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# ACTION RULES & META ACTIONS


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# Introduction : Action Rule

An action rule /Ras & Wieczorkowska, PKDD 2000/ is a rule extracted from an information system that *describes a possible transition of objects from one state to another* with respect to a distinguished attribute called a decision attribute

Information System

| | A | B | D |
|---|---|---|---|
| | a1 | b2 | d1 |
| | a2 | b2 | |
| | a2 | b2 | d2 |

Assumption:

- attributes are partitioned into stable and flexible

Values of attributes can be changed

# Introduction : Action Rule

Action rule is defined as a term $[(\omega) \wedge (\alpha \rightarrow \beta)] \rightarrow (\varphi \rightarrow \psi)$

conjunction of fixed condition features shared by both groups

proposed changes in values of flexible features

desired effect of the action

Information System

| | A | B | D |
|---|---|---|---|
| | a1 | b2 | d1 |
| | a2 | b2 | |
| | a2 | b2 | d2 |

# Example

| X | a | b | c | d |
|---|---|---|---|---|
| $x_1$ | 0 | S | 0 | L |
| $x_2$ | 0 | R | 1 | L |
| $x_3$ | 0 | S | 1 | L |
| $x_4$ | 0 | R | 1 | L |
| $x_5$ | 2 | P | 2 | L |
| $x_6$ | 2 | P | 2 | L |
| $x_7$ | 2 | S | 2 | H |

Decision Table

{a, c} - stable attributes,

{b, d} - flexible attributes,

d - decision attribute.

Rules discovered:

$r_1 = [\quad (b, P) \rightarrow (d, L)]$

$r_2 = [(a, 2) \wedge (b, S) \rightarrow (d, H)]$

Action rule:

$[(a, 2) \wedge (b, P \rightarrow S)](x) \Rightarrow [(d, L \rightarrow H)](x)$

# Application domain: *Customer Attrition*

**Facts:**

- **On average, most US corporations lose half of their customers every five years (Rombel, 2001).**
- **Longer a customer stays with the organization, the more profitable he or she becomes (Pauline, 2000; Hanseman, 2004).**
- **The cost of attracting new customers is five to ten times more than retaining existing ones.**
- **About 14% to 17% of the accounts are closed for reasons that can be controlled like price or service (Lunt, 1993).**

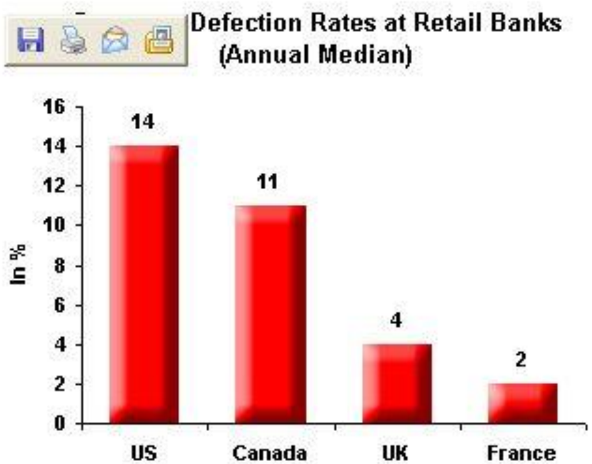
**Action:**

**Reducing the outflow of the customers by 5% can double a typical company's profit (Rombel, 2001).**

# Customer Attrition in Retail Banking: US, Canada, UK, France

## Report Published by Celent

**Fighting customer attrition is a top priority at US and Canadian banks, and for good reason: customer defection rates are up to 7 times higher in the US and Canada than in Western Europe. A new Celent report explains why.**




In a new report, ***"Customer Attrition in Retail Banking: the US, Canada, the UK, and France,"*** Celent analyzes why customer defection rates are so much higher in the US and Canada than the UK and France.

The report, which is based on a survey of more than 30 banks, reviews trends affecting customer attrition in the UK and France, analyzes how Canadian and American bankers have addressed attrition so far, and reports on their success. The report also suggests tactics that North American banks should pursue to get a 10% defection rate or better.

Over the past two years, half of top US and Canadian banks have seen their defection rates decrease by an average of 10%, while defection rates have remained the same for 40%. Only 10% have experienced increased attrition. The best organizations in the US and Canada have achieved a 12% customer defection rate.

Methods for Action Rules Extraction:

1] Rule-based

Prior extraction of classification rules is needed

Example – DEAR /Tsay & Ras, tree-based strategy/

2] Object-based

Action rules are extracted directly from DB

Example – ARED /similar to Apriori/

Ref 1: "*Action rules discovery: System DEAR2, method and experiments*", L.-S. Tsay, Z.W. Ras, Journal of Experimental and Theoretical Artificial Intelligence, Taylor & Francis, Vol. 17, No. 1-2, 2005, 119-128

Ref 2: "*Association Action Rules*", Z.W. Ras, A. Dardzinska, L.-S. Tsay, H. Wasyluk, IEEE/ICDM Workshop on Mining Complex Data (MCD 2008), Pisa, Italy, ICDM Workshops Proceedings, IEEE Computer Society, 2008, 283-290

# Action Rules Discovery (Preprocessing)

## Partition decision table S

| a | b | c | e | f | d |
|---|---|---|---|---|---|
| 2 | 1 | 1 | 7 | 8 | 1 |
| 2 | 5 | 4 | 6 | 8 | 1 |
| 1 | 1 | 4 | 9 | 4 | 2 |
| 1 | 4 | 5 | 8 | 7 | 2 |
| 2 | 1 | 5 | 2 | 8 | 3 |
| 2 | 1 | 4 | 2 | 8 | 3 |
| 1 | 2 | 4 | 7 | 1 | 2 |
| 2 | 1 | 1 | 6 | 8 | 1 |
| 3 | 2 | 4 | 6 | 8 | 2 |
| 3 | 3 | 5 | 7 | 4 | 2 |
| 3 | 3 | 5 | 6 | 2 | 3 |
| 2 | 5 | 4 | 6 | 8 | 3 |

Stable:{ a, b}

Flexible: {c, e, f}

Reclassification direction:

2→ 1 or 3 →1

Splitting the node using the stable attribute Dom(a) = {1,2,3} & Dom(b) = {1,2,3,4,5}

a = 1

$T_1$

| b | c | e | f | d |
|---|---|---|---|---|
| 1 | 9 | 4 | 4 | 2 |
| 4 | 5 | 8 | 7 | 2 |
| 2 | 4 | 7 | 1 | 2 |

All objects have the same decision value, so this sub-table is not analyzed any further

a = 2

$T_2$

| b | c | e | f | d |
|---|---|---|---|---|
| 1 | 1 | 7 | 8 | 1 |
| 5 | 4 | 6 | 8 | 1 |
| 1 | 5 | 2 | 8 | 3 |
| 1 | 4 | 2 | 8 | 3 |
| 1 | 1 | 6 | 8 | 1 |
| 5 | 4 | 6 | 8 | 3 |

All objects have the same value 8 for attribute f, so it is crossed out from the sub-table ( this condition is used for stable attributes as well)

a = 3

$T_3$

| b | c | e | f | d |
|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 2 |
| 3 | 5 | 7 | 4 | 2 |
| 3 | 5 | 6 | 2 | 3 |

None of the objects contain the desired class “1”, so this sub-table stops splitting any further

b = 1

$T_4$

| c | e | d |
|---|---|---|
| 1 | 7 | 1 |
| 5 | 2 | 3 |
| 4 | 2 | 3 |
| 1 | 6 | 1 |

b = 5

$T_5$

| c | e | d |
|---|---|---|
| 4 | 6 | 1 |
| 4 | 6 | 3 |

All the flexible values are the same for both objects , therefore this sub-table is not analyzed any further

# DEAR1 - Rule-Based Action Rules Discovery

Set of rules R with supporting objects

| Objects | a | b | c | d |
|---|---|---|---|---|
| x1, x2, x3, x4 | 0 | | | L |
| x1, x3 | | | 0 | L |
| x2, x4 | | 2 | | L |
| x2, x4 | | | 1 | L |
| x5, x6 | | 3 | | L |
| x7, x8 | 2 | 1 | | H |
| x7, x8 | | 1 | 2 | H |

**Stable Attribute: {a, c}**

**Flexible Attribute: b**

**Decision Attribute: d**

*(d, L)-tree T2*

| Objects | a | b | c |
|---|---|---|---|
| x1, x2, x3, x4 | 0 | | |
| x1, x3 | | | 0 |
| x2, x4 | | 2 | |
| x2, x4 | | | 1 |
| x5, x6 | | 3 | |

a = ?

| Objects | b | c |
|---|---|---|
| x1, x3 | | 0 |
| x2, x4 | 2 | |
| x2, x4 | | 1 |
| x5, x6 | 3 | |

a = 0

| Objects | b | c |
|---|---|---|
| x1, x2, x3, x4 | | |

c = ?

| Objects | b |
|---|---|
| x1, x2, x3, x4 | |

T6

c = ?

| Objects | b |
|---|---|
| x2, x4 | 2 |
| x5, x6 | 3 |

T3

c = 0

| Objects | b |
|---|---|
| x1, x3 | |

T4

c = 1

| Objects | b |
|---|---|
| x2, x4 | |

T5

*(d, H)-tree T1*

| Objects | a | b | c |
|---|---|---|---|
| x7, x8 | 2 | 1 | |
| x7, x8 | | 1 | 2 |

a = 2

| Objects | b | c |
|---|---|---|
| x7, x8 | 1 | |

c = ?

| Objects | b |
|---|---|
| x7, x8 | 1 |

T1

a = ?

| Objects | b | c |
|---|---|---|
| x7, x8 | 1 | 2 |

c = 2

| Objects | b |
|---|---|
| x7, x8 | 1 |

T2

(T3, T1) : $(a = 2) \wedge (b, 2 \rightarrow 1) \Rightarrow (d, L \rightarrow H)$

$(a = 2) \wedge (b, 3 \rightarrow 1) \Rightarrow (d, L \rightarrow H)$

# System DEAR2

| | Objects | a | b | c | d |
|---|---|---|---|---|---|
| r1 | x1, x2, x3, x4 | 0 | | | L |
| r2 | x1, x3 | | | 0 | L |
| r3 | x2, x4 | | 2 | | L |
| r4 | x2, x4 | | | 1 | L |
| r5 | x5, x6 | | 3 | | L |
| r6 | x7, x8 | 2 | 1 | | H |
| r7 | x7, x8 | | 1 | 2 | H |

Set of rules R with supporting objects

Stable Attribute: b

Flexible Attribute: {a, c}

Decision Attribute: d

| Objects | a | b | c | d |
|---|---|---|---|---|
| x1, x2, x3, x4 | 0 | | | L |
| x1, x3 | | | 0 | L |
| x2, x4 | | 2 | | L |
| x2, x4 | | | 1 | L |
| x5, x6 | | 3 | | L |
| x7, x8 | 2 | 1 | | H |
| x7, x8 | | 1 | 2 | H |

b = 2

| Objects | a | c | d |
|---|---|---|---|
| x1, x2, x3, x4 | 0 | | L |
| x1, x3 | | 0 | L |
| x2, x4 | | | L |
| x2, x4 | | 1 | L |

b = 3

| Objects | a | c | d |
|---|---|---|---|
| x1, x2, x3, x4 | 0 | | L |
| x1, x3 | | 0 | L |
| x2, x4 | | 1 | L |
| x5, x6 | | | L |

b = 1

| Objects | a | c | d |
|---|---|---|---|
| x1, x2, x3, x4 | 0 | | L |
| x1, x3 | | 0 | L |
| x2, x4 | | 1 | L |
| x7, x8 | 2 | | H |
| x7, x8 | | 2 | H |

d = L

| Objects | a | c |
|---|---|---|
| x1, x2, x3, x4 | 0 | |
| x1, x3 | | 0 |
| x2, x4 | | 1 |

d = H

| Objects | a | c |
|---|---|---|
| x7, x8 | 2 | |
| x7, x8 | | 2 |

$(b = 1) \wedge (a, 0 \rightarrow 2) \Rightarrow (d, L \rightarrow H)$
$(b = 1) \wedge (c, 0 \rightarrow 2) \Rightarrow (d, L \rightarrow H)$
$(b = 1) \wedge (c, 1 \rightarrow 2) \Rightarrow (d, L \rightarrow H)$

# Cost of Action Rule

Action rule r:

$[(b_1, v_1 \to w_1) \wedge (b_2, v_2 \to w_2) \wedge \ldots \wedge (b_p, v_p \to w_p)](x) \Rightarrow (d, k_1 \to k_2)(x)$

The cost of r in S:

$$cost_S(r) = \sum\{\wp_S(v_i , w_i) : 1 \leq i \leq p\}$$

Action rule r is feasible in S, if $cost_S(r) < \wp_S(k_1 , k_2)$.

For any feasible action rule r, the cost of the conditional part of r is lower than the cost of its decision part.

# Cost of Action Rule

Example:

$$r = [(b_1, v_1 \rightarrow w_1) \wedge \ldots \wedge (b_j, v_j \rightarrow w_j) \wedge \ldots \wedge ( b_p, v_p \rightarrow w_p)](x) \Rightarrow (d, k_1 \rightarrow k_2)(x)$$

In $R_S[(b_j, v_j \rightarrow w_j)]$ we find

$$r_1 = [(b_{j1}, v_{j1} \rightarrow w_{j1}) \wedge (b_{j2}, v_{j2} \rightarrow w_{j2}) \wedge \ldots \wedge( b_{jq}, v_{jq} \rightarrow w_{jq})](x) \Rightarrow (b_j, v_j \rightarrow w_j)(x)$$

Then, we can compose $r$ with $r_1$ and the same replace term $(b_j, v_j \rightarrow w_j)$ by term from the left hand side of $r_1$:

$$[(b_1, v_1 \rightarrow w_1) \wedge \ldots \wedge [(b_{j1}, v_{j1} \rightarrow w_{j1}) \wedge (b_{j2}, v_{j2} \rightarrow w_{j2}) \wedge \ldots \wedge ( b_{jq}, v_{jq} \rightarrow w_{jq})] \wedge \ldots \wedge( b_p, v_p \rightarrow w_p)](x) \Rightarrow (d, k_1 \rightarrow k_2)(x)$$

# Support, Confidence

## Decision System $S$

| $X$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $x_1$ | $a_1$ | $b_1$ | $c_1$ | $d_1$ |
| $x_2$ | $a_2$ | $b_1$ | $c_2$ | $d_1$ |
| $x_3$ | $a_2$ | $b_2$ | $c_2$ | $d_1$ |
| $x_4$ | $a_2$ | $b_1$ | $c_1$ | $d_1$ |
| $x_5$ | $a_2$ | $b_3$ | $c_2$ | $d_1$ |
| $x_6$ | $a_1$ | $b_1$ | $c_2$ | $d_2$ |
| $x_7$ | $a_1$ | $b_2$ | $c_2$ | $d_1$ |
| $x_8$ | $a_1$ | $b_2$ | $c_1$ | $d_3$ |

$r=[(a, a_2 \rightarrow a_1)*(b, b_1 \rightarrow b_1)] \rightarrow (d, d_1 \rightarrow d_2)$

rule

$(Y_1, Y_2)$ $(Z_1, Z_2)$

$Y_1 = \{x_2, x_4\}$
$Z_1 = \{x_1, x_2, x_3, x_4, x_5, x_7\}$

$Y_2 = \{x_1, x_6\}$
$Z_2 = \{x_6\}$

$$\sup(r) = card(Y_1 \cap Z_1)$$

$$conf(r) = \left[\frac{card(Y_1 \cap Z_1)}{card(Y_1)}\right] * \left[\frac{card(Y_2 \cap Z_2)}{card(Y_2)}\right]$$

$sup(r) = 2$
$conf(r) = 1/2$

**Meta-actions** /A. Tuzhilin (2006)/

Actions which trigger changes of flexible attributes either directly or indirectly because of correlations among certain attributes in the system.

**Example 1:** Taking a drug /Lamivudine is used for treatment of chronic hepatitis B. It *improves the seroconversion of e-antigen positive hepatitis B* and also *improves histology staging of the liver* but at the same time it can cause a number of other symptoms. This is why doctors have to order certain lab tests to check patient's response to that drug.

**Example 2:** Classification attributes are: *Explain difficult concepts effectively, Stimulate student interest in the course, Provide sufficient feedback.*
Meta-actions : *Change the content of the course, Change the textbook of the course, …*

$$[(b_1, v_1 \rightarrow w_1) \wedge (b_2, v_2 \rightarrow w_2) \wedge \ldots \wedge (b_p, v_p \rightarrow w_p)](x) \Rightarrow (d, k_1 \rightarrow k_2)(x)$$

# Action Rules - continuation

Influence Matrix

$A_1, A_2, A_3, \ldots, A_m$ – attributes
$M_1, M_2, M_3, \ldots, M_n$ – meta actions
$E_{ij}$ – atomic action or NULL, $i \le m, j \le n$

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | ….. | $A_m$ |
|---|---|---|---|---|---|---|
| $M_1$ | $E_{11}$ | $E_{12}$ | $E_{13}$ | $E_{14}$ | | $E_{1m}$ |
| $M_2$ | $E_{21}$ | $E_{22}$ | $E_{23}$ | $E_{24}$ | | $E_{2m}$ |
| $M_3$ | $E_{31}$ | $E_{32}$ | $E_{33}$ | $E_{34}$ | | $E_{3m}$ |
| $M_4$ | $E_{41}$ | $E_{42}$ | $E_{43}$ | $E_{44}$ | | $E_{4m}$ |
| ….. | | | | | | |
| $M_n$ | $E_{m1}$ | $E_{m2}$ | $E_{m3}$ | $E_{m4}$ | | $E_{mn}$ |

Atomic actions triggered by $M_3$

Action rule: $r = [t \Rightarrow (d, d_1 \rightarrow d_2)]$, where $t$ – action term

# Action Rules - continuation

Meta-actions-based decision system $S(d)=(X, A \cup \{d\}, V)$,
with $A = \{A_1, A_2, \ldots, A_m\}$

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | ….. | $A_m$ |
|---|---|---|---|---|---|---|
| $M_1$ | $E_{11}$ | $E_{12}$ | $E_{13}$ | Null | | $E_{1m}$ |
| $M_2$ | Null | $E_{22}$ | $E_{23}$ | $E_{24}$ | | $E_{2m}$ |
| $M_3$ | $E_{31}$ | $E_{32}$ | Null | Null | | Null |
| $M_4$ | $E_{41}$ | Null | $E_{43}$ | $E_{44}$ | | $E_{4m}$ |
| ….. | | | | | | |
| $M_n$ | $E_{m1}$ | Null | $E_{m3}$ | $E_{m4}$ | | $E_{mn}$ |

Influence Matrix

Meta-action $M_3$ triggers two atomic actions:

$\{E_{32} = [a_2 \rightarrow a_2'],$
$E_{31} = [a_1 \rightarrow a_1']\}$

Candidate action rule -

$\mathbf{r} = [(A_1, a_1 \rightarrow a_1') \wedge (A_2, a_2 \rightarrow a_2') \wedge (A_4, a_4 \rightarrow a_4')] \Rightarrow (d, d_1 \rightarrow d_1')$

We say that r is valid in S with respect to meta-action $M_3$ if the atomic actions triggered by $M_3$ do not contradict with atomic actions in r.

# Action Rules Discovery - Example

| | **a** | **b** | **c** |
|---|---|---|---|
| $M_1$ | | $b_1$ | $c_2 \to c_1$ |
| $M_2$ | $a_2 \to a_1$ | $b_2$ | |
| $M_3$ | $a_1 \to a_2$ | | $c_2 \to c_1$ |
| $M_4$ | | $b_1$ | $c_1 \to c_2$ |
| $M_5$ | | $b_2$ | $c_1 \to c_2$ |
| $M_6$ | $a_1 \to a_2$ | $b_1$ | $c_1 \to c_2$ |

Influence Matrix for S(d)

$r = [[(b, b_1) \wedge (c, c_1 \to c_2)]\ ) \Rightarrow (d, d_1 \to d_2)]$, valid with respect to $M_4$, $M_6$

# Application Domain

Database HEPAR (Medical Center of Postgraduate Education, Warsaw, Poland) prepared by Dr. med. Hanna Wasyluk

- 758 patients described by 106 attributes (including 31 laboratory tests with values discretized to: “below normal”, “normal”, “above normal”). 14 attributes are stable. Two tests are invasive tests: HBsAg [in tissue], HBcAg [in tissue]. There are 7 decision values (attribute d):

  I. Acute hepatitis
  IIa. Subacute hepatitis (types BC)
  IIb. Subacute hepatitis (alcohol-abuse)
  IIIa. Chronic hepatitis (curable)
  IIIb. Chronic hepatitis (non-curable)
  IV. Cirrhosis hepatitis
  V. Liver cancer

  We ask for re-classifications: IIb $\rightarrow$ I, IIIa $\rightarrow$ I

# Application Domain

## Chosen d-reduct has no invasive tests, 11% incomplete data

***{m, n, q, u, y, aa, ah, ai, am, an, aw, bb, bg, by, cj, cm}***

| | |
|---|---|
| m | Bleeding |
| n | Subjaundice symptoms |
| q | Eructation |
| u | Obstruction |
| y | Weight loss |
| aa | Smoking |
| ah | History of viral hepatitis (stable) |
| ai | Surgeries in the past (stable) |
| am | History of hospitalization (stable) |
| an | Jaundice in pregnancy |
| aw | Erythematous dermatitis |
| bb | Cysts |
| bg | Sharp liver edge (stable) |
| bm | Blood cell plaque |
| by | Alkaline phosphatase |
| cj | Prothrombin index |
| cm | Total cholesterol |
| **d** | **Decision attribute** |

# Application Domain: Database HEPAR

- No history of viral hepatitis but with history of surgery and hospitalization, sharp
- liver edge normal, no subjaundice symptoms, total cholesterol normal,
- erythematous dermatitis normal, weight normal, no cysts , patient does not smoke

- $[(ah=1) \wedge (ai=2) \wedge (am=2) \wedge (bg=1)] \wedge (cm=1) \wedge (aw=1) \wedge (u, \rightarrow 1) \wedge (bb=1) \wedge (aa=1)$
- $\wedge (q, \rightarrow 1) \wedge (m, \rightarrow 1) \wedge (n=1) \wedge (bm, \rightarrow up) \wedge (y=1) \wedge (by, \rightarrow up)$
- $\rightarrow (d, IIIa \rightarrow I)$

- $[(ah=1) \wedge (ai=2) \wedge (am=2) \wedge (bg=1)] \wedge (cm=1) \wedge (aw=1) \wedge (u, \rightarrow 1) \wedge (bb=1) \wedge (aa=1)$
- $\wedge (q, \rightarrow 1) \wedge (m, \rightarrow 1) \wedge (n=1) \wedge (bm, \rightarrow up) \wedge (y=1) \wedge (by, \rightarrow down)$
  $\rightarrow (d, IIIa \rightarrow I)$
- Two quite similar action rules have been constructed:
  *By getting rid of obstruction, eructation, bleeding, by decreasing the blood cell plaque and by changing the level of alkaline phosphatase we should be able reclassify the patient from IIIa group to I.*

Attribute values of *total cholesterol, weight, and smoking* have to stay unchanged.

# Cost of Meta Action Rules

| | Meta Actions (drugs) | |
|---|---|---|
| *Action Rule II requires getting rid of:* | | |
| - *obstruction (u)* | HEPATIL | TRIPHALA |
| - *eructation (q)* | HEPATIL | HEPARGEN |
| - *bleeding (m)* | HEPATIL | HEPARGEN |
| *decreased blood cell plaque (bm)* | HEPATIL | HEPARGEN |
| *increased level of alkaline phosphatase (by)* | HEPATIL | HEPARGEN |

***[(ah=1)∧ (ai=2)∧ (am=2)∧ (bg=1)]****∧(cm=1)∧(aw=1)∧(u, → 1)∧ (bb=1)∧ (aa=1) ∧ (q, → 1)∧(m, → 1)∧(n=1)∧(bm, → up)∧(y=1)∧* ***(by, → down)*** **→ *(d, IIIa → I )***

==========================================

***[(ah=1)∧ (ai=2)∧ (am=2)∧ (bg=1)]****∧(cm=1)∧(aw=1)∧ (bb=1)∧ (aa=1) ∧(n=1)∧(y=1) ∧ [obstruction, eructation, bleeding, increased blood cell plaque, increased level of alkaline phosphatase ] ∧ [Take HEPATIL]* → ***(d, IIIa → I )***

***[(ah=1)∧ (ai=2)∧ (am=2)∧ (bg=1)]****∧(cm=1)∧(aw=1)∧ (bb=1)∧ (aa=1) ∧ (n=1) ∧ (y=1) ∧ [obstruction, eructation, bleeding, increased blood cell plaque, increased level of alkaline phosphatase ] ∧ [Take HEPERGEN & TRIPHALA]* → ***(d, IIIa → I )***

## Action Reducts

X2={x1,x2,x3,x4}, X1={x5,x6,x7,x8}

| X | B | C | E | D |
|---|---|---|---|---|
| x1 | b2 | c1 | e1 | d2 |
| x2 | b1 | c3 | e2 | d2 |
| x3 | b1 | c1 | | d2 |
| x4 | b1 | c3 | e1 | d2 |
| x5 | b1 | c1 | e1 | d1 |
| x6 | b1 | c1 | e1 | d1 |
| x7 | b2 | | e2 | d1 |
| x8 | b1 | c2 | e2 | d1 |

**Discernable Table**

| | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x5 | b2 | c3+e2 | 0 | c3 |
| x6 | b2 | c3+e2 | 0 | c3 |
| x7 | c1+e1 | b1+c3 | b1+c1 | b1+c3+e1 |
| x8 | b2+c1+e1 | c3 | c1 | c3+e1 |

b2 is needed to discern x1 from x5

Action Reducts:

R(x1)=b2[c1+e1][b2+c1+e1]= b2c1 + b2e1

R(x2)=c3, R(x3)=NIL,

R(x4)=c3

Action Rules:

(B, → b2)(C, → c1) ⇒ (D, d1 → d2)

(B, → b2)(E, → e1) ⇒ (D, d1 → d2)

(C, → c3) ⇒ (D, d1 → d2)

*Questions?*

*Thank You*