The Personality and Scientific Achievements of Professor Helena Rasiowa

A Personal View

from the Perspective of Influence
Professor Helena Rasiowa
on our Personal Life,
Scientific and Professional Development

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Women from the Circle of the Lwów-Warsaw School (LWS) Conference of the Center for Research on the Tradition of LWS and the Commission on the History of Science of the Polish Academy of Arts and Sciences, February 13, 2025.

Professor Rasiowa taught us, among other things, methods of finding answers to the following three questions of great importance in the life of each of us

1. How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

2. How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

3. How can we apply in practice (especially in complex IT projects) achievements in mathematical logic (especially algebraic methods in logic) and achievements in IT foundations?

How can we support personal, scientific,

and

professional development and good cooperation among people

(especially in extremely difficult situations)?

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

- **Principles of Normal Life Management in the Shadow of Suffering and Life-Threatening**:
  - WWII, including:
    - Underground Education
    - the Warsaw Uprising
  - Illness and an Unfinished Book...
- **Principles of striving for "normal" functioning under the prevailing conditions of difficult external ideological conditions** (as opposed to the internal hierarchy of values)
- **Ability to reconcile family life with professional work and peer activities**
- **Reconciling intense professional work with numerous managerial positions**
- **Reconciling outstanding scientific achievements with numerous international scientific activities**

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

In 1942, she established scientific contact with S. Mazurkiewicz and Cz. Białobrzeski, where she took exams at **the underground university**.

At the same time, under the supervision of **J. Łukasiewicz and B. Sobociński**, she was preparing her master's thesis on mathematical logic titled "**Axiomatization of a Certain Partial System of Implicational Deduction Theory**."

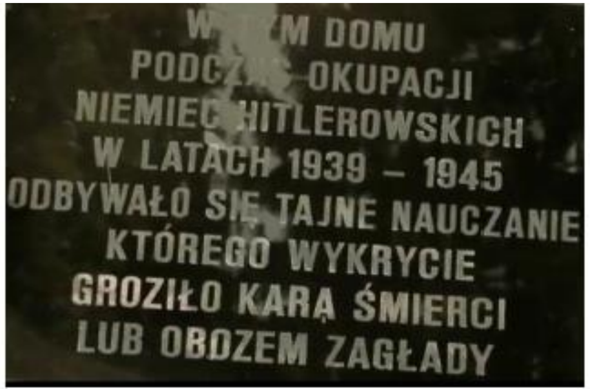
The manuscript of the thesis was lost in the Warsaw Uprising, but in 1945 she reconstructed it and in 1946 obtained her master's degree.

Helena Rasiowa

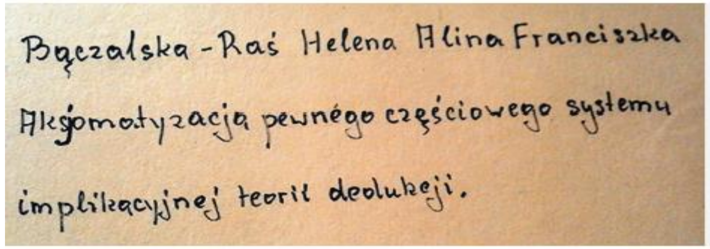
ALGEBRAIC MODELS OF LOGICS

Warszawa 2001

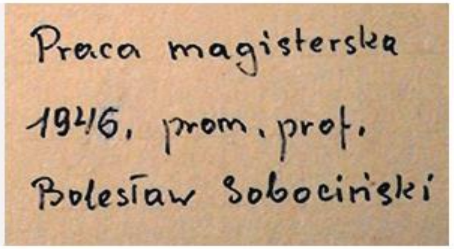
Unfinished Book



A commemorative plaque on a Warsaw house marks where underground education took place (1939-1945); ***discovery by the Germans meant death or concentration camp for teachers and students***.



Bączalska-Raś Helena Alina Franciszka
Aksjomatyzacja pewnego częściowego systemu
implikacyjnej teorii dedukcji.



Praca magisterska
1946, prom. prof.
Bolesław Sobociński

Stanisław Domoradzki:
ANTIQUITATES MATHEMATICAE Vol. 12(1) 2018, p. 271—283

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

- *Mathematics and logic was Her initial Passion*
- *World War II Did Not Prevent Her from Pursuing This Passion*

Helena Rasiowa

- She herself has survived with her mother in a cellar covered by ruins of the demolished building.
- She was alone in a cellar in Old Town in Warsaw covered by ruins, and when after several days she was able by a miracle to emerge alive, she saw on the other side of the street her mother, who could not instantly recognize her daughter because Rasiowa was wholly covered by dust.
- The two women left Warsaw and survived thanks to the help of other members of their family.

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

1976 -...

Mathematical Foundations of Computer Science

Computation Theory

Logics of Programs and Their Applications

FREUDENSCHAFTSVERTRAG between
the University of Warsaw and Humboldt University

Concurrency, Specification & Programming (CS&P)
1992 – 2023 continuation by prof. Ludwik Czaja)

Prof. Helmut Thiele

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

Helena Rasiowa

Actively maintaining

and systematically strengthening positive personal interpersonal relationships

among

colleagues and friends

How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

Learning through examples from her own life

The principle of striving for the most "normal" functioning possible in the prevailing conditions of difficult external ideological conditions (contrary to the internal hierarchy of values)

Helena Rasiowa

- One of the lessons of the Warsaw Uprising": striving for the best possible approximation of conciliatory solutions within the limits set by:
  - external conditions
  - hierarchy of life values (especially moral and ethical).
- High Quality of Time Management (one's own and others') enabling intensive professional work, fulfilling numerous managerial functions and active family life
- Outstanding scientific achievements and a strong international scientific position
- Franciscan Tertiary (Third Order member)
- High value of teamwork
- Martial Law in Poland in the 1980s...

How can we develop AI technologies
and their applications
using algebraic logic, new computational models
and reasoning about their computations?

Some of the inspirations for Helena Rasiowa's work on computational and AI models

At the beginning of her scientific career, Helena Rasiowa was particularly interested in the **research and development of algebraic and topological (geometric) methods in classical logic and non-classical logics**.

Her exceptionally large contribution in this field is still very highly appreciated.

*This trend of scientific activity refers to the first conceptual approaches to AI, and especially to the concept of "**calculemus" ("let us calculate")** by Gottfried Leibniz, derived from his work "The Art of Discovery" (1685).*

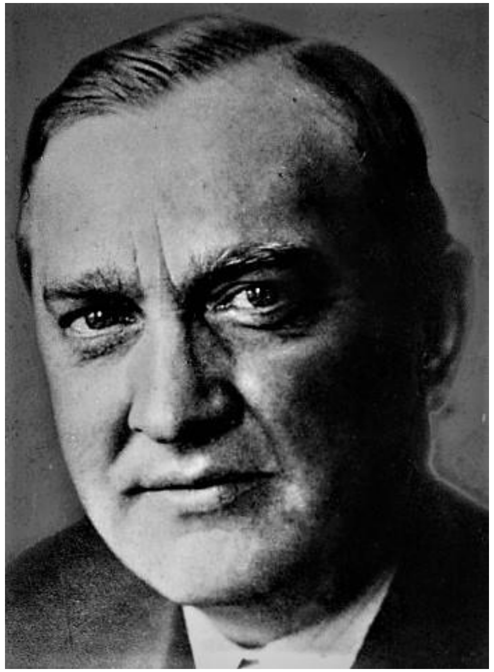
Key aspects of the "calculemus" concept:

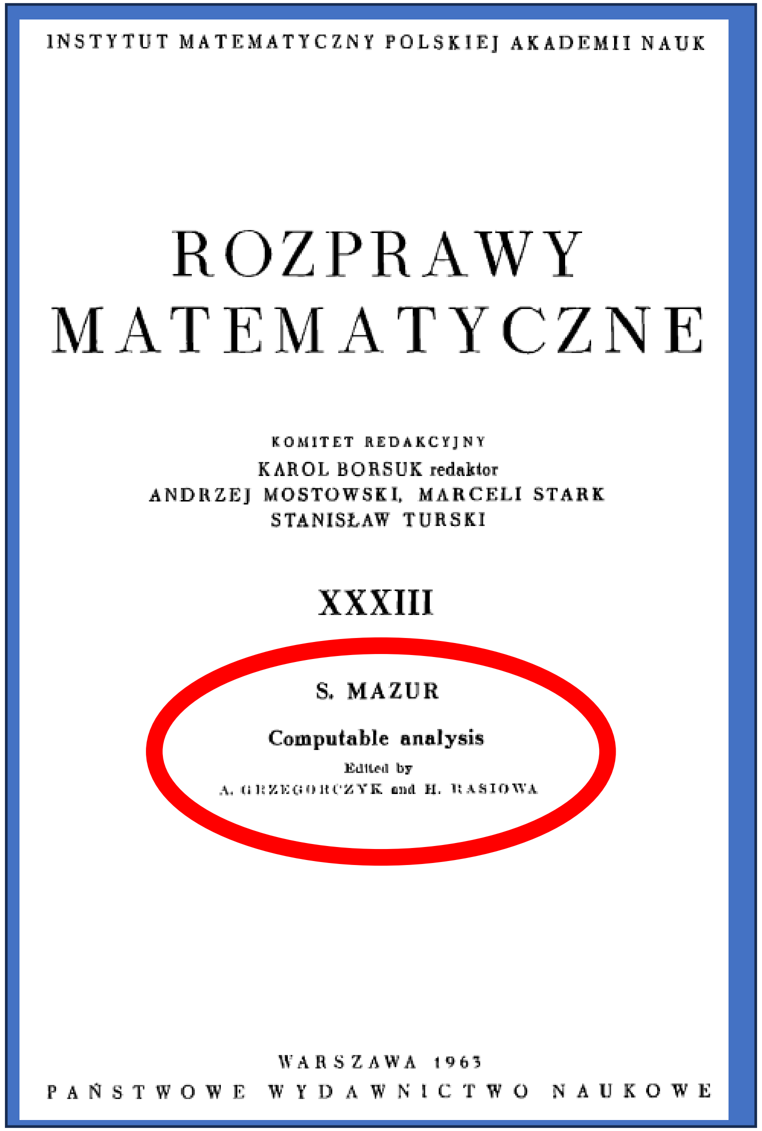
- **Formalization of language:** Leibniz proposed the creation of a universal symbolic language (*characteristica universalis*) in which every concept could be uniquely represented by a symbol. This would enable precise manipulation of ideas, similar to how algebra operates on numbers.
- **Logic as a computational tool:** Leibniz believed that logic could be reduced to a form of calculation. Instead of engaging in endless discussions, a dispute could be reduced to a set of symbols and logical rules, and then, by applying these rules, a clear conclusion could be reached. It should be similar to **arithmetic calculations**.
- **Reduction of disputes to argument evaluation:** According to the idea of "*calculemus*," in the event of a dispute, instead of engaging in subjective arguments and emotional discussions, the parties to the dispute could present their positions in a formalized language. Then, by applying logical and mathematical rules, it would be possible to "calculate" which side is right.
- **Universal tool:** Leibniz believed that "*calculemus*" could be applied not only in mathematics and logic, but also in philosophy, law, theology, and even politics. He believed that through it, it would be possible to resolve disputes in an objective and impartial manner.

Some of the inspirations from Lwów-Warsaw School for Rasiowa's work on computational and AI models

Inspired by Łukasiewicz, Leśniewski, and Tarski, as well as their disciples, Helena Rasiowa became a leading figure in Polish logic

Some of the inspirations from Lwów-Warsaw School for Rasiowa's work on computational and AI models





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STANISŁAW TURSKI

XXXIII

S. MAZUR

Computable analysis

Edited by
A. GRZEGORCZYK and H. RASIOWA

WARSZAWA 1963
PAŃSTWOWE WYDAWNICTWO NAUKOWE

Banach-Mazur „computable analysis” (1936-1939) was one of the inspirations for Helena Rasiowa's interest in computational models

Notes of the lectures of S. Mazur "**Computable Analysis**" in the academic year 1949-1950. These lectures contain:

- Systematic exposition of the results obtained by S. Banach and S. Mazur in 1936 - 1939
- A detailed examination of recursive real numbers, recursive sequences, functionals, and real functions.
- The results concerning general recursive mathematical objects obtained by Mazur himself after the war.

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

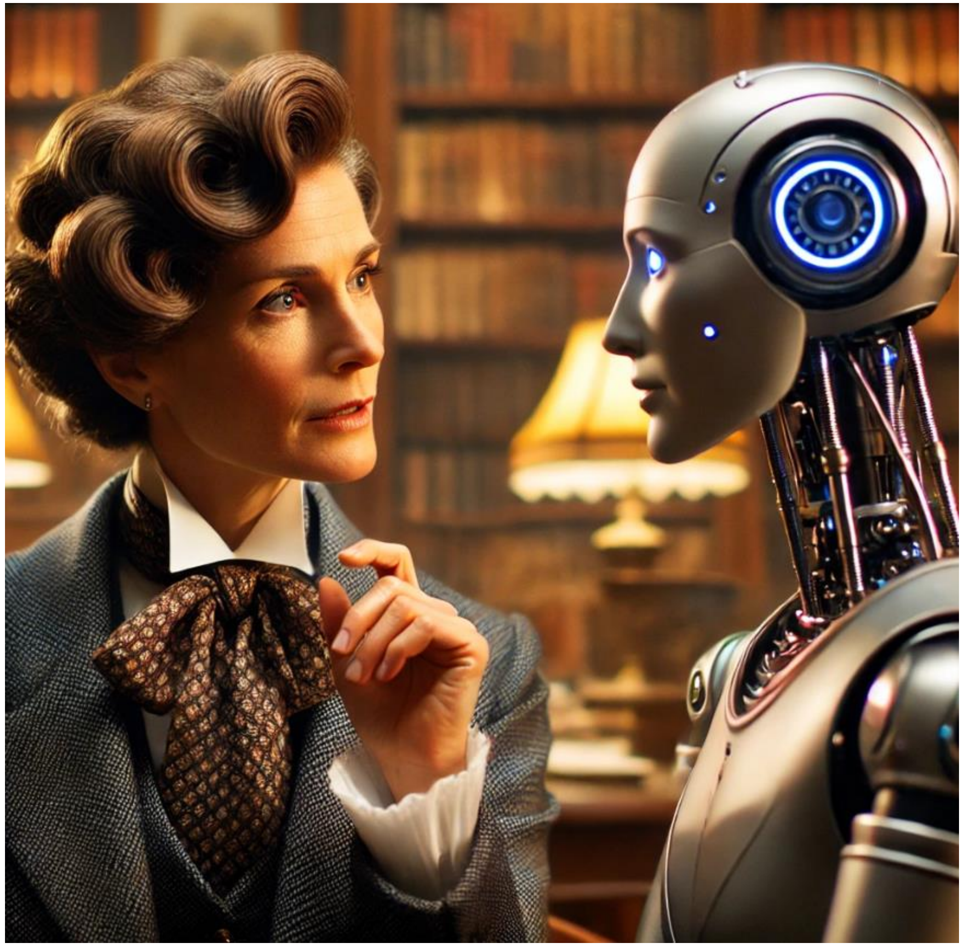
Since we met Professor Rasiowa (AS in 1963 and AJ in 1972), she strongly believed **in the pivotal role artificial intelligence would play in shaping the future of civilization**.

Therefore, she viewed her research—and that of her group, which **focused on developing methods essential for advancing AI** (at the beginning, primarily understood as automated reasoning and problem-solving)—as a top priority.

She particularly believed that future AI development and application breakthroughs would require appropriate tools for:

1. the **algebraization and geometrization of logic,**
2. the **development of innovative computational models that utilize appropriate algorithmic logic to provide rigorous reasoning methods about their computations**.

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?



„I've always thought of AI [artificial intelligence] as the most profound technology humanity is working on.

More profound than fire or electricity or anything that we've done in the past"

Sundar Pichai, the CEO of Google and its parent company Alphabet

CBC NEWS https://www.cbsnews.com/news/how-googles-dont-be-evil-motto-has-evolved-for-ai-age-60-minutes-2023-06-11/

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

In simple terms, the essence of the current AI/LLM revolution lies in computers' ability to:

1. process textual meaning,
2. carry out different types of reasoning,
3. and conduct natural conversations,

while these capabilities are simultaneously being developed for image and sound.

This computer’s ability to process meaning, carry out different types of reasoning, and conduct natural conversation's ability is largely **due to advanced machine learning models that construct a representation of meaning (semantics) from the properties of the text itself (primarily provided during model training**).

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

To summarize:

1. Contemporary AI/LLM models' functional understanding and reasoning capabilities are mainly realized by **representing and reasoning about the semantic aspects of language through algebraization and geometrization of language structures**

2. This space is generated from the syntactic properties of word co-occurrence within training data, without any direct interaction with the physical world.

For a better understanding of this phenomenon, let us consider a ***brief history of the evolution of representing and reasoning about the semantic aspects of language through the algebraization and geometrization of language structures***

A Simplified Evolution of Representing and Reasoning about the Semantic Aspects of Language through Algebraization and Geometrization of Language Structures

Table of Key Figures and Achievements in Semantic Representation and Reasoning in Mathematical Logic through Algebraization and Geometrization

Key persons	Field of Study	Key Achievements	Seminal Works" (Year)
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Leibniz	1-st idea of algebraic formalization of logic	The idea of calculemus, i.e., **whenever there is a dispute, let's compute**	The Art of Discovery (1685)
Boole	An algebraization of propositional logic	The concept of Boolean Algebra	An Investigation of the Laws of Thought (1854)
Gödel	Completeness and Incompleteness	Completeness Theorem for Predicate Logic	Die Vollständigkeit der Axiome des logischen Funktionenkalküls (1930)
Tarski & Lindenbaum	Propositional Logic	Algebraization od propositional calculus (Lindenbaum-Tarski Algebra)	Tarski, Grundzüge des Systemenkalküls, (1936)
Tarski Tsao-Chen	Propositional Logic	Topology of sematic for propositional calculus	Tarski: Der Aussagenkalkul und die Topologie (1938) Tsao-Chen: Algebraic postulates and a geometric interpretation for the Lewis calculus of strict implication (1938)
Rasiowa & Sikorski	**Classical Predicate Logic**	**The Rasiowa-Sikorski Lemma provided an algebraic framework for the study of models in predicate logic, particularly through Boolean canonical models**	**A Proof of the Completeness Theorem of Gödel by Means of Boolean Algebras (1950)**
Rasiowa & Sikorski	Algebraic Approach to Logic	Algebraization of Model Theory for classical, intuitionistic, and modal predicate calculi	The Mathematics of Metamathematics (1963)
Rasiowa	Multi-Valued Predicate Logic	Algebraic Model Theory for Multi-Valued Predicate Logic	An Algebraic Approach to Non-Classical Logics (1976)

This provides the conceptual backdrop for the later developments in NLP

A slightly more detailed Evolution of Representing and Reasoning about the Semantic Aspects of Language through Algebraization and Geometrization of Language Structures

Table of Key Figures and Achievements in Semantic Representation and Reasoning in Mathematical Logic through Algebraization and Geometrization

Key Person	Field of Study	Key Achievements	Seminal Work (Year)
Gottfried Wilhelm Leibniz	Philosophy, Mathematics, Logic	The idea of calculemus, i.e., whenever there is a dispute, let's compute Idea of calculus ratiocinator (universal logical language) and characteristica universalis (universal system of signs). Postulate of calculemus. Early concepts of algebraizing logic.	Dissertatio de arte combinatoria (1666); De Arte Combinatoria (fragments); The Art of Discovery (1685)
George Boole	Mathematics, Logic	Creation of Boolean algebra, an algebraic system of propositional logic. Formalization of logic using algebraic operations (conjunction, disjunction, negation).	The Mathematical Analysis of Logic (1847); An Investigation of the Laws of Thought (1854)
Gottlob Frege	Logic, Philosophy of Mathematics	Creation of the first formalized system of predicate logic. Distinction between syntax and semantics. Introduction of quantifiers, variables, functions, and relations. Definition of interpretation.	Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens (1879); Foundations of Arithmetic (1884)
Ernst Schröder	Algebraic Logic	Developed algebraic logic, expanded Boole's work; introduced relational algebra concepts, influenced Tarski.	Vorlesungen über die Algebra der Logik (1890–1905)
Leopold Löwenheim	Mathematical Logic	Proof of Löwenheim's theorem (later generalized by Skolem). First explicit construction of a model from a consistent set of formulas.	Über Möglichkeiten im Relativkalkül (1915)
Thoralf Skolem	Mathematical Logic, Model Theory	Simplification and generalization of the proof of Löwenheim's theorem. Introduction of Skolem normal forms (skolemization). Development of the concept of satisfiability.	Logisch-kombinatorische Untersuchungen über die Erfüllbarkeit oder Beweisbarkeit mathematischer Sätze... (1920)
Kurt Gödel	Mathematical Logic	Proof of the completeness theorem for first-order logic. Proof of the incompleteness theorems.	Die Vollständigkeit der Axiome des logischen Funktionenkalküls (1930); Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I (1931)

A slightly more detailed Evolution of Representing and Reasoning about the Semantic Aspects of Language through Algebraization and Geometrization of Language Structures

Table of Key Figures and Achievements in Semantic Representation and Reasoning in Mathematical Logic through Algebraization and Geometrization

Key Person	Field of Study	Key Achievements	Seminal Work (Year)
Jan Łukasiewicz	Logic, Philosophy	Introduction of three-valued logic (true, false, possible/indeterminate). Development of many-valued logics. Initiation of research into semantics for non-standard logics.	O logice trójwartościowej (Zur dreiwertigen Logik), Ruch Filozoficzny (1920)
Alfred Tarski	Mathematical Logic, Model Theory, Semantics	Formal definition of truth in a model (Tarski's semantics). Development of model theory.	Der Wahrheitsbegriff in den formalisierten Sprachen (1933, Polish; 1935, German); Logic, Semantics, Metamathematics (collection of papers); Introduction to Logic (1941)
Alfred Tarski & Adolf Lindenbaum	Logic, Algebra, Formal Semantics	Introduction of the Lindenbaum-Tarski algebra for propositional logic.	Grundzüge des Systemenkalküls (1936)
Tarski Tsao-Chen	Propositional Logic	Topology of sematic for propositional calculus	Tarski: Der Aussagenkalkul und die Topologie (1938) Tsao-Chen: Algebraic postulates and a geometric interpretation for the Lewis calculus of strict implication (1938)
Leon Henkin	Mathematical Logic, Model Theory	New proof of the completeness theorem for first-order logic, based on the construction of a Henkin model. Generalization to higher-order logics.	The Completeness of the First-Order Functional Calculus (1949)
Helena Rasiowa & Roman Sikorski	Mathematical Logic, Algebra, Model Theory	Algebraic proof of the completeness theorem for first-order logic. Rasiowa-Sikorski lemma. Semantic for intuitionistic and modal predicate calculi	A Proof of the Completeness Theorem of Gödel (1950) The Mathematics of Metamathematics (1963)
Helena Rasiowa	Mathematical Logic, Algebra	Development of an algebraic theory of models for many-valued predicate logics. Systematic algebraic approach to the semantics of non-classical logics.	An Algebraic Approach to Non-Classical Logics (1974);
William Lawvere Myles Tierney	Category Theory, Algebraic Logic, Geometry	Topos theory as a tool for algebraization of logic.	Quantifiers and Sheaves (1971) Sheaf Theory and the Continuum Hypothesis (1972)

A Simplified Evolution of Representing and Reasoning about the Semantic Aspects of Language through Algebraization and Geometrization of Language Structures

From Statistical Beginnings to Contextual Embeddings: The Algebraization and Geometrization of Semantic Representation and Reasoning in Natural Language Processing

The development of methods for representing and reasoning about the semantic aspects of text in natural language has evolved, starting from the statistical analysis of word occurrence in text, to advanced models that learn context and relationships between text fragments. The main stages of this evolution are (approximately):

1960s:	Vector space model (Salton: SMART, TF-IDF)
2010s:	Simple word embeddings (e.g., word2vec, GloVe)
Late 2010s:	Contextual embeddings (BERT and related models, e.g., ELMo, RoBERTa)
From the 2020s to the present:	Large language models (LLMs), such as GPT-4, Gemini, Claude, and Deep-Seek v3, use advanced architectures based on LSTM/RNN, attention, and transformers. These models integrate various machine learning techniques, including reinforcement learning (RL), to improve their language abilities

Syntactic Properties as the Basis for Constructing Semantic Distance for Reasoning (Examples from AI/LLM and Algebraic Models)

CONCLUSION:

AI vector spaces and canonical Boolean models are constructed by analyzing their respective languages' syntactic properties.

AI/LLM Perspective:

1. Large Language Models (LLMs) like GPT or BERT analyse massive corpora to derive **vector spaces** based on **syntactic co-occurrences** of words.
2. Meaning is **not directly programmed** or extracted from the real world but emerges from the structure of language itself.

Algebraic Logic Perspective:

1. **Canonical Boolean models** are constructed using algebraic methods that rely entirely on **syntactic properties of logical formulas**.
2. Meaning is assigned **via algebraic operations on syntactic structures**, not by external interpretation.

Key insight: Both AI and algebraic logic derive **semantic structure** purely from **syntactic rules**, demonstrating a deep methodological similarity.

Syntactic Properties as the Basis for Constructing Semantic Distance for Reasoning (Examples from AI/LLM and Algebraic Models)

Closer points ⇒ greater logical/semantic similarity

AI/LLM Perspective:

1. A **“ball” of radius R** around a vector P in an embedding space identifies all items that are semantically close to P within distance R.
2. **Example**: Words near *“king”* might be: queen, prince, royal, while *“cat”* might be far away.

Algebraic Logic Perspective:

1. In a **metrizable Stone space**, small “balls” (or **clopen neighborhoods**) around a point P (i.e., an **ultrafilter/model**) capture other models that **agree on many formulas**.
2. **Closer models ⇒ share more semantic properties (the truth of sentences).**

1. **Key insight:** This parallel underscores how semantic distance applies in both logic-based model spaces and AI embedding spaces.

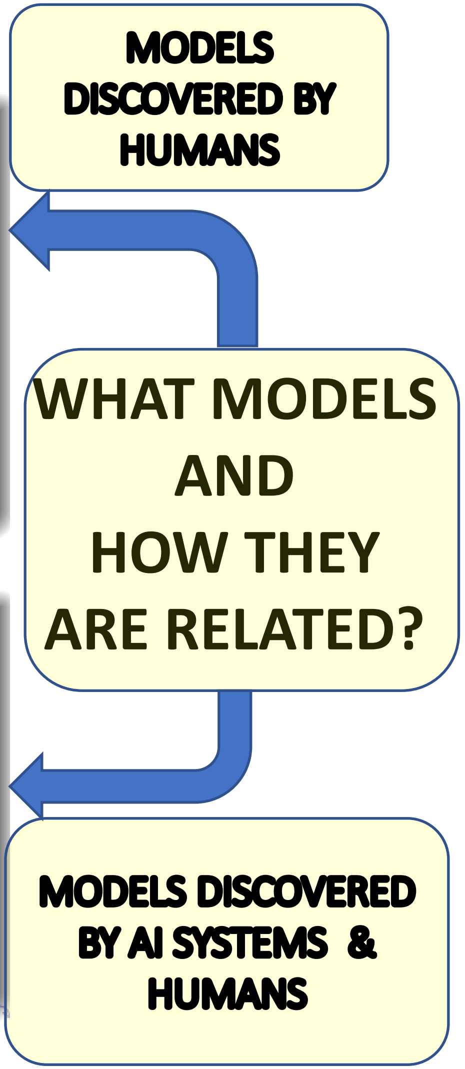
Syntactic Properties as the Basis for Constructing Semantic Distance for Reasoning (Examples from AI/LLM and Algebraic Models)

Algebraic Logic (Countable First-Order Setting)

- For a **countable** first-order language, the *Lindenbaum Boolean algebra* is countable.
- Its **Stone space** thus has a countable base of clopen sets, making it: **second-countable**,**compact**, and **Hausdorff**
- These properties ensure it is **metrizable**.
- Consequently, each model (or ultrafilter) can be considered a **point in a metric space**.

AI (e.g., NLP)

- In many modern AI systems (especially **NLP**), words, phrases, or entire documents are mapped to vectors in high-dimensional metric spaces.
- **Distances** (e.g., Euclidean or cosine) measure semantic similarity or dissimilarity.
- Like logical models in a Stone space, these embeddings capture how "close" or "far apart" concepts are, reflecting meaningful semantic relationships.



Conclusion: A Fundamental Conceptual Connection Between AI/LLM and Algebraic Models

- **AI and algebraic logic share a core principle**: syntactic analysis constructs structured semantic representations and forms the foundation for knowledge manipulation.
- **Canonical Boolean models** in logic serve as a mathematical counterpart to **vector spaces** in AI.
- **This perspective bridges logic, mathematics, and AI**, opening pathways for deeper interdisciplinary research.

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

Personal conclusions of AS & AJ: Two crucial conclusions from the perspective of Helena Rasiowa's research and those associated with it (collaborators and students).

Looking Back:

1. The scientific achievements of H. Rasiowa and R. Sikorski demonstrated the fundamental importance of methods for representing and reasoning about the semantic aspects of language within mathematical logic. They did this through the algebraization and geometrization of language structures.

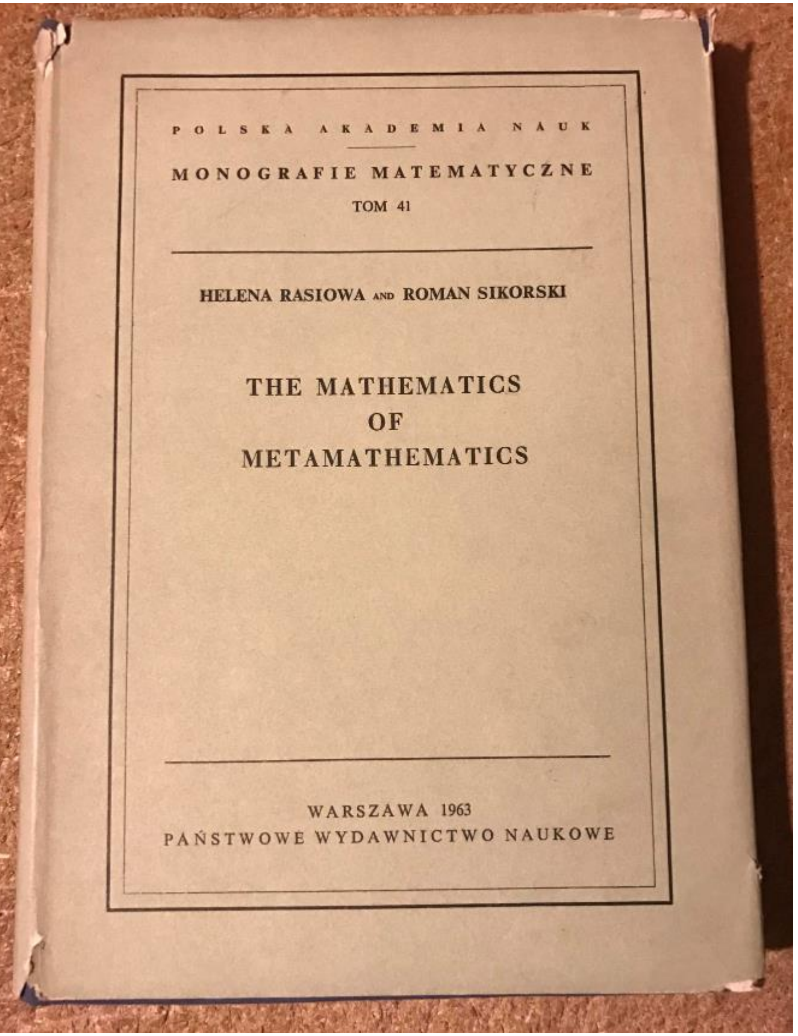
2. AI/LLMs have shown the crucial role of methods for representing and reasoning about the semantic aspects of natural language by algebraizing and geometrizing language structures *within natural language*.

3. Both approaches to logic (formalized and natural language) currently do not incorporate processes of interaction with the physical world aimed at understanding real-world phenomena.

Looking Ahead:

Based on our experience (AS & AJ), we believe that next-generation computational AI models will require the inclusion of perception. We define perception as a process of interaction with the physical world aimed at understanding situations within it (Wisdom Technology / IGrC).

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?



•A Tribute to Professor Helena Rasiowa

Melvin Fitting

•It is not surprising that, among logicians of my generation, Poland and algebra were thought of as synonymous.

….

•It was a strange book. On the one hand, it was painfully exhaustive and thorough in the presentation of material. Page after page enumerated detailed results almost in the form of a catalog, dry and utilitarian. On the other hand, the ideas were of a sort I had never come across before, and I was enchanted. This way of using algebra, producing known results in classical logic, then applying similar techniques to non-classical logics to get new results - it all seemed like magic. Profound results fell out so effortlessly, it seemed. How could one read this book and remain unaffected?

•….

•I once heard Dana Scott criticize „The Mathematics of Metamathematics" because, while it took an algebraic approach to logic, it did not carry the work further and consider set theory. If it had, forcing would have been discovered years earlier than it was. This is not, at heart, a criticism, but a tribute. The building of mathematics always goes on. Foundations, Firmly laid, enable later construction, and the foundations laid by that book were powerfully firm.

•….

•http://melvinfitting.org/bookspapers/pdf/papers/Rasiowa.pdf

How can we develop AI technologies and their applications
using algebraic logic, new computational models and reasoning about their computations?

Rasiowa - Pawlak School
of
Artificial Intelligence

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

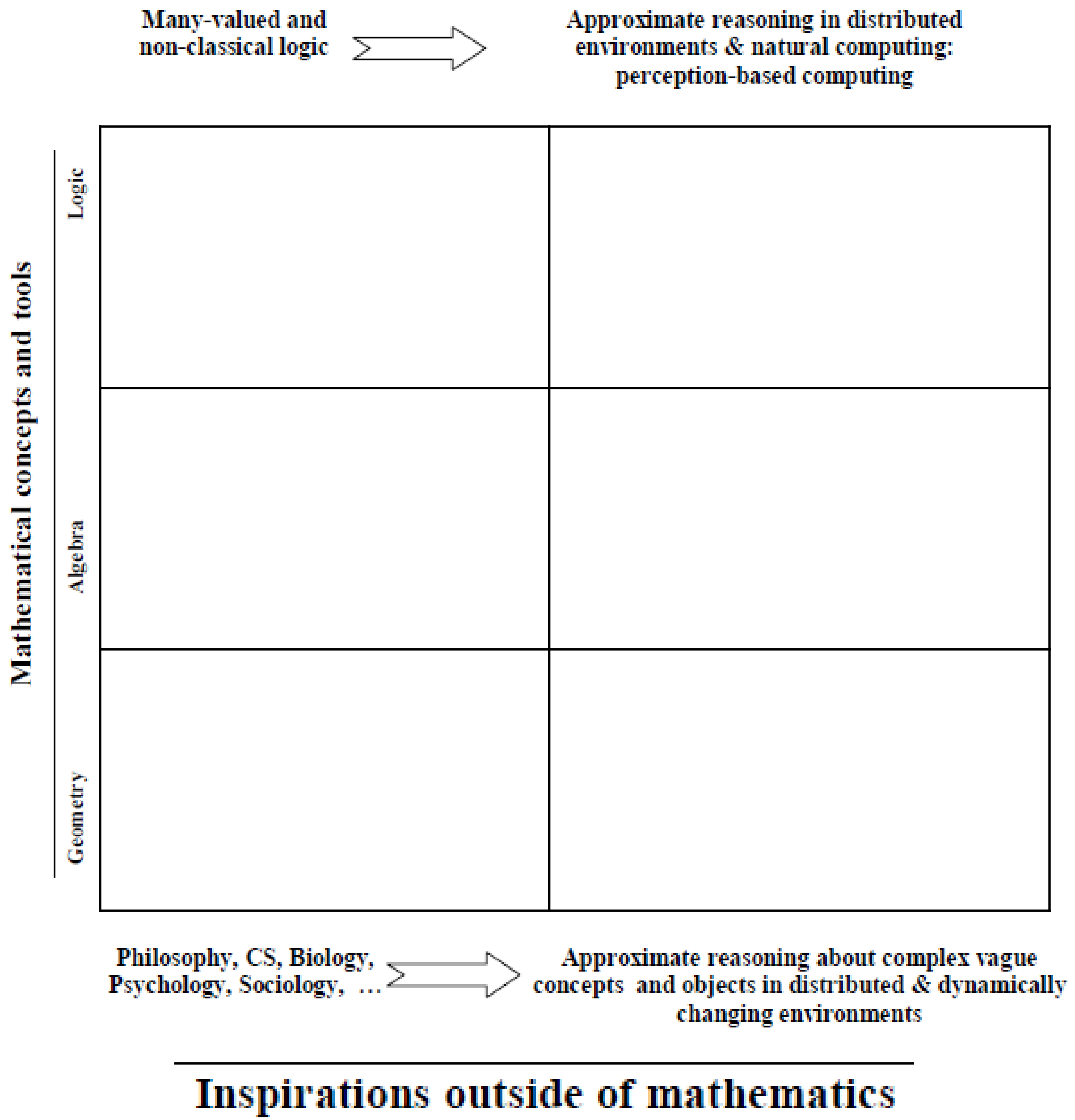
It is impossible to briefly present all the important achievements of the Rasiowa - Pawlak School of AI. Unfortunately, we do not yet have a complete documented history of the school. In order to change this, we decided to start working in this direction and prepared our very personal point of view in the paper:

A. Jankowski, A. Skowron: Logic for Artificial Intelligence: Rasiowa Pawlak School Perspective, w: Andrzej Mostowski and Foundational Studies, IOS Press, Amsterdam, 2008, 106-143.

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

Rasiowa - Pawlak School of AI

Framework presentation of the evolution of computation models at the Rasiowa - Pawlak School of AI to build new AI technologies (and their applications)



Rasiowa - Pawlak School of AI

Framework presentation of the evolution of computation models at the Rasiowa - Pawlak School of AI to build new AI technologies (and their applications)

	Many-valued and non-classical logic ⇒	**Approximate reasoning in distributed environments & natural computing: perception-based computing**
Logic	➢ Computability, uncertainty, natural deduction, algebraic semantics and language algebraic properties of different types of logic, especially in: • intuitionistic, • modal, • Post, intermediate, • with strong negation • implicative • algorithmic • program • non-Fregean • with infinite logical operators ➢ Abstract logics, relationship between them and characterization of classical and other logics ➢ Hierarchy of metalogics ➢ Logical aspects of programming paradigms ➢ Interpretation of logical operators in models of computation (generalized quantifiers, model operators, Post operators)	➢ Evolution of concepts: hierarchy of metalogics created by interactions with environment. ➢ Society of agents, represented by a set of modal operators. ➢ Consensus and emotional states as modal operators over non-classical truth values. ➢ Logic for distributed systems ➢ Reasoning under uncertainty in distributed systems. ➢ Vague concept approximation ➢ Boolean approximate reasoning: RSES ➢ Conflicts, negotiations, cooperation ➢ RS, FS, combination with nonmonotonic reasoning ➢ Approximate reasoning about knowledge ➢ Common sense reasoning ➢ Perception logic: evolving system of interacting local logics ➢ Computational models based on perception ➢ Computational models of behavior ➢ Learning and adaptation ➢ Autonomous computing
Algebra	➢ Algebraic models for non-classical and abstract predicate calculus (Q-algebras), generalization of Rasiowa – Sikorski Lemma ➢ Lattice theory, Boolean, Heyting, Brouwer, Post and other algebras ➢ Syntax and semantics as adjoint: concepts (Galois connections) ➢ Topos theory approach ➢ Internal representation of deduction by sheaves over closure spaces	➢ Algebraic structures for reasoning under uncertainty ➢ RS algebras, FS algebras ➢ Relational calculi ➢ Partial algebras ➢ Calculi of approximation spaces ➢ Mereological calculi of information granules
Geometry	➢ Topological properties of spaces of models and concepts ➢ “Distance” between theories which represent knowledge of agents ➢ Geometry of computations ➢ Cantor Space, as a geometric space of models for classical propositioinal calculus ➢ Topological interpretation of modal operators ➢ Closure spaces as generalized geometric spaces ➢ Heuristics based on geometry of computation space	➢ Measures of proximity (similarity): states and set of states of computations and concepts ➢ Similarity of cases and case-based reasoning ➢ Geometry of concepts ➢ Similarity of theories ➢ Granular space, information granulation and granular computing ➢ Discovery of granularity levels from data, e.g., relevant multi-valued logics.
	Philosophy, CS, Biology, Psychology, Sociology, … ⇒	**Approximate reasoning about complex vague concepts and objects in distributed & dynamically changing environments**

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

•Reference to the role of Fundamenta Mathematicae in the development of the Polish School of Mathematics through the initiative to establish Fundamenta Informaticae

- Long-term vision of the growing role of IT foundations research and applications
- Referencing the legacy of the Polish School of Logic
- Support and direct involvement in research on computational models with particular emphasis on their use in artificial intelligence logic applications

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

It is difficult to present a complete list of researchers whose investigations were substantially influenced in different periods of their scientific activity by Helena Rasiowa or Zdzisław Pawlak. Among them are

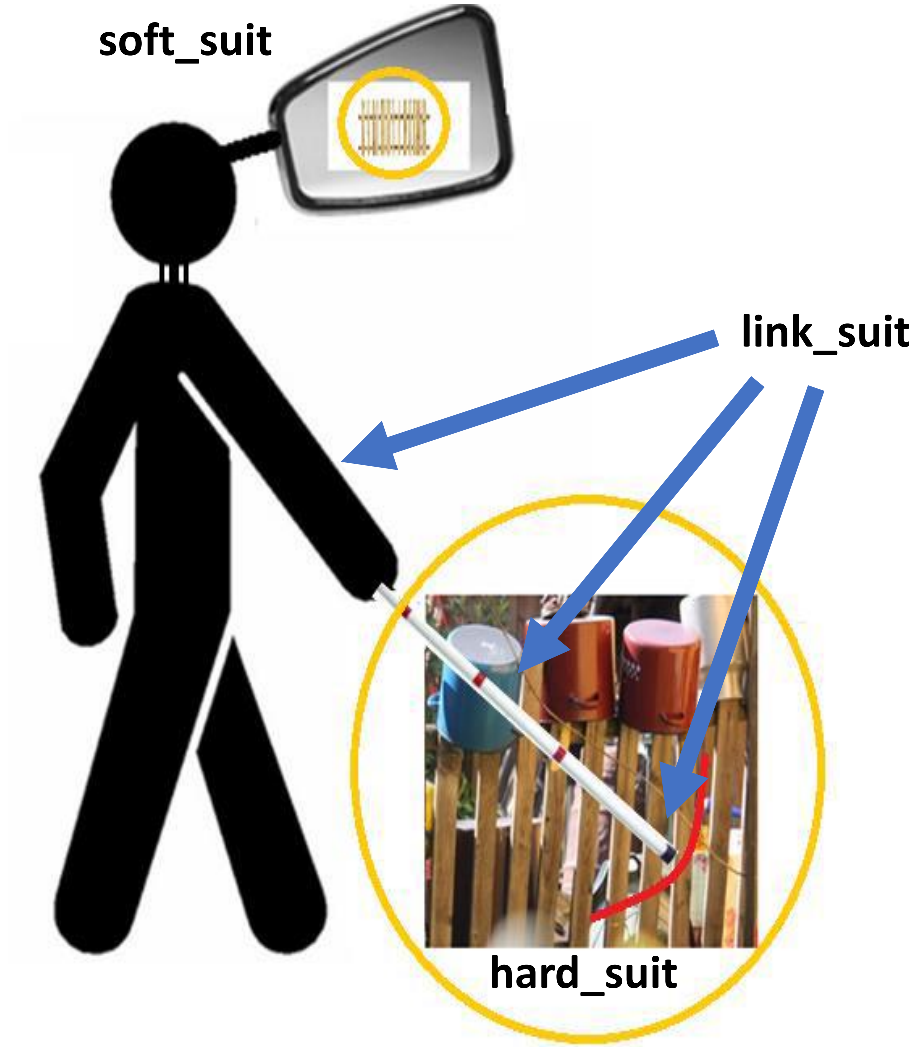
Lech Banachowski, Mohua Banerjee, Wiktor Bartol, Jan Bazan, Malcolm Beynon, Leonard Bolc, Gianpiero Cattaneo, Mihir Kumar Chakraborty, Newton da Costa, Andrzej Czyżewski, Wiktor Dańko, Piotr Dembiński, Patrick Doherty, Jan Doroszewski, Albert Dragalin, Didier Dubois, Ivo Duntsch, George Epstein, Anna Gomolińska, Jerzy Grzymała-Busse, Petr Hajek, Tsutomu Hosoi, Masahiro Inuiguchi, Andrzej Jankowski, Jouni Jarvinen, Jan Komorowski, Beata Konikowska, Bożena Kostek, Antoni Kreczmar, Churn J. Liau, Tsau Young Lin, Witold Lipski, Wing Liu, Witold Łukaszewicz, Larisa Maksimowa, Victor Marek, Antoni Mazurkiewicz, Ernestina Menasalvas, Grażyna Mirkowska-Salwicka, Mikhail Moshkov, Adam Mrózek, Maciej Mączyński, Daniele Mundici, Hung Son Nguyen, Cat Ho Nguyen, Sinh Hoa Nguyen, Tuan Trung Nguyen, Damian Niwiński, Hiroakira Ono, Ewa Orłowska, Sankar K. Pal, Eleonora Perkowska, James F. Peters, Jan Plaza, Lech Polkowski, Henri Prade, Andrzej Proskurowski, Halina Przymusińska, Slavian Radev, Sheela Ramanna, Zbigniew Raś, Cecylia Rauszer, Grzegorz Rozenberg, Leszek Rudak, Andrzej Salwicki, Giovanni Sambin, Dana Scott, Maria Semeniuk-Polkowska, Roman Sikorski, Dimiter Skordev, Andrzej Skowron, Roman Słowiński, Jerzy Stefanowski, Jaros law Stepaniuk, Zbigniew Suraj, Roman Suszko, Piotr Synak, Roman Swiniarski, Andrzej Szałas, Marcin Szczuka, Dominik Ślęzak, Helmut Thiele, Jerzy Tiuryn, Tadeusz Traczyk, Boris Trahtenbrot, Shusaku Tsumoto, Paweł Urzyczyn, Dimiter Vakarelov, Alicja Wakulicz-Deja, Stanisław Waligórski, Quoyin Wang, Anita Wasilewska, Jakub Wróblewski, Wei-Zhi Wu, Urszula Wybraniec-Skardowska, JingTao Yao, YiYu Yao, Marek Zawadowski, Ning Zhong, Wojciech Ziarko, Tomasz Zieliński.

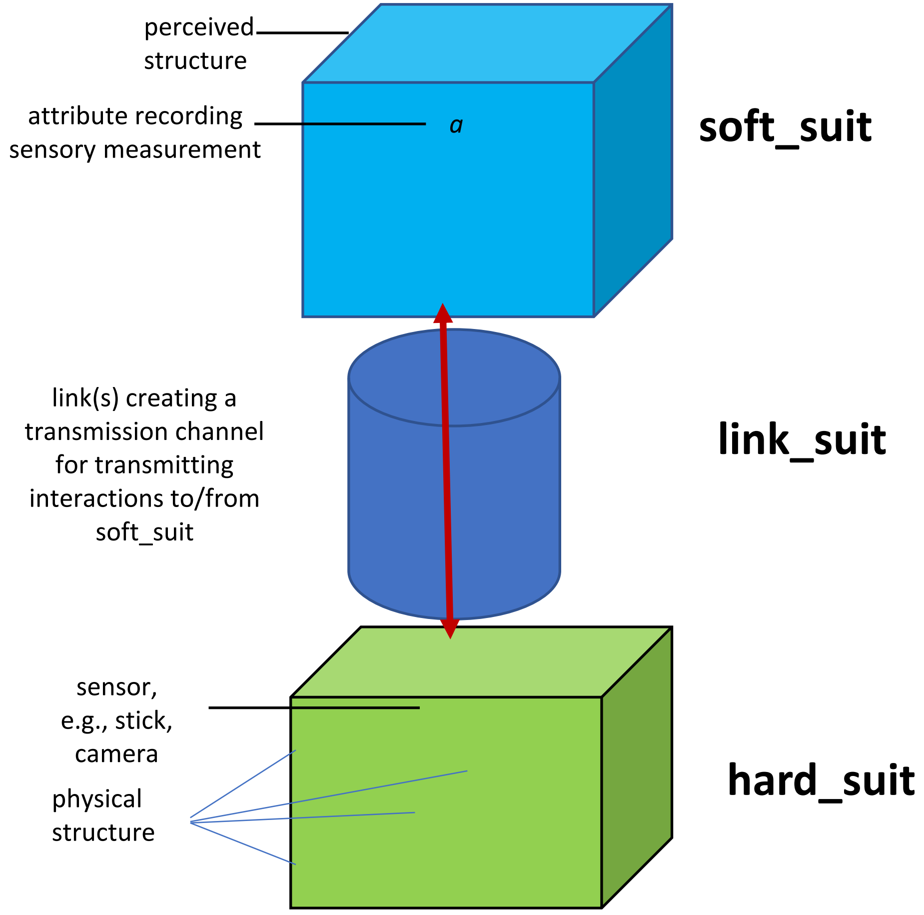
Main idea od the WisTech/IGrC approach

Wisdom = Interactions + Adaptive Judgement + Knowledge

Wisdom Technology/IGrC (WisTech/IGrC)= Computational models of complex granules (c-granules) enabling agents to implement **intelligent interactions** with the environment **based on the adaptive judgment** of interaction outcomes (relative to the changing hierarchy of needs) **using acquired knowledge**.

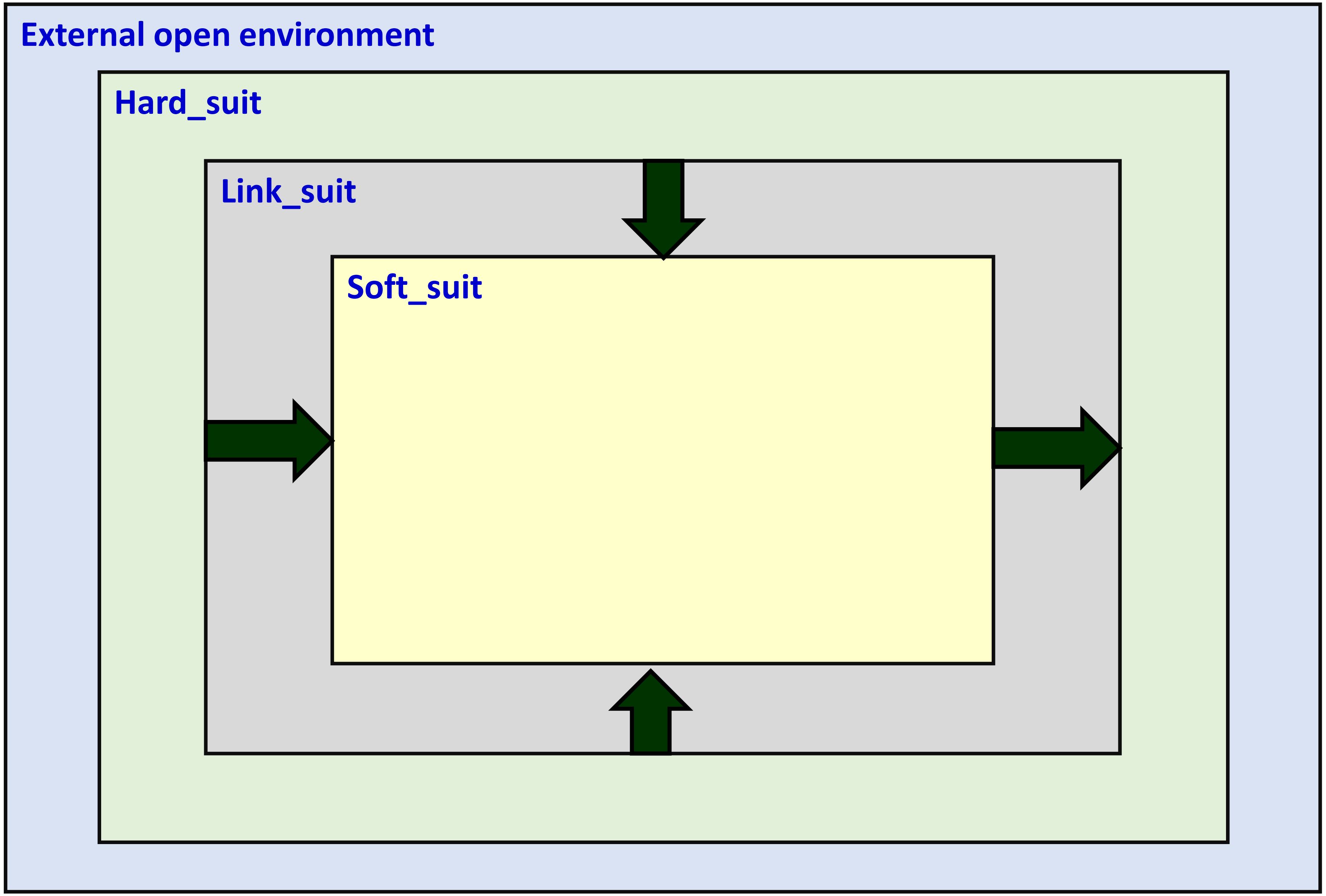
How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?





How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

General scheme to computational models of c-granules

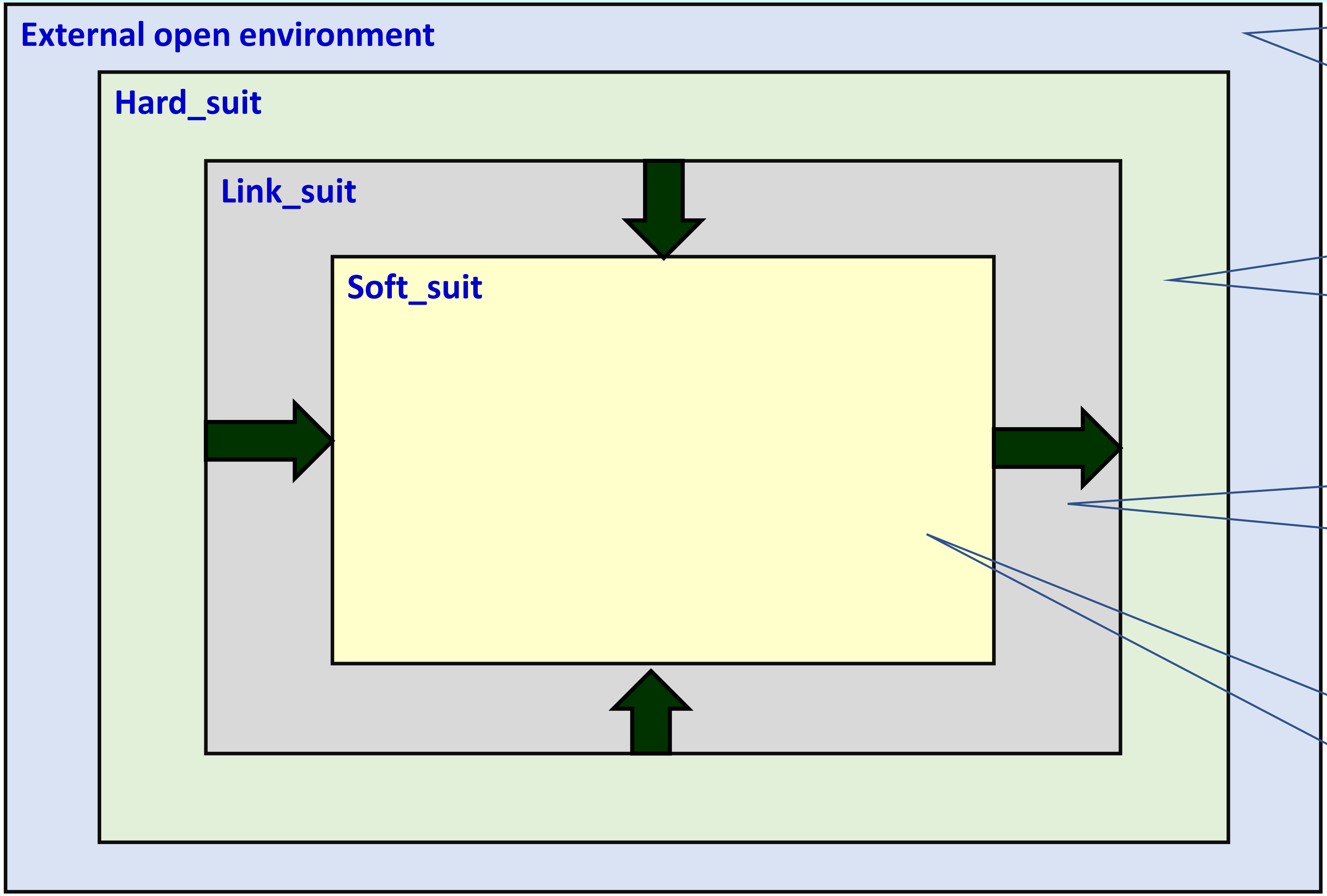


Some examples of c-granules

- ✓ Logical gate
- ✓ Computer processor
- ✓ Decision table
- ✓ Decision tree
- ✓ Rule-based classifier
- ✓ Fuzzy controller
- ✓ Neuron
- ✓ Neural network
- ✓ Brain
- ✓ …

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

A proposal of computational model of a generic c-granule for AI. applications



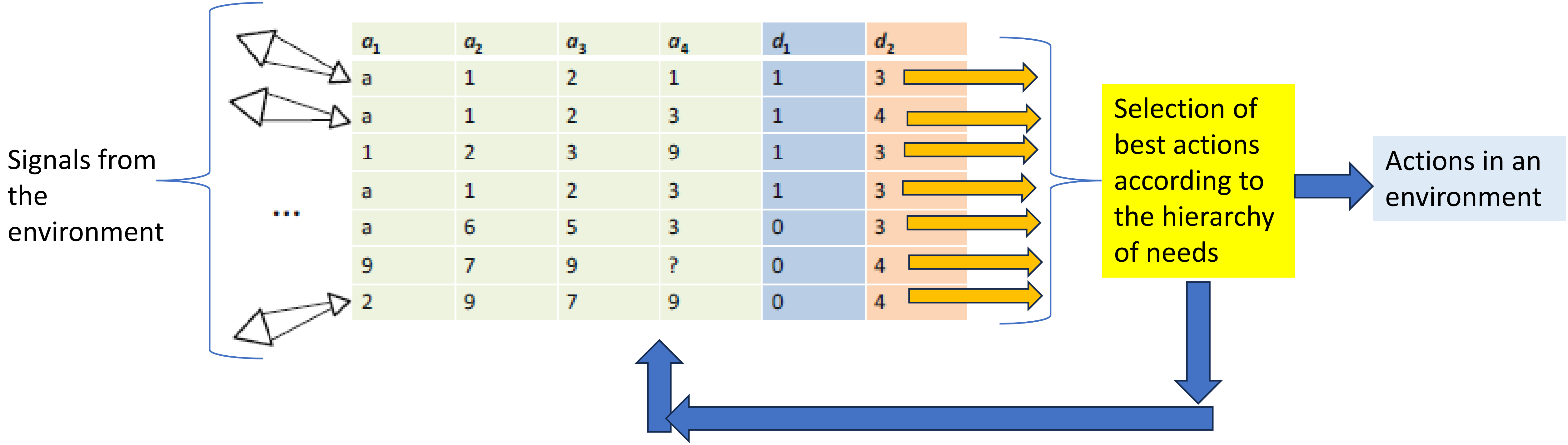
Open and unpredictable external world. The world is in many interactions with hard_suit. However, for simplicity, we assume the external world is available for simplified c-granules only through hard_suit (via links).

Family of Relational Scenes. In general, we assume that objects, relations, and functions of the hard_suit are c-granules equipment with unique addresses and their sorts/types

Specialized types of links for transferring signals. Components of links and their functionalities are expressed by a multi-sorted many-valued predicate logic and addresses of components of scenes

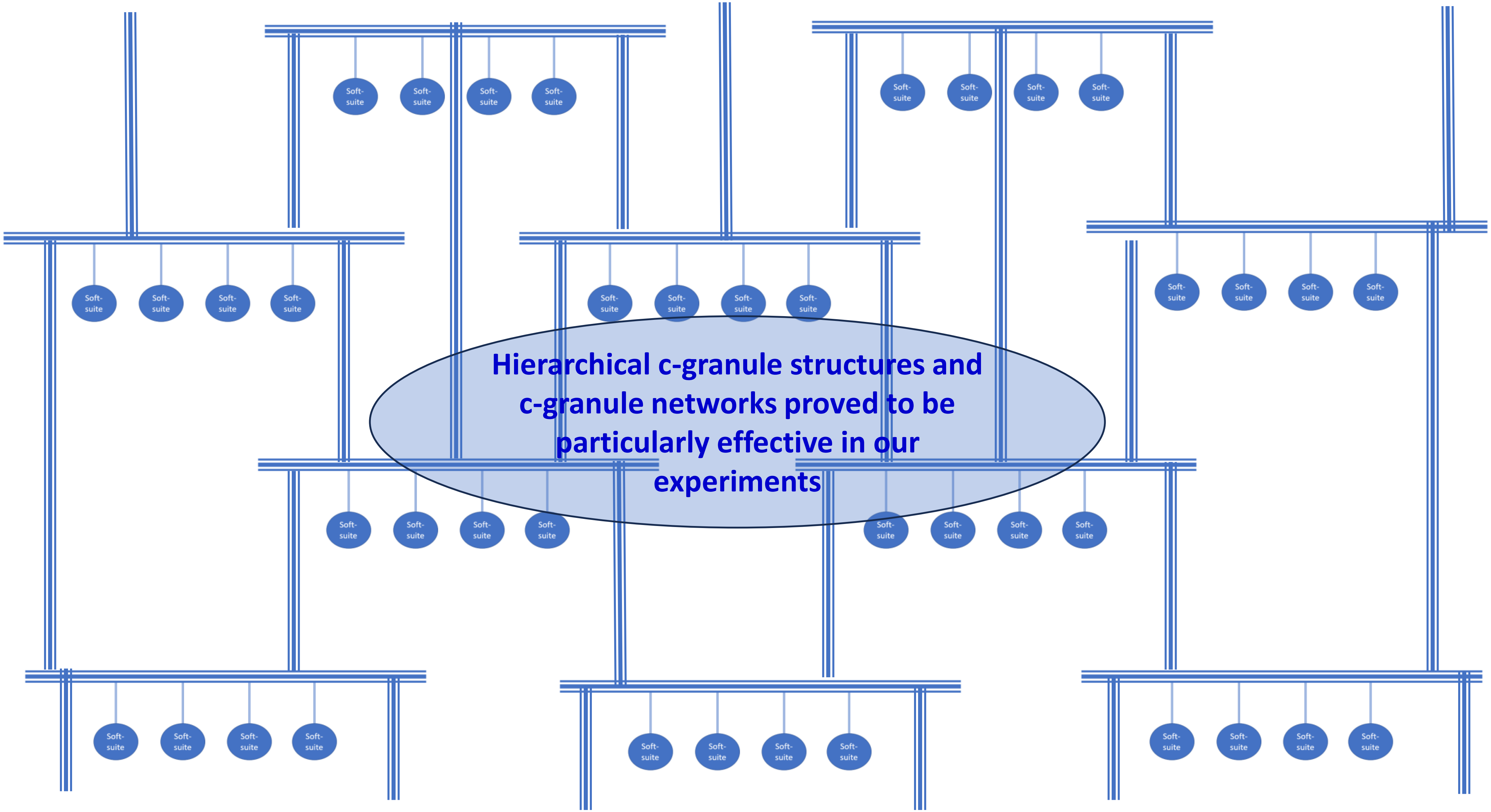
Algorithms (in general concurrent, i.e., swimlanes) process received signals (via links) and sending signals (via links)

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?



a_1	a_2	a_3	a_4	d_1	d_2
a	1	2	1	1	3
a	1	2	3	1	4
1	2	3	9	1	3
a	1	2	3	1	3
a	6	5	3	0	3
9	7	9	?	0	4
2	9	7	9	0	4

How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?



How can we apply in practice
(especially in complex IT projects)
achievements in mathematical logic
(especially algebraic methods in logic)
and achievements in IT foundations?

How can we apply in practice (especially in complex IT projects) achievements in mathematical logic (especially algebraic methods in logic) and achievements in IT foundations?

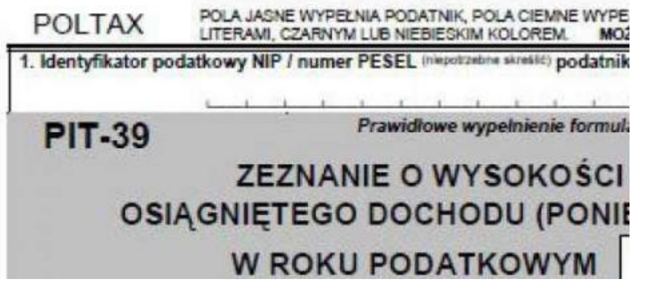
- Professor Rasiowa taught us, among other things, methods for decomposing complex problems into simpler ones. These methods included **developing a flexible ontology for the problem and its application domain**. This, in turn, allowed us to use algebraic methods for solutions and, **when possible, to leverage known analogous problems in mathematics.**

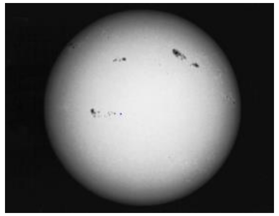
- Thanks to this, it was easier for us to apply in practice the achievements of logic and achievements in the foundations of IT.

- **Approximation of complex vague concepts - connections with rough sets and reasoning on them.**

How can we apply in practice (especially in complex IT projects) achievements in mathematical logic (especially algebraic methods in logic) and achievements in IT foundations?

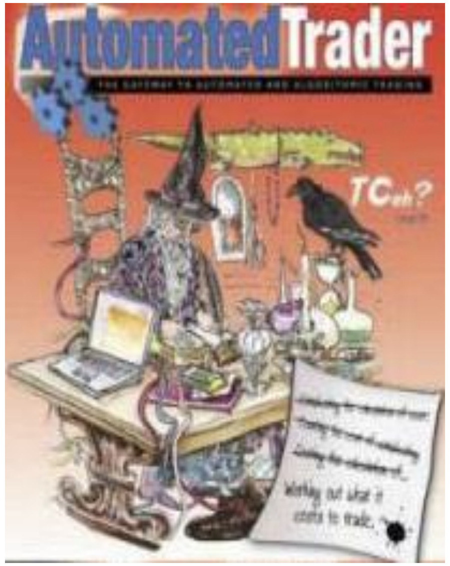
- As a result, we were able to initiate, implement and monitor hundreds of complex projects. Particularly interesting among them are the projects:
- UAV
- POLTAX
- Excavio
- MerixAlgo
- Tradix
- Rough ICE
- Sunspot Classification….





These projects are described in books such as:

- Doherty P, Łukaszewicz W, Skowron A, Szałas A. (2006) Knowledge engineering: a rough set approach. Studies in fuzziness and soft computing, vol 202. Springer, Heidelberg
- A. Jankowski A. Interactive Granular Computations in Networks and Systems Engineering: A Practical Perspective. Lecture Notes in Networks and Systems, Springer, Heidelberg,
- (2017).Bazan,. J.: Hierarchical Classifiers for Complex Spatio-temporal Concepts, Lecture Notes in Computer Science 5390, Transactions on Rough Sets IX, 474--750, (2008).T
- Trung Thanh Nguyen, Claire P. Willis, Derek J. Paddon, Hung Son Nguyen, On Learning of Sunspot Classification, Intelligent Information Processing and Web Mining pp 59-68, (2004)



SUMMARY

- It should be emphasized that our presentation is personal and very condensed and certainly does not exhaust all important elements of the profile and scientific achievements of Professor Helena Rasiowa
- We allow ourselves to present slightly more aspects in our publications:
- A. Jankowski, A. Skowron. Logic for artificial intelligence: The Rasiowa - Pawlak School perspective. In: A. Ehrenfeucht, V. Marek, M. Srebrny (Eds.), Andrzej Mostowski and Foundational Studies, IOS Press, Amsterdam. 2008, pp. 106–143.
- A. Jankowski, A. Skowron. Helena Rasiowa (1917-1994). In: A. Garrido, U. Wybraniec-Skardowska (Eds.), Lvov-Warsaw School. Past and Present. Studies in Universal Logic series (ed. Beziau, J.-Y.) Birkhaüser Publishing Ltd. (Springer), Basel (2017), World Scientific, Basel. 2017, pp. 1–9.
- A. Skowron, E. Orlowska, A. Jankowski. Helena Rasiowa (1917-1994). In: A. K. Wróblewski (Ed.), Portraits of Scholars. Professors of the University of Warsaw after 1945, University of Warsaw, Warsaw. 2017, pp. 564–571.
- Helena Rasiowa (1917-1994). Life and Personality. Antiquitates Mathematicae XIV(1) (2020) 161–185.

SUMMARY

Professor Rasiowa Taught Us methods of finding answers to the following three questions of great importance in the life of each of us

1. How can we support personal, scientific, and professional development and good cooperation among people (especially in extremely difficult situations)?

2. How can we develop AI technologies and their applications using algebraic logic, new computational models and reasoning about their computations?

3. How can we apply in practice (especially in complex IT projects) achievements in mathematical logic (especially algebraic methods in logic) and achievements in IT foundations?

SUMMARY

THE AUTHORS ARE VERY GRATEFUL TO PROFESSOR HELEN RASIOWA FOR TEACHINGS THAT WERE AND ARE STILL VERY HELPFUL IN THEIR PERSONAL, SCIENTIFIC AND PROFESSIONAL LIVES