectively define the *fluid-scaled process* and *fluid limit* in terms of initial backlog b_{j_n} as $\bar{x}^{b_{j_n}}(t) := X^{b_{j_n}}(b_{j_n}t)/b_{j_n}$ and $\bar{x}(t) := \lim_{n\to\infty} X^{b_{j_n}}(b_{j_n}t)/b_{j_n}$ where $t \geq 0$. Then, using the techniques of [27, Th. 4.1], we can show that, for almost all sample paths and for all positive sequence $b_j \to \infty$, there exists a subsequence b_{j_n} with $n \to \infty$ such that, the following convergence holds uniformly over compact interval:

$$\frac{1}{b_{j_n}} Q_f^{b_{j_n}}(b_{j_n}t) \to \bar{q}_f(t); \tag{8}$$

$$\frac{1}{b_{j_n}} \int_0^{b_{j_n}t} \pi^{b_{j_n}}(\tau)d\tau \to \bar{s}_\pi(t); \tag{9}$$

$$\frac{1}{b_{j_n}} \int_0^{b_{j_n}t} Y_f^{b_{j_n}}(\tau)d\tau \to \bar{y}_f(t). \tag{10}$$

Similarly, the following fluid limits hold at every continuous point of the limit function (which are denoted by "$\Rightarrow$"):

$$\frac{1}{b_{j_n}} W_f^{b_{j_n}}(b_{j_n}t) \Rightarrow \bar{w}_f(t); \tag{11}$$

$$\frac{1}{b_{j_n}} U_f^{b_{j_n}}(b_{j_n}t) \Rightarrow \bar{u}_f(t), \tag{12}$$

where $U_f(t) \triangleq t - W_f(t)$ denotes the time when the HoL packet of Q_f arrives, and $\bar{u}_f(t)$ denotes the corresponding fluid limit. With the aid of fluid-scaled processes and fluid limits, in the following Lemma 1, we examine the association details for the fluid model in Eqs. (7a)-(7f).

Lemma 1: The fluid model, i.e., Eqs. (7a)-(7f), is associated with the queueing network model, i.e., Eqs. (5)-(6).

Proof: According to Definition 7, we first (i) show that for the queueing network model, i.e., Eqs. (5)-(6), with delay-based scheduling and HT traffic, every positive sequence b_j possesses a subsequence b_{j_n} as initial backlog, on which the fluid limit exists and is u.o.c. Next, we (ii) prove that those fluid limits, i.e., Eqs. (8)-(12), satisfy the FM, i.e., Eqs. (7a)-(7f).

(i): First of all, consider the initial condition of FM. It is assumed the existence of a vector $q \in \mathbb{R}_+^F$ and of a sequence of positive numbers $\{\epsilon_{b_{j_n}}; j \in \mathbb{N}\}$ with respect to initial backlog b_{j_n} that satisfy $\max_{f\in F} |\bar{q}_f^{b_{j_n}}(0) - q_f| \leq \epsilon_{b_{j_n}}$. This implies that the fluid limit (of queue backlog) converges to the initial condition at time zero, i.e., $\bar{q}_f(0) = q_f, \forall f \in F$.

Moreover, to ensure the existence of a positive probability of a certain sample path set with hybrid HT and LT flows, we define $G_{b_{j_n}}$ with initial backlog b_{j_n} as the set on which the strong law of large number (SLLN) holds for the arrivals and service time [33]. Specifically, for $\omega \in G_{b_{j_n}}$ and $f \in F$, we have

$$\lambda_f = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} A_f(\tau), \quad \pi_f = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \pi_f(\tau) \quad (13)$$

where A_f and π_f follow Eqs. (5)-(6). Also, there exists a sequence of deviation terms $\delta_{b_{j_n}} \in \mathbb{R}_+$ so that as $b_{j_n} \to \infty$, it gives $\delta_{b_{j_n}} \to 0$ and

$$\Pr(\sup_{t\leq b_{j_n}} \frac{1}{b_{j_n}} \max_{f\in F} |\sum_{\tau=0}^{t-1} A_f(\tau) - \lambda_f t| < \delta_{b_{j_n}}) \to 1. \quad (14)$$

As a result, a fluid limit of the queueing network, with queueing network process $X(\cdot)$ defined in Definition 7, can be any limit

$$\bar{x}(t) = \lim_{n\to\infty} \frac{1}{b_{j_n}} X^{b_{j_n}}(b_{j_n} t), \quad (15)$$

for any choice of $\omega \in G_{b_{j_n}}$ and any sequence b_{j_n} satisfying the above setup. Also, the u.o.c. convergence for each of fluid limit components of $\bar{x}(t)$ is established by following [27, Th. 4.1], as mentioned.

(ii): Next, we prove that Eqs. (7a)-(7e) belong to *basic fluid model equations* with sample paths in $G_{b_{j_n}}$. It is indicated by [32] that every fluid limit satisfies the basic fluid model equations. The details in our case are given as follows. First, to show Eq. (7a) with $f \in F$, we use the limits involving $\sum_{\tau=0}^{t-1} A_f^{b_{j_n}}(\tau)$, $\sum_{\tau=0}^{t-1} \pi_f^{b_{j_n}}(\tau)$, and $\sum_{\tau=0}^{t-1} Y_f^{b_{j_n}}(\tau)$ in Eq. (13) and Eqs. (9)-(10) as $n \to \infty$. They imply that

$$\frac{1}{b_{j_n}} \sum_{\tau=0}^{t-1} A_f^{b_{j_n}}(b_{j_n}\tau) \to \lambda_f t,$$

$$\frac{1}{b_{j_n}} \sum_{\tau=0}^{t-1} \pi_f^{b_{j_n}}(b_{j_n}\tau) - Y_f^{b_{j_n}}(b_{j_n}\tau) \to \sum_{\pi\in S} \bar{s}_\pi(t)\pi_f - \bar{y}_f(t)$$

for all t, as $n \to \infty$. A further subsequence is chosen so that $\bar{q}_f(0)$ exists. Hence, the limit $\bar{q}_f(t)$ exists and Eq. (7a) holds. Since the summations of $A_f^{b_{j_n}}(\cdot)$, $\pi_f^{b_{j_n}}(\cdot)$, and $Y_f^{b_{j_n}}(\cdot)$ are monotone with u.o.c. convergence, convergence to $\bar{q}_f(\cdot)$ is also u.o.c.

Next, Eqs. (7b)-(7d) as the defining relation for the idling service $Y_f(t)$ and their u.o.c. convergence follow quickly from Eqs. (9)-(10) and the previous limits, i.e., $\sum_{\tau=0}^{t-1} \pi_f^{b_{j_n}}(\tau)$ and $\sum_{\tau=0}^{t-1} Y_f^{b_{j_n}}(\tau)$. In particular, summations in these limits imply non-decreasing properties in Eq. (7c).

Third, to show Eq. (7e) with $f \in F$, suppose that for all $t \in [t_1, t_2]$, $w_f(t) > 0$. Since $w_f(\cdot)$ is continuous, it is bounded away from zero on the interval. Convergence to $\bar{w}_f(\cdot)$ is u.o.c., and so for sufficiently large n, $W_f^{b_{j_n}}(b_{j_n} t) > 0$ on $[t_1, t_2]$ as well. Due to the working-conserving discipline of investigated delay-based scheduling in Eq. (6), we must have $\sum_{\tau=0}^{t_1-1} Y_f^{b_{j_n}}(\tau) = \sum_{\tau=0}^{t_2-1} Y_f^{b_{j_n}}(\tau)$. Since the same equality also holds in the limit as $n \to \infty$, this implies Eq. (7e), as desired.

Based on the above accomplishments, to show that the FM is associated with the queueing network model with delay-based scheduling and hybrid traffic, we still need to verify that Eq. (7f) is satisfied by all fluid limits. Specifically, pick a regular time t, and suppose that $\sum_{f\in F} w_f^{\alpha_f}(t)\pi_f < \max_{\sigma\in S} \sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f$. Then, pick some small interval $I = [t, t+\delta]$ and n sufficiently large such that

$$\sum_{f\in F} [\bar{w}_f^{b_{j_n}}(\tau)]^{\alpha_f} \pi_f < \max_{\sigma\in S} \sum_{f\in F} [\bar{w}_f^{b_{j_n}}(\tau)]^{\alpha_f} \sigma_f, \quad \forall \tau \in I.$$

Rewriting this in terms of the unscaled queueing system, we have the following as $\sum_{f\in F} [W_f^{b_{j_n}}(b_{j_n}\tau)]^{\alpha_f}\pi_f < \max_{\sigma\in S} \sum_{f\in F} [W_f^{b_{j_n}}(b_{j_n}\tau)]^{\alpha_f}\sigma_f$ for all $\tau \in I$. By considering a general delay-based policy in Eq. (6), π will not be chosen throughout this entire interval. This implies that after scaling we have $\bar{s}_\pi^{b_{j_n}}(t+\delta/2) - \bar{s}_\pi^{b_{j_n}}(t) = 0$, where $\delta/2$ is selected to sidestep any discretization problems, and taking the limit gives $\bar{s}_\pi(t+\delta/2) = \bar{s}_\pi(t)$. Since $\bar{s}_\pi$ is assumed to be differentiable at t; the derivative must be zero. To this end, we have proved that Eqs. (7a)-(7f) associate with the corresponding queueing model. ∎

Based on the studied FM, i.e., Eqs. (7a)-(7f), and the established association in Lemma 1, in the following Lemma 2, we introduce an extended FM equation that characterizes a linear relation of fluid model solutions between queue length $q_f(t)$ and HoL packet delay $w_f(t)$.

Lemma 2: For any fixed time $\tau_F > 0$, the condition $\sum_{\pi\in S} s_\pi(\tau_F)\pi_f - y_f(\tau_F) > q_f(0), \forall f \in F$ is equivalent to the condition $u_f(\tau_F) > 0, \forall f \in F$, where $u_f(t)$ denotes the fluid model solution of stochastic process $U_f(t)$. Moreover, if such a condition holds, we have

$$q_f(t) = \lambda_f w_f(t), \quad \forall f \in F, \quad (16)$$

for all time $t \geq \tau_F$.

Proof: Regarding the first part, i.e., the two conditions are equivalent, we have that

$$U_f(t) = \inf\{\tau \le t | Q_f(0) + \sum_{m=0}^{\tau-1} A_f(m) > \sum_{m=0}^{t-1} \pi_f(m) - Y_f(m)\} \quad (17)$$

from the definition of $U_f(t)$. Combining this with the association of FM and queueing model, it is straightforward to yield the equivalence of two conditions.

Next, we focus on the second part of the proof, i.e., if $\sum_{\pi\in S} s_\pi(\tau_F)\pi_f - y_f(\tau_F) > q_f(0), \forall f \in F$, then Eq. (16) follows. Suppose that $\sum_{\pi\in S} s_\pi(\tau_f)\pi_f - y_f(\tau_f) > q_f(0)$ for a certain flow $f \in F$. Then, by the definition of $u_f(t)$, we have $\sum_{\pi\in S} s_\pi(t)\pi_f - y_f(t) = q_f(0) + \lambda_f u_f(t)$ for all $t \ge \tau_f$. This, combining with Eq. (7a), implies that $q_f(t) = [q_f(0)+\lambda_f t] - [q_f(0)+\lambda_f u_f(t)] = \lambda_f w_f(t), \forall t \ge \tau_f$. The existence of such $\tau_F > 0$ for all $f \in F$ can be further obtained through similar arguments in [6]. ■

Note that Lemma 2 relates queue lengths and HoL delays in the fluid domain and thus plays a crucial role in the analysis of delay-based policies. These novel results are widely used in the rest of the paper.

C. Network Instability Analysis of DMWS

In the following, we first show the sufficient conditions of having unbounded average delay for HT traffic through the argument with fictitious queue. Then, through fluid model, we prove the network instability the DMWS policy by showing the LT traffic may experiences unbounded average queueing delay as well.

Theorem 1 (Unbounded Delay of HT Traffic): Under the DMWS policy, HT traffic flow h has unbounded average delay (i.e., $E[W_h] = \infty, \forall h \in HT$), if the HT traffic flow $A_h(t)$ is with tail index smaller than two, i.e.,

$$\kappa(A_h(t)) < 2. \quad (18)$$

Proof: We construct a fictitious queue $\tilde{h}$ with queue length $Q_{\tilde{h}}(t)$ and HoL packet waiting time $W_{\tilde{h}}(t)$ at time slot t, which has the same packet arrivals and initial queue lengths as HT traffic $h \in HT$, but is served at unit rate whenever it is not empty. It is easy to verify that during the same time interval, more packets are served in fictitious queue $\tilde{h}$ than in queue h (i.e., $Q_h(t)$ dominates $Q_{\tilde{h}}(t)$ at all time slots). This implies

$$\Pr(Q_h(t) > q) \ge \Pr(Q_{\tilde{h}}(t) > q), \quad \forall q \in \mathbb{Z}_+,\ t \in \mathbb{Z}_+. \quad (19)$$

Furthermore, since the arriving traffic is assumed admissible in Section II-B and queue length processes converge in distribution, we have that $E[Q_h] \ge E[Q_{\tilde{h}}]$ by taking $t \to \infty$. So it is sufficient to show that $E[Q_h] = \infty$ as long as $E[Q_{\tilde{h}}]$ is infinite for the result of $E[Q_h] = \infty$. This follows immediately from the Pollaczeck-Khinchine (P-K) formula, as queue $\tilde{h}$ can be seen as a M/G/1 queue with variance $\sigma^2_{S_{\tilde{h}}}$ of service time. If the HT arrival $A_h(t)$ has a tail index smaller than two (i.e., $A_h(t) \in \mathcal{RV}(\beta), \beta < 2$), then the variance $\sigma^2_{S_{\tilde{h}}}$ is infinite. Moreover, we have

$$E[Q_{\tilde{h}}] = \rho_{\tilde{h}} + \frac{\lambda_h^2 \sigma^2_{S_{\tilde{h}}} + \rho^2_{\tilde{h}}}{2(1-\rho_{\tilde{h}})} \quad (20)$$

where $\rho_{\tilde{h}} = \lambda_h E[S_{\tilde{h}}]$. It implies that $E[Q_h]$ is infinite. Similar results can be obtained for the HoL packet delay. In particular, $E[W_h] \ge E[W_{\tilde{h}}]$ and $E[W_{\tilde{h}}] = \infty$ from P-K formula, which also implies that $E[W_h]$ is infinite. ■

While Theorem 1 shows HT traffic can experience unbounded average delay under DMWS policy, Theorem 2 below shows that DMWS policy can lead to unbounded queueing delay for LT traffic; thus it is not throughput optimal with respect to moment stability.

Theorem 2 (Network Instability): Consider the DMWS policy under single-hop hybrid HT and LT traffic, where LT flows conflict with HT flows.[1] Then, LT traffic flow l has unbounded delay (i.e., $E[W_l] = \infty, \forall l \in LT$), if a HT traffic flow $A_h(t)$ is with tail index smaller than two, i.e.,

$$\min_{f\in F} \kappa(A_f(t)) < 2. \quad (21)$$

Proof: Intuitively, when LT traffic competes the service with HT traffic using DMWS policy, LT traffic will not be served until the bulky arrivals of HT traffic are served because those arrivals trend to have longer waiting time in the queue. In this case, the queueing delay of the LT traffic will be at the same order as the HT traffic, which has unbounded queueing delay as shown in Theorem 1. In the following, the formal proof is based on the sample-path analysis.

Assume the considered queueing system is in the steady state, where the queue length process is a positive recurrent regenerative process. Define T as the time interval between two consecutive instances when all queues are empty. Then, we have $E[T] < \infty$, due to the positive recurrent system. Consider the set $\mathcal{E}_{d_{j_n}}$ of sample paths, where at time slot zero, queue h, having the HT arrival with the smallest tail index, i.e., $\kappa(A_h(t)) = \min_{f\in F} \kappa(A_f(t))$, receives a file of size $d_{j_n} > 0$ packets, and all other queues receive no traffic. Let $\{Q^{d_{j_n}}(\cdot); j \in \mathbb{N}\}$ be the sequence of stochastic queue backlog processes. It implies that a sequence of single-hop queueing systems, indexed by $j \in \mathbb{N}$, with initial queue lengths $Q_h^{d_{j_n}}(0) = d_{j_n}$ and $Q_l^{d_{j_n}}(0) = 0$, $\forall l \neq h$ is examined. Moreover, as the arrival processes of different traffic flows are mutually independent, the probability of having the set $\mathcal{E}_{d_{j_n}}$ is as follows

$$\Pr(\mathcal{E}_{d_{j_n}}) = \Pr(A_h(0) > d_{j_n}) \prod_{l\neq h} \Pr(A_l(0) = 0) \ge \Pr(A_h(0) > d_{j_n}) \prod_{l\neq h} (1 - \Pr(A_l(0) > 0)). \quad (22)$$

$\Pr(\mathcal{E}_{d_{j_n}})$ is positive because d_{j_n} is in the support of $A_h(0)$, and $\Pr(A_l(0) > 0) < 1$.

Furthermore, we define a new set of sample paths $\hat{\mathcal{E}}_{d_{j_n}} = G_{b_{j_n}} \bigcap \mathcal{E}_{d_{j_n}}$, where $G_{b_{j_n}}$ defined with Eqs. (13)-(14) is a set

[1]Two flows are conflicted if there is no schedule that includes both flows [23].

sequence of sample paths of arrival processes that satisfy the SLLN. Due to the i.i.d. nature of arriving traffic, it implies that

$$\Pr(\hat{\mathcal{E}}_{d_{j_n}}) = \Pr(G_{b_{j_n}})\Pr(\mathcal{E}_{d_{j_n}}) > 0. \tag{23}$$

Consider a sample path in $\hat{\mathcal{E}}_{d_{j_n}}$ and that other flows ($\forall l \in F$, $l \neq h$) conflict with HT flow h. Let $T_{d_{j_n}}$ be the first time slot when the HoL packet delay of queue h becomes less than or equal to the HoL delay of one of the queues $l \in F$ and $l \neq h$, i.e.,

$$T_{d_{j_n}} := \min\{t > 0 | W_l(t) \geq W_h(t), l \in F, l \neq h\}. \tag{24}$$

Under the DMWS policy, queue l receives no service until time $T_{d_{j_n}}$, and queue h is served at an unit rate while all initial traffic of queue h is served at time slot $T_{d_{j_n}}$. The reason is that all packets from initial traffic d_{j_n} of queue h have one time slot more delay than any other HoL packet in any LT queue. Hence, queue h keeps sending until all d_{j_n} packets are sent out. This implies that $\forall l \neq h$

$$Q_l(T_{d_{j_n}}) = \sum_{t=0}^{T_{d_{j_n}}-1} A_l(t); \quad T_{d_{j_n}} \geq d_{j_n}. \tag{25}$$

For sample paths in $G_{b_{j_n}}$ from the SLLN with Eqs. (13)-(14), we have $\sum_{t=0}^{T_{d_{j_n}}-1} A_l(t) \geq \lambda_l T_{d_{j_n}} - \delta_{b_{j_n}}$, with probability 1. Thus, there exist positive constants $0 < c < 1$ and d_0 such that for every sample path in $\hat{\mathcal{E}}_{d_{j_n}}$, $Q_l(T_{d_{j_n}}) \geq c d_{j_n}\lambda_l$, $\forall d_{j_n} \geq d_0$, $l \neq h$. Following the property of regenerative process in steady-state with cycle length T, we have

$$\begin{aligned}\Pr(Q_l > \frac{c d_{j_n}\lambda_l}{2}) &= \lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{I}_{\{Q_l(\tau) > \frac{c d_{j_n}\lambda_l}{2}\}} \\ &= \frac{E[\sum_{t=0}^{T}\mathbb{I}_{\{Q_l(t) > \frac{c d_{j_n}\lambda_l}{2}\}}]}{E[T]}.\end{aligned} \tag{26}$$

Moreover, by Eq. (23), it implies that

$$\begin{aligned}E[\sum_{t=0}^{T}\mathbb{I}_{\{Q_l(t) > \frac{c d_{j_n}\lambda_l}{2}\}}] &\geq E[\mathbb{I}_{\hat{\mathcal{E}}_{d_{j_n}}}\sum_{t=0}^{T}\mathbb{I}_{\{Q_l(t) > \frac{c d_{j_n}\lambda_l}{2}\}}] \\ &\geq \Pr(G_{b_{j_n}})\Pr(A_h(0) > d_{j_n})\prod_{l\neq h}(1-\lambda_l) \\ &\quad \times \sum_{t=\frac{c d_{j_n}}{2}}^{c d_{j_n}}\Pr(Q_l(t) > \frac{c d_{j_n}\lambda_l}{2} | \hat{\mathcal{E}}_{d_{j_n}}).\end{aligned} \tag{27}$$

By the queueing dynamic in Eq. (5) and $Q_l(0) = 0$, we have $Q_l(t) = \sum_{\tau=0}^{t-1}[A_l(\tau) - \pi_l(\tau)\mathbb{I}_{\{Q_l(\tau)>0\}}]$. This implies that

$$\begin{aligned}&\lim_{d_{j_n}\to\infty}\frac{1}{d_{j_n}}\sum_{t=\frac{c d_{j_n}}{2}}^{c d_{j_n}}\Pr(Q_l(t) > \frac{c d_{j_n}\lambda_l}{2} | \hat{\mathcal{E}}_{d_{j_n}}) \\ &\quad \geq \lim_{d_{j_n}\to\infty}\frac{1}{d_{j_n}}\sum_{t=\frac{c d_{j_n}}{2}}^{c d_{j_n}}\Pr(\sum_{\tau=0}^{t-1}A_l(\tau) > \frac{c d_{j_n}\lambda_l}{2}) = \frac{c}{2}.\end{aligned} \tag{28}$$

The last equality in Eq. (28) holds due to the fact that HT traffic flow h occupies the entire service of a DMWS schedule during the time interval $0 \leq t \leq c d_{j_n}/2$. Combining Eq. (26) with Eqs. (27)-(28), it follows from the assumption $\kappa(A_h(t)) = \min_{f\in F}\kappa(A_f(t)) < 2$ that

$$\lim_{d_{j_n}\to\infty}\frac{\log[\Pr(Q_l > \frac{c d_{j_n}\lambda_l}{2})]}{\log[\frac{c d_{j_n}\lambda_l}{2}]} \geq -\min_{f\in F}\kappa(A_f(t)) + 1 \geq -1$$

which, by applying moment theorem [34], implies $E[Q_l]$ is infinite. By Little's law, we thus have $E[W_l] = \infty$. ∎

IV. Delay-Based Maximum Power-Weight Scheduling (DMPWS)

In this section, we study the throughput optimality of the proposed DMPWS policy for hybrid HT and LT traffic. Then, we propose a DMPWS variant, namely IU-DMPWS, which only requires infrequent HoL delay updates and show that this IU-DMPWS policy still guarantees throughput optimality under hybrid traffic, facilitating the practical implementation due to less signaling overhead.

A. Delay-Based Maximum Power-Weight Scheduling (DMPWS) Policy

Intuitively speaking, the key problem of DMWS is that both LT and HT traffic flows are assigned with the same priority or weight (i.e., $\alpha_f = 1$ for all $f \in F$ in Eq. (6)). In this case, the HT traffic flow may receive more services because it has the higher probability to have long queues and high HoL delay, which leads to long waiting time in the queue for the LT traffic. To solve this problem, DMPWS assigns different weights to different flows. More specifically, as the name suggests, the *power-weight* of a feasible schedule is the sum of the HoL packet delay $W_f(t)$ up to α_f order, and the DMPWS policy activates a schedule that has maximum power-weight at any given time slot. Specifically, under the DMPWS policy, the scheduling vector $S(t)$ satisfies Eq. (6), i.e.,

$$S(t) = \operatorname*{argmax}_{\pi\in S}\{\sum_{f\in F}W_f^{\alpha_f}(t)\pi_f(t)\}$$

and the power-weight assigned to each flow should be set to be proportional to the maximum order of the finite moments of the arrival processes $A(t)$ to ensure the network stability. In particular, we set the power-weight of flow $f \in F$ as follows:

$$\alpha_f = \begin{cases}\kappa(A_f(t)) - 1, & \forall f \in HT; \\ c_f > 2, & \forall f \in LT,\end{cases} \tag{29}$$

where $\kappa(\cdot)$ is the tail coefficient defined in Eq. (3) and c_f is an arbitrary constant larger than two. Such power-weight assignment is inspired by the queue length-based maximum power-weight scheduling (QMPWS) algorithm [25], [26], which can achieve throughput-optimality with respect to moment stability under hybrid HT and LT traffic. Because of the linear relation between queue backlog and queueing delay in fluid domain, it is expected that the proposed DMPWS policy with the designated power-weight in Eq. (29) is also able to provide the desired throughput optimality under HT environment. However, the analysis tools for QMPWS cannot be applied in DMPWS because DMPWS has to be analyzed in the fluid domain. The detailed proofs are shown in the following Section IV-B and Section IV-C.

B. Throughput Optimality of DMPWS

To prove the throughput optimality of DMPWS, it is equivalent to show that DMPWS can achieve moment stability, i.e., all the LT traffic flows have bounded average queueing delay, as long as the incoming traffic flows are within the network capacity region. Towards this, we first show that DMPWS can achieve steady-state stability by proving that the corresponding fluid model, i.e., Eqs. (7a)-(7f) and Eq. (29), is stable. With such a condition satisfied, we then show that DMPWS can achieve moment stability by exploiting fluid model-based moment analysis under the condition that incoming traffic flows are within the network capacity region.

In the following, we first prove the steady-state stability in Theorem 4 by the Lyapunov technique in fluid domain and by exploiting Theorem 3.

Theorem 3 [32]: The queueing network is steady-state stable (i.e., positive Harris recurrent) whenever an associated fluid model is stable (i.e., there exist time $T > 0$ such that $q_f(t) = 0$ for all $f \in F$ and $t \geq T$).

Theorem 4 (Steady-State Stability): If the incoming traffic rates reside in the network capacity region, the corresponding queueing system is steady-state stable with the DMPWS policy under hybrid HT and LT traffic.

Proof: The idea is to show that a Lyapunov function in fluid domain of the system has a negative drift, which implies that the FM is stable and so is the original queueing system. We define the Lyapunov function as follows:

$$L(q(t)) = \sum_{f\in F} L_f(q_f(t)) = \sum_{f\in F} \frac{q_f(t)^{\alpha_f+1}}{(1+\alpha_f)\lambda_f^{\alpha_f}}, \qquad (30)$$

where $q(t)$ is assumed Lipschitz continuous. It is sufficient to show that for any $\chi_2 > 0$, there exists $\chi_3 > 0$ so that at any regular time $t \geq \tau_F$, $L(q(t)) \geq \chi_2$ implies $\frac{d^+}{dt^+}L(q(t)) \leq -\chi_3$. Choose $\chi_4 > 0$ such that $L(q(t)) \geq \chi_2$ implies $\max_{f\in F} w_f^{\alpha_f}(t) \geq \chi_4$. Since $q(t)$ is differentiable for any regular time $t \geq \tau_F$, the derivative of $L(q(t))$ is given as

$$\begin{aligned}
\frac{d^+}{dt^+}L(q(t)) &= \sum_{f\in F} \frac{q_f(t)^{\alpha_f}}{\lambda_f^{\alpha_f}}(\lambda_f - \sum_{\pi\in S}\frac{ds_\pi(t)}{dt}\pi_f + \frac{dy_f(t)}{dt}) \\
&= \sum_{f\in F}\lambda_f w_f^{\alpha_f}(t) - \sum_{f\in F} w_f^{\alpha_f}(t)\sum_{\pi\in S}\frac{ds_\pi(t)}{dt}\pi_f \qquad (31)\\
&= \sum_{f\in F}\lambda_f w_f^{\alpha_f}(t) - \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sum_{\sigma\in S}\frac{ds_{\sigma_f}(t)}{dt}\sigma_f \qquad (32)\\
&= \sum_{f\in F} w_f^{\alpha_f}(t)\lambda_f - \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f \qquad (33)\\
&\leq \sum_{f\in F} w_f^{\alpha_f}(t)\sum_{\pi\in S}\gamma_\pi\pi_f - \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f \qquad (34)\\
&\leq \sum_{\pi\in S}\gamma_\pi \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f - \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f \qquad (35)\\
&\leq (\sum_{\pi\in S}\gamma_\pi - 1)\max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(t)\sigma_f \qquad (36)\\
&\leq -\chi_4(1 - \sum_{\pi\in S}\gamma_\pi) := -\chi_3 < 0 \qquad (37)
\end{aligned}$$

where $\frac{d^+}{dt^+}L(q(t)) = \lim_{\delta\downarrow 0}\frac{L(q(t+\delta)) - L(q(t))}{\delta}$, and the first equality is from queueing dynamic in Eq. (7a). Eq. (31) is from Eq. (7e) and the FM Little's law in Eq. (16). Eq. (32) and Eq. (33) result from the DMPWS policy defined by Eq. (7f) and Eq. (29); Eq. (34) follows from the assumption that arrival rate vector λ is strictly inside stability region Φ given in Definition 4. Note that Eq. (36) gives the drift inequality as $\frac{d^+}{dt^+}L(q(t)) \leq -(1 - \sum_{\pi\in S}\gamma_\pi)\max_{\sigma\in S}\sum_{f\in F} q_f^{\alpha_f}(t)\frac{\sigma_f}{\lambda_f^{\alpha_f}}$, which implies that the fluid model solution $q(t)$ will drain in finite time without inducing an exponential decay. Also, Eq. (37) indicates that $L(q(t)) \geq \chi_2$ implies $\frac{d^+}{dt^+}L(q(t)) \leq -\chi_3$ for all regular time $t \geq \tau_F$. Hence, it immediately follows that for any $\chi_1 > 0$, there exists finite $T_1 \geq \tau_F > 0$ such that $\|q(t)\|_1 \leq \chi_1$ for all $t \geq T_1$. Moreover, given the traffic rate vector λ, we set a sufficiently small number $\epsilon > 0$ such that $(1+\epsilon)\lambda$ is also strictly inside the region Φ, and choose $\chi_1 < \epsilon\min_{f\in F}\lambda_f$. As a result, for all $t \geq T_1$, we have $\sum_{\pi\in S}\frac{ds_\pi(t)}{dt}\pi_f \geq (1+\epsilon)\lambda_f - \chi_1$ by Definition 4 and $\frac{d^+}{dt^+}q_f(t) = \lambda_f - \sum_{\pi\in S}\frac{ds_\pi(t)}{dt}\pi_f \leq -\epsilon\lambda_f + \chi_1 < 0$ if $q_f(t) > 0$ from Eq. (7a) and Eq. (7e). This implies that there exists a finite $T^* > T_1 \geq \tau_F > 0$ such that $\|q(t)\|_1 = 0$ for $t \geq T^*$. The discussed FM is thus stable, which by Theorem 3, implies that its associated queueing network model is also stable. ∎

Remark: Theorem 4 proves that DMPWS can achieve steady-state stability, which only means that the queue length process converges in distribution. However, under HT environments, this does not imply that the converged distribution of queue lengths has finite mean. For example, by letting all queues have the same weight (e.g., $\alpha_f = 1, \forall f \in F$), DMPWS becomes conventional DMWS and Theorem 4 still holds. This means DMWS can achieve steady-state stability as long as the incoming traffic rates are within the network capacity region. However, even if such steady-state stability holds, as proved in Theorem 2, DMWS can have unbounded average queueing delay for all LT traffic flows and thus is not throughput-optimal (with respect to moment stability). In the following Theorem 5, we show that the DMPWS policy is throughput-optimal, which ensures that the queue length and queueing delay of all LT traffic flows are of finite mean as long as traffic rates are within the network capacity region.

Theorem 5 (Throughput Optimality): Consider the DMPWS policy under hybrid HT and LT traffic. The corresponding queueing system is throughput-optimal with respect to moment stability by ensuring that all LT traffic flows have bounded average queue length and queueing delay (i.e., $E[Q_f] < \infty$ and $E[W_f] < \infty, \forall A_f(t) \in LT$) and all HT traffic flows have bounded queue-length moments of maximum-order (i.e., $E[Q_f^{\kappa(A_f(t))-1}] < \infty, \forall A_f(t) \in HT$).

Proof: We extend and employ the fluid model-based moment analysis in [35] to study the impact of HT traffic on the moment boundness of LT traffic as follows. First of all, the stable fluid model from DMPWS in Theorem 4 implies that there exists $T^* > \tau_F$ such that $\lim_{b_{j_n}\to\infty} \frac{1}{b_{j_n}} \|Q^{b_{j_n}}(b_{j_n}T^*)\|_1 = 0$, with probability 1. In addition, since $\frac{1}{b_{j_n}} Q_f^{b_{j_n}}(b_{j_n}T^*) \leq 1 + \frac{1}{b_{j_n}} \sum_{\tau=0}^{b_{j_n}T^*} A_f(\tau)$ and $E[A_f^{\alpha_f+1}(t)] < \infty$ by Eq. (29) and Eq. (3), it follows from [36] that the collection of random variables $\{\frac{1}{b_{j_n}^{\alpha_f+1}} Q_f^{b_{j_n}}(b_{j_n}T^*)^{\alpha_f+1} : b_{j_n} \geq 1\}$ is uniformly integrable. Hence, we have the following: $\forall f \in F$

$$\lim_{b_{j_n}\to\infty} \frac{1}{b_{j_n}^{\alpha_f+1}} E[Q_f(b_{j_n}T^*)^{\alpha_f+1}] = \lim_{b_{j_n}\to\infty} \frac{1}{b_{j_n}^{\alpha_f+1}} \int_{Q_f(b_{j_n}T^*)} y^{\alpha_f+1} \Pr(dy) = 0, \quad (38)$$

where dy is within a Borel field on Q_f.

Next, we show that the mean return time to a compact set has a strong bound with respect to each flow queue. Specifically, regarding flow $f \in F$, given some $\delta > 0$ and a subset M_f of the state space Q_f, we define the return time as $\tau_{M_f}(\delta) := \inf\{t \geq \delta : Q_f(t) \in M_f\}$. By following Eq. (38), there exists a compact set of the form $M_f := \{b_{j_n} : b_{j_n} \leq B\}$ such that

$$E[Q_f(b_{j_n}T^*)^{\alpha_f+1}] \leq \frac{1}{2} b_{j_n}^{\alpha_f+1}, \quad b_{j_n} \in M_f^c, \quad (39)$$

where M_f^c is the complement of set M_f. Letting $t(b_{j_n}) = T^* \max(B, b_{j_n})$, we rewrite Eq. (39) as

$$\int_{Q_f(t(b_{j_n}))} y^{\alpha_f+1} \Pr(dy) = \int_{M_f^c} \cdots + \int_{M_f} \cdots \leq \frac{1}{2} b_{j_n}^{\alpha_f+1} + c\mathbb{I}_{M_f},$$

where c is a finite constant. Moreover, since $Q_f^{b_{j_n}}(t) \leq Q_f^{b_{j_n}}(0) + \sum_{\tau=0}^{t} A_f(\tau)$ and $E[A_f^{\alpha_f}(t)] < \infty$, by [36], there exists a constant c_1 so that

$$E[(\sum_{\tau=0}^{t} A_f(\tau))^{\alpha_f}] \leq c_1(t^{\alpha_f} + 1), \quad t \geq 0.$$

Hence for constants $c_0, c_2, c_3 \leq \infty$, using the strong Markov property, we have

$$E[\int_0^{t(b_{j_n})} Q_f(t)^{\alpha_f} dt] \leq c_2 t(b_{j_n})(Q_f(0)^{\alpha_f} + t(b_{j_n})^{\alpha_f}) \leq c_0(b_{j_n}^{\alpha_f+1} + 1),$$

which shows that $E[\int_0^{t(b_{j_n})}(1+Q_f(t)^{\alpha_f})dt] \leq c_3(b_{j_n}^{\alpha_f+1}+1)$. Since $t(b_{j_n}) \geq \tau_{M_f}(T^*B)$, by Fubini's theorem, we yield the following: for some constants $c_{\alpha_f+1} < \infty$, $\delta > 0$, and a compact set $M_f \subset Q_f$,

$$E[\int_0^{\tau_{M_f}(\delta)} (1 + Q_f(t)^{\alpha_f})dt] \leq c_{\alpha_f+1}(b_{j_n}^{\alpha_f+1} + 1). \quad (40)$$

By applying the strong Markov property and [35, Proposition 5.4], we have that for constant $\chi < \infty$ and $t > 0$,

$$\begin{aligned} &E[\int_t^{t+\tau_{M_f}(\delta)} (1 + Q_f(s)^{\alpha_f})ds] \\ &= E[\int_0^{\tau_{M_f}(\delta)} \ldots] - E[\int_0^{t} \ldots] + E[\int_{\tau_{M_f}(\delta)}^{t+\tau_{M_f}(\delta)} \ldots] \\ &\leq E[\int_0^{\tau_{M_f}(\delta)} (1+Q_f(t)^{\alpha_f})dt] - \int_0^{t} E[1 + Q_f(s)^{\alpha_f}]ds + \chi t. \end{aligned} \quad (41)$$

Then, by combining Eq. (40) and Eq. (41), it implies that there exists a constant $\chi_{\alpha_f} < \infty$ for flow $f \in F$ and $b_{j_n} \in Q_f$ such that

$$\frac{1}{t} \int_0^t E[Q_f(s)^{\alpha_f}]ds \leq \chi_{\alpha_f}(\frac{1}{t} b_{j_n}^{\alpha_f+1} + 1), \quad t > 0. \quad (42)$$

That is, for each initial condition, we have

$$\limsup_{t\to\infty} \frac{1}{t} \int_0^t E[Q_f(s)^{\alpha_f}]ds \leq \chi_{\alpha_f}, \quad f \in F.$$

With the fact that a finite union of compact sets in $\mathbb{R}^1$ is compact, we obtain the following:

$$\limsup_{t\to\infty} \frac{1}{t} \int_0^t E[\sum_{f\in F} Q_f(s)^{\alpha_f}]ds < \infty. \quad (43)$$

We further claim that $E[\sum_{f\in F} Q_f^{\alpha_f}] < \infty$ from the contradiction arguments as follows. Suppose that $E[\sum_{f\in F} Q_f^{\alpha_f}] = \infty$. It implies that for every constant $M > 0$, there exists a time $T > 0$ so that $\int_{T\times\Omega} \frac{1}{T|\Omega|} \sum_{f\in F} Q_f(t,\omega)^{\alpha_f} > M$. Hence, $\frac{1}{T}\int_0^T E[\sum_{f\in F} Q_f(t)^{\alpha_f}] > M$. Since it is true for any $M > 0$, it implies that $\limsup_{t\to\infty} \frac{1}{t}\int_0^t E[\sum_{f\in F} Q_f(s)^{\alpha_f}]ds = \infty$, which brings the contradiction. This indicates that for LT flow l, we have $E[Q_l^2] < \infty$ and $E[Q_l] < \infty$, which, by Little's law, yields $E[W_l] < \infty$. For HT flow h, we have $E[Q_h^{\alpha_h}] = E[Q_h^{\kappa(A_h(t))-1}] < \infty$ and end the proof. ■

C. Infrequent Queue State Measurements

Both DMWS and DMPWS policies depend on queue state information (particularly, the HoL packet waiting time) of all flow queues. However, it is impractical to acquire such information at every time slot due to high signaling overhead. To address such challenges, we propose a variant DMPWS policy, IU-DMPWS, which only requires infrequent queue information updates and prove that the IU-DMPWS policy is throughput-optimal under hybrid HT and LT traffic. It is worth to note that such infrequent updates of queue information have been exploited for conventional QMWS [30], [31]. However, the impact on the proposed DMPWS is still unknown and is quite involved to analyze.

Let the time slots be grouped into *interval* of time T^I. It implies that the $(k+1)$th interval consists of slots $kT^I, \ldots, (k+1)T^I - 1$. Although queue states can change in each slot, these are measured only at the beginning of each interval (i.e., at the beginning of slot kT^I, for $k = 0, 1, \ldots$). Therefore, the interval length T^I denotes the duration between successive sampling instances of the HoL packet delay, and the IU-DMPWS policy follows the DMPWS and only updates the schedule at each time interval T^I. Specifically, under the

IU-DMPWS policy, the scheduling vector $S(t)$ belongs to the set:

$$S(kT^I + l) = \underset{\pi\in S}{\operatorname{argmax}}\{\sum_{f\in F} W_f^{\alpha_f}(kT^I)\pi_f(t)\}$$

where $k = 0, 1, \dots$ and $l = 0, 1, \dots, T^I - 1$. If the set on the right-hand side includes multiple schedules, one of them is chosen uniformly at random. Similarly, the FM equation of the IU-DMPWS policy is yield as

$$\sum_{f\in F} w_f^{\alpha_f}(kT^I)\pi_f < \max_{\sigma\in S}\sum_{f\in F} w_f^{\alpha_f}(kT^I)\sigma_f \Rightarrow \frac{ds_\pi(kT^I+n)}{dt} = 0, \quad \forall \pi\in S,\ 0\le n\le T^I-1. \tag{44}$$

It implies that schedules without the maximum power-weight at the beginning of a interval receive no service during that interval. Moreover, the queue dynamic equation is also rewritten as

$$Q((k+1)T^I) = Q(kT^I) + A(kT^I) - T^I[S(kT^I) - Y(kT^I)],$$

where its counterpart FM is provided as follows:

$$q_f((k+1)T^I) - q_f(kT^I) = \lambda_f T^I - \sum_{l=0}^{T^I-1}(\sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f - \frac{dy_f(kT^I)}{dt}), \quad \forall f\in F. \tag{45}$$

Therefore, given the IU-DMPWS policy in fluid domain, we prove the throughput optimality of IU-DMPWS under the hybrid traffic in the following Theorem 6. It is easy to see that the DMPWS policy is a special case of the IU-DMPWS policy, by considering the case $T^I = 1$.

Theorem 6: The IU-DMPWS policy is throughput-optimal.

Proof: To show the throughout optimality of IU-DMPWS, it is sufficient to show that the corresponding FM still provides a negative Lyapunov drift, and then follows the same analysis steps in Section IV-B to obtain the desired results. However, the difficulty here is that instead of dealing with the continuous (regular) time system as in the DMPWS case, we build a discrete-time system for the IU-DMPWS policy and need to utilize the difference equation for characterizing the negative drift. In particular, using the same Lyapunov function in Eq. (30), we consider the difference of Lyapunov drift upon the successive time intervals in fluid domain by exploiting the first-order Taylor expansion of Lyapunov function [22], [23] as follows.

Case (i) $1 \le \alpha_f$*:* Consider the first-order Taylor expansion around $q_f(kT^I)$,

$$\begin{aligned}
&L_f(q_f((k+1)T^I)) \\
&\quad= \frac{1}{(1+\alpha_f)\lambda_f^{\alpha_f}}[q_f(kT^I) + \lambda_f T^I \\
&\qquad - T^I(\sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f - \frac{dy_f(kT^I)}{dt})]^{\alpha_f+1} \\
&\quad= \frac{q_f^{\alpha_f+1}(kT^I)}{(1+\alpha_f)\lambda_f^{\alpha_f}} + \Delta_f(kT^I)w_f^{\alpha_f}(kT^I) + \frac{\Delta_f^2(kT^I)}{2\lambda_f^{\alpha_f}}\alpha_f \\
&\qquad\times \gamma^{\alpha_f-1}(kT^I)
\end{aligned}$$

where $\Delta_f(kT^I) = \lambda_f T^I - T^I(\sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f - \frac{dy_f(kT^I)}{dt})$ and $\gamma(kT^I) \in [q_f(kT^I) - T^I(\sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f - \frac{dy_f(kT^I)}{dt}), q_f(kT^I) + \lambda_f T^I]$. Therefore, by the fact that $\Delta_f^2(kT^I) \le (\lambda_f T^I)^2 + (T^I)^2$ and $(q_f(kT^I) + \lambda_f T^I)^{\alpha_f-1} < 2^{\alpha_f-1}(q_f^{\alpha_f-1}(kT^I) + (\lambda_f T^I)^{\alpha_f-1})$, we have the following:

$$\begin{aligned}
&L_f(q_f((k+1)T^I)) - L_f(q_f(kT^I)) \\
&\quad= \Delta_f(kT^I)w_f^{\alpha_f}(kT^I) + \frac{\Delta_f^2(kT^I)}{2\lambda_f^{\alpha_f}}\alpha_f\gamma^{\alpha_f-1}(kT^I) \\
&\quad\le \Delta_f(kT^I)w_f^{\alpha_f}(kT^I) + 2^{\alpha_f-2}\alpha_f\Delta_f^2(kT^I) \\
&\qquad\times[q_f^{\alpha_f-1}(kT^I) + (\lambda_f T^I)^{\alpha_f-1}]\lambda_f^{-\alpha_f}.
\end{aligned}$$

Then, the Lyapunov drift upon successive intervals becomes

$$\begin{aligned}
&\frac{1}{T^I}[L(q((k+1)T^I)) - L(q(kT^I))] \\
&\quad= \sum_{f\in F}\frac{\Delta_f(kT^I)w_f^{\alpha_f}(kT^I)}{T^I}[1 + 2^{\alpha_f-2}\alpha_f\Delta_f(kT^I) \\
&\qquad\times(\frac{1}{q_f(kT^I)} + \frac{(T^I)^{\alpha_f-1}}{\lambda_f w_f^{\alpha_f}(kT^I)})]
\end{aligned} \tag{46}$$

Case (ii) $\alpha_f \le 1$*:* Consider the zeroth-order Taylor expansion around $q_f(kT^I)$ (i.e., the mean value theorem),

$$\begin{aligned}
L_f(q_f((k+1)T^I)) &= \frac{1}{(1+\alpha_f)\lambda_f^{\alpha_f}}[q_f(kT^I) + \Delta_f(kT^I)]^{\alpha_f+1} \\
&= \frac{q_f^{\alpha_f+1}(kT^I)}{(1+\alpha_f)\lambda_f^{\alpha_f}} + \frac{\Delta_f(kT^I)\gamma^{\alpha_f}(kT^I)}{\lambda_f^{\alpha_f}}.
\end{aligned}$$

Similarly, the difference of Lyapunov functions is

$$\begin{aligned}
&L_f(q_f((k+1)T^I)) - L_f(q_f(kT^I)) = \frac{\Delta_f(kT^I)\gamma^{\alpha_f}(kT^I)}{\lambda_f^{\alpha_f}} \\
&\quad\le \Delta_f(kT^I)[w_f^{\alpha_f}(kT^I) + (T^I)^{\alpha_f}] \\
&\quad= T^I[\lambda_f - (\sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f - \frac{dy_f(kT^I)}{dt})] \\
&\qquad\times[w_f^{\alpha_f}(kT^I) + (T^I)^{\alpha_f}]
\end{aligned}$$

Hence, the Lyapunov drift for this case is obtained as

$$\begin{aligned}
&\frac{1}{T^I}[L(q((k+1)T^I)) - L(q(kT^I))] \\
&\quad= \sum_{f\in F}[(\lambda_f - \sum_{\pi\in S}\frac{ds_\pi(kT^I)}{dt}\pi_f + \frac{dy_f(kT^I)}{dt})w_f^{\alpha_f}(kT^I) \\
&\qquad\times(1 + \frac{(T^I)^{\alpha_f}}{w_f^{\alpha_f}(kT^I)})]
\end{aligned} \tag{47}$$

Summarizing our findings from **Case** (i) and **Case** (ii), Eq. (46) and Eq. (47) imply that

$$\frac{1}{T^I}[L(q((k+1)T^I)) - L(q(kT^I))] < 0,$$

following similar arguments in the proof of Theorem 4 with the IU-DMPWS policy in Eq. (44). Regarding this negative Lyapunov drift, it is easy to show that there exists a finite $k^*T^I > \tau_F$ such that $\|q(mT^I)\|_1 = 0$ for $m \ge k^*$. Therefore, the FM is stable, which by Theorem 3, implies that its

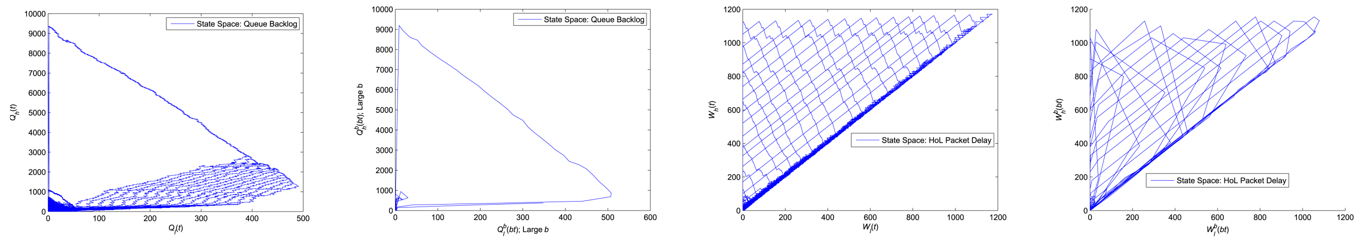


Fig. 1. Simulation of the models for hybrid HT and LT traffic with various initial starting points. (a) Stochastic model: queue lengths. (b) Fluid model: queue lengths. (c) Stochastic model: HoL delay. (d) Fluid model: HoL packet delay.

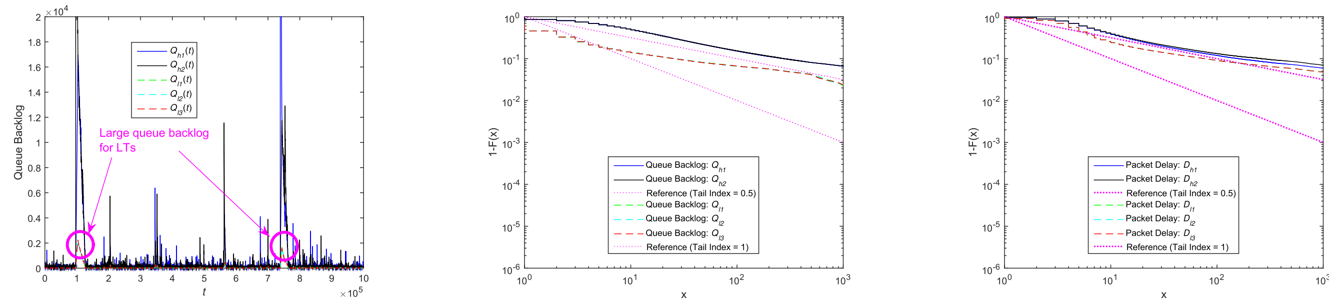


Fig. 2. Queue lengths and packet delay under the DMWS policy, where network is unstable. (a) Evolution of queue lengths. (b) Tail distribution of queue lengths. (c) Tail distribution of packet delay.

associated queueing network model is stable. The rest of the proof for throughput optimality follows exactly the same steps as for the DMPWS policy, and thus is omitted here. Note that for Eq. (43) in IU-DMPWS, it follows from more general results in [37] for discrete-time countable Markov chains. ■

V. Performance Evaluation

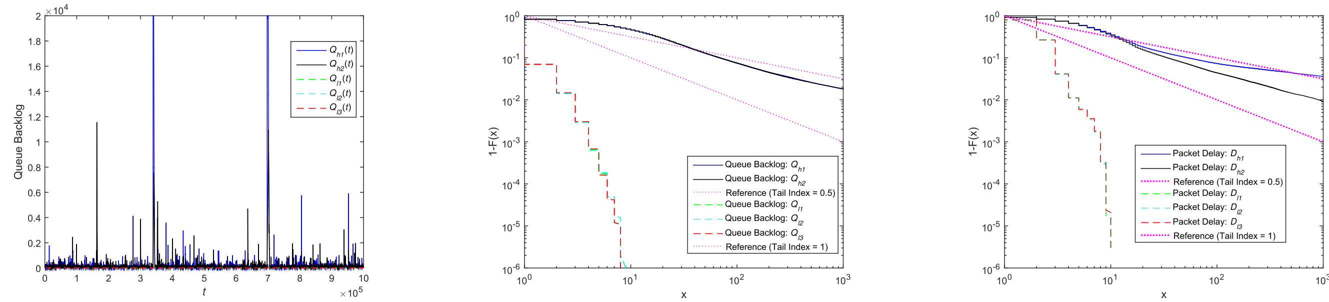
In this section, we provide simulation results to verify our theoretical analysis. We choose Pareto and log-normal distributions to represent HTs, and Poisson distribution to depict LTs. A random variable $X \in PAR(\alpha, x_m)$, if it follows Pareto distribution with parameters α and x_m (i.e., $P(X > x) = (x_m/x)^\alpha$). Also, a random variable $X \in \ln N(\mu, \sigma^2)$, if it follows log-normal distribution with parameters μ and σ (i.e., $P(X > x) = 1/2 - 1/2 \times \text{erf}[(\ln x - \mu)/\sqrt{2}\sigma]$). On the other hand, a random variable $X \in Poisson(\lambda)$, if it follows Poisson distribution with parameter λ (i.e., $P(X > x) = 1 - e^{-\lambda}\sum_{i=0}^{\lfloor x \rfloor} \lambda^i/i!$, where $\lfloor x \rfloor$ is the floor function). In the following, we first consider a scenario with two flows (i.e., $f \in \{h, l\}$) in the evaluation of fluid model in Section V-A, and then investigate a scenario with five flows (i.e., $f \in \{h1, h2, l1, l2, l3\}$) for the performance comparison under various scheduling policies in Section V-B and Section V-D.

A. Evaluation of Stochastic Model and Fluid Model

We first compare the behavior of a FM and a stochastic model under the DMWS policy. Consider a LT traffic with traffic arrival process $A_l(t) \in Poisson(3)$ and a HT traffic with $A_h(t) \in PAR(1.5, 1)$ sharing a single schedule under DMWS. The goal is to compare the behaviors of the original system and its fluid-limit approximation. Towards this, we simulate the stochastic system and the FM, starting from various initial values of queue lengths (i.e., $(Q_l(0), Q_h(0))$). Figure 1 shows evolution results of both queue lengths and HoL packet delay. Specifically, by comparing Figure 1a with Figure 1b (and Figure 1c with Figure 1d for HoL delay), except for fluctuations because of arrival variations, queue-length evolution of the FM successfully mimics that of the stochastic model independent of initial conditions. This suggests that queue behavior under various scheduling policies can be well approximated by the behavior of the corresponding FM.

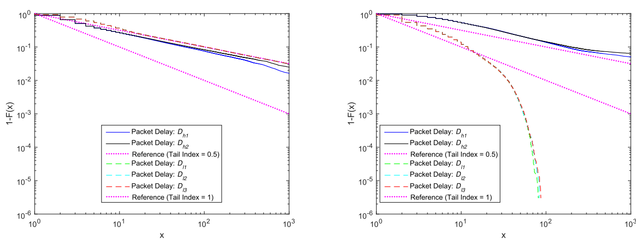
B. Throughput Optimality of DMPWS

We consider a scenario where two HT flows and three LT flows sharing a single schedule (i.e., $A_{h1}(t), A_{h2}(t) \in PAR(1.5, 1)$ and $A_{l1}(t), A_{l2}(t), A_{l3}(t) \in Poisson(3)$). All the following tail distribution results are plotted on log-log coordinates, by which HT distribution with tail index κ can manifest itself as a straight line with the slope equal to $-\kappa$. We first investigate the performance of hybrid HT and LT traffic under the DMWS policy in Figure 2. Figure 2a shows that large queue lengths are introduced for LT traffic due to the scheduling sharing with HT flows, particularly when HT flows have large queue lengths themselves. Moreover, as shown in Figure 2b and Figure 2c that under the DMWS policy, both queue lengths and packet delay of all LT flows follow heavy tailed distribution with a tail index smaller than one, since their tail distributions decay slower than the reference Pareto distribution with tail index one. This means the LT traffic flows also have unbounded queue lengths and delay.



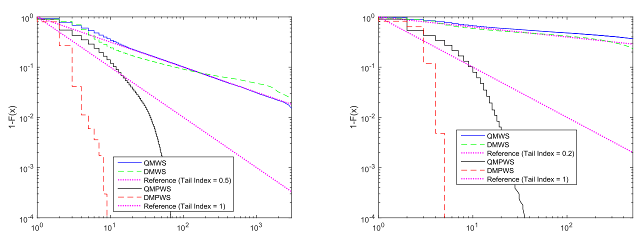
(a) (b) (c)

Fig. 3. Queue lengths and packet delay under the DMPWS policy, where network is stable. (a) Evolution of queue lengths. (b) Tail distribution of queue lengths. (c) Tail distribution of packet delay.



(a) (b)

Fig. 4. Packet delay under queue length-based scheduling. (a) Tail distribution of packet delay under QMWS. (b) Tail distribution of packet delay under QMPWS.



(a) (b)

Fig. 5. Packet delay of LT traffic under the QMWS, QMPWS, DMWS, and DMPWS policies. (a) $A_{h1}(t), A_{h2}(t) \in PAR(1.5, 1)$. (b) $A_{h1}(t), A_{h2}(t) \in PAR(1.2, 1)$.

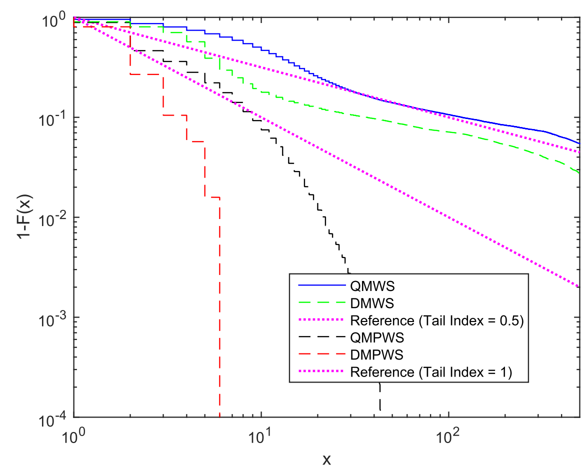
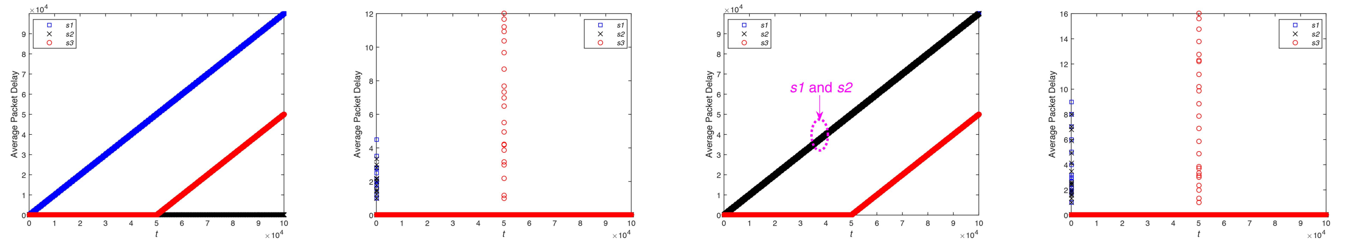


Fig. 6. Packet delay of LT traffic under the QMWS, QMPWS, DMWS, and DMPWS policies. A scenario that a LT flow conflict with four HT flows (i.e., $A_{l1}(t) \in Poisson(3)$ and $A_{h1}(t), A_{h2}(t), A_{h3}(t), A_{h4}(t) \in PAR(1.5, 1)$).

This is consistent with Theorem 2, which indicate under the DMWS policy with hybrid traffic, the LT flows necessarily has infinite average HOL delay (thus infinite average packet delay) and it results in the unstable network.

We next show that the throughput optimality and bounded delay can be achieved with hybrid traffic by applying the DMPWS policy with Eq. (29). More specifically, we assign the queues of HT flows and LT flows with weight 0.5 and 2, respectively. As indicated by Theorem 4 and Theorem 5, under such settings, the DMPWS policy can guarantee that LT traffic flows have bounded average queue lengths and delay, which cannot be achieved by applying the DMWS policy. In particular, Figure 3a shows that there is no large queue length for LT flows during the evolution. Figure 3b and Figure 3c further indicate that for both the tail distributions of queue lengths and delay of LT flows have a slope or decaying rate greater than one, which implies that the queue lengths and packet delay of LT flows will have finite mean and thus bring the stable queueing network.

To further illustrate the merits of delay-based designs, we extend the discussion by comparing packet delay of DMWS and DMPWS policies with their queue length-based counterparts. More specifically, QMWS policy adopts the queue length as flow weight, and QMPWS policy based on queue lengths utilizes the same power-weight as DMPWS (i.e., HT and LT flows with weight 0.5 and 2, respectively, in our discussed scenario). It is shown in Figure 4a that under the QMWS policy, both LT and HT flows have infinite average delay due to their heavy tailed distributions with a tail index smaller than one. In particular, these distributions exhibit themselves as a straight line parallel to that of the reference Pareto distribution with tail index 0.5. On the other hand, in Figure 4b that under QMPWS, the bounded delay for LT flows can be guaranteed as similar to the cases of DMPWS policy. Moreover, we provide the comprehensive delay comparison among these policies in Figures 5-6. As mentioned previously, while QMWS and DMWS make LT flows experience heavy-tailed queueing delay, in Figure 5, both QMPWS and DMPWS can support bounded delay for those LT flows. The DMPWS policy can further reduce much more delay as



(a) (b) (c) (d)

Fig. 7. Average packet delay of three short-lived flows A_{s1}, A_{s2}, A_{s3} that conflict with two HT flows A_{h1}, A_{h2} under the QMPWS and DMPWS policies, respectively. (a) QMPWS with $A_{h1}(t), A_{h2}(t) \in PAR(1.5, 1)$. (b) DMPWS with $A_{h1}(t), A_{h2}(t) \in PAR(1.5, 1)$. (c) QMPWS with $A_{h1}(t), A_{h2}(t) \in \ln N(3, 1)$. (d) DMPWS with $A_{h1}(t), A_{h2}(t) \in \ln N(3, 1)$.

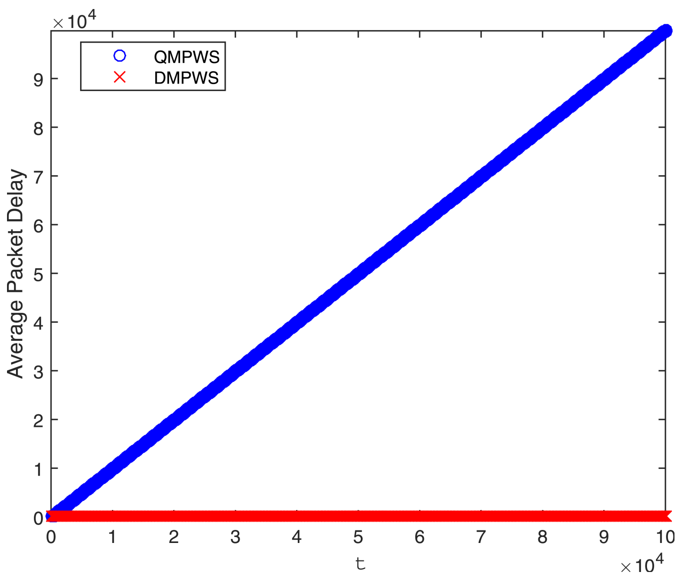


Fig. 8. Average packet delay of a short-lived flow A_{s1} that conflicts with four HT flows $A_{h1}, A_{h2}, A_{h3}, A_{h4}$ under the QMPWS and DMPWS policies, respectively, where $A_{s1}(t) \in EXP(1/3)$, starting around time zero for 10 time slots; $A_{h1}(t), A_{h2}(t), A_{h3}(t), A_{h4}(t) \in PAR(1.5, 1)$.

compared to the QMPWS policy. Furthermore, we consider a scenario where four HT flows and a LT flow sharing a single schedule in Figure 6. Similar results are obtained as DMPWS provides the shortest delay for LT traffic among all scheduling policies.

C. Performance Under Flow Dynamics

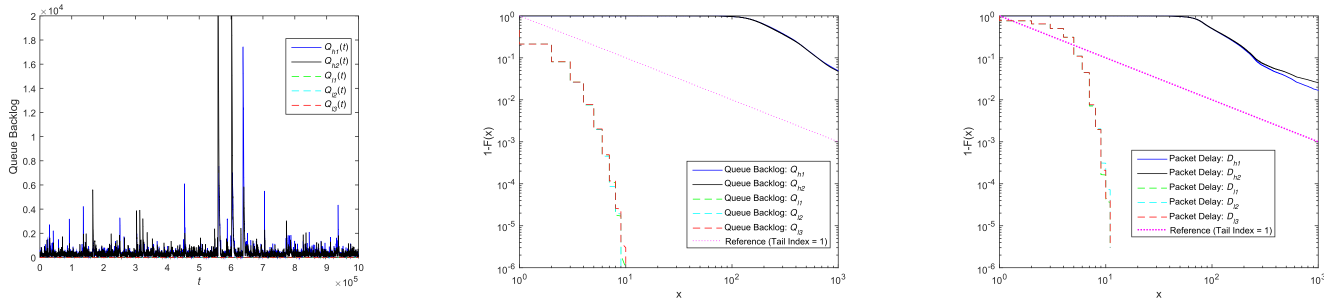
As the main motivation for delay-based scheduling, flow dynamics, where certain flows only have a finite number of packets to transmit, easily make queue length-based scheduling lose throughput optimality and result in the well-known last packet problem. To this end, in this section, we evaluate the delay performance of short-lived flows that conflict with HT flows under QMPWS and DMPWS, respectively.

We consider a scenario where two HT flows $\{h1, h2\}$ and three short-lived flows $\{s1, s2, s3\}$ sharing a schedule, i.e., $A_{h1}(t), A_{h2}(t) \in PAR(1.5, 1)$ or $\ln N(3, 1)$; $A_{s1}(t), A_{s2}(t) \in EXP(1/3)$, starting from time zero for 10 time slots; $A_{s3}(t) \in EXP(1/3)$, starting from time 5×10^4 for 10 time slots. A random variable $X \in EXP(\lambda)$, if it follows exponential distribution with parameter λ (i.e., $P(X > x) = e^{-\lambda x}$). Also, QMPWS and DMPWS policies assign weight 0.5 for HTs and weight 2 for short-lived flows. Figure 7 shows average packet delay of short-lived flows under the two scheduling policies, where Figures 7a-7b are for HTs with Pareto distributions and Figures 7c-7d are for HTs with log-normal distributions. In Figure 7a, the results show that under QMPWS, the delay of short-lived flows will linearly increase as time proceeds, no matter the flows start from the beginning or the middle of the evaluated time period. The reason is that these short-lived flows cannot bring sufficiently large queue lengths to establish desired queueing dynamics. (Note that Figure 7a implies that under QMPWS there might be a chance for a single short-lived flow, starting from the beginning, to exit the system. This will happen only when the following two conditions sustain simultaneously in the very beginning of queueing dynamics: (i) the conflicting HT flows have not built-up large queue lengths and (ii) the duration of the short-lived flow is extremely short.) On the other hand, under DMPWS in Figure 7b, the delay of short-lived flows can be maintained in very small values with some small bursts at time around flow arrivals. This indicates the advantage of delay-based scheduling with respect to flow dynamics, as the delay (or packet weighting time) now is the trigger for queueing dynamics, and short-lived flows will have chances to exit the system when time evolves.

Figures 7c-7d also shows the delay performance with the consideration of HT flows from log-normal distributions. Similarly, the results show that DMPWS can still provide bounded delay for all short-lived flows that conflict with HT flows, while QMPWS cannot guarantee bounded delay anymore. At this time, no short-lived flow can exit the system when conflicting with HT flows from heavy tailed distributions, i.e., log-normal distributions. Furthermore, we consider a scenario where four HT flows and a short-lived flow sharing a single schedule in Figure 8. The results still show that DMPWS preserves bounded average delay for short-lived traffic flow when it competes with HT traffic flows. Hence, all of above numerical results suggest the preference of DMPWS than QMPWS when considering flow dynamics.

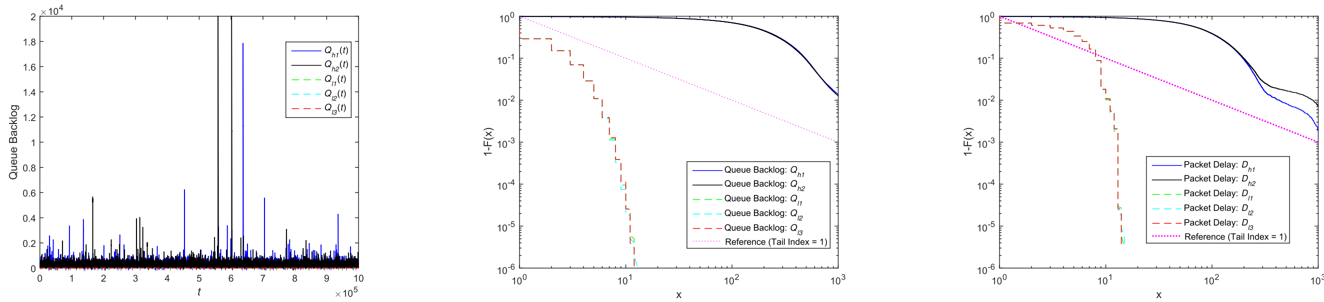
D. Performance of Infrequent Measurement Update

While we successfully demonstrate the superiority of DMPWS, in the following, we evaluate the impact of



(a) (b) (c)

Fig. 9. Queue lengths and packet delay under the IU-DMPWS policy with $T^I = 2$. (a) Evolution of queue lengths. (b) Tail distribution of queue lengths. (c) Tail distribution of packet delay.



(a) (b) (c)

Fig. 10. Queue lengths and packet delay under the IU-DMPWS policy with $T^I = 4$. (a) Evolution of queue lengths. (b) Tail distribution of queue lengths. (c) Tail distribution of packet delay.

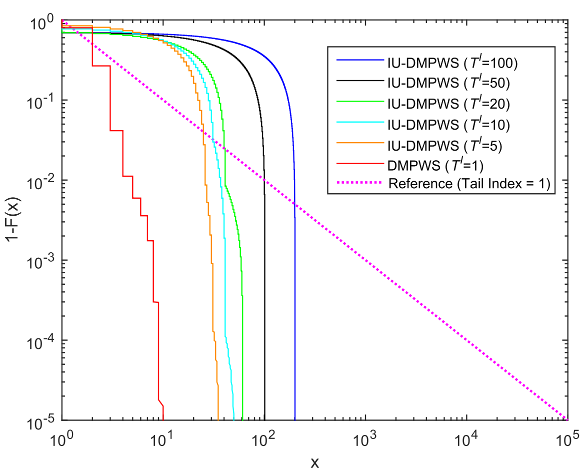


Fig. 11. Packet delay of LT traffic under the IU-DMPWS policy with various update frequencies T^I.

infrequent update of state measurements (i.e., from $T^I = 1$ to $T^I > 1$) by simulating the IU-DMPWS policy. Figure 9 and Figure 10 show the performance of IU-DMPWS policies with $T^I = 2$ and $T^I = 4$, respectively. Both policies can provide bounded queue lengths and delay for LT flows and achieve the stable network. It is consistent with Theorem 6 that such an infrequent update will not affect the throughput optimality and network stability. Moreover, we further compare the packet delay under IU-DMPWS policies with various update frequency in Figure 11. It is shown that as the update frequency increases, the delay of LT flows will also slightly increase but still have finite mean, which confirms our theoretical finding. These outstanding performances accompanied with infrequent updating facilitate practical implementation of IU-DMPWS.

VI. Conclusion

This paper develops a throughput-optimal and delay-based maximum power-weight scheduling for single-hop flows with heavy-tailed traffic. Delay-based scheduling provides a simple way to reduce packet delay that plagues its queue length-based counterpart. We first prove the network instability of the conventional DMWS policy. Then, we propose a DMPWS and its variant IU-DMPWS and prove their throughput optimality with respect to moment stability. The IU-DMPWS policy is favored among these scheduling algorithms for practical implementation because of its immunity to the impact of HT traffic along with limited signaling overhead.

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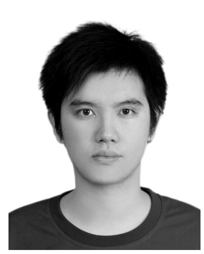

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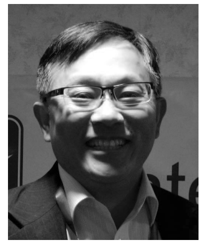

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